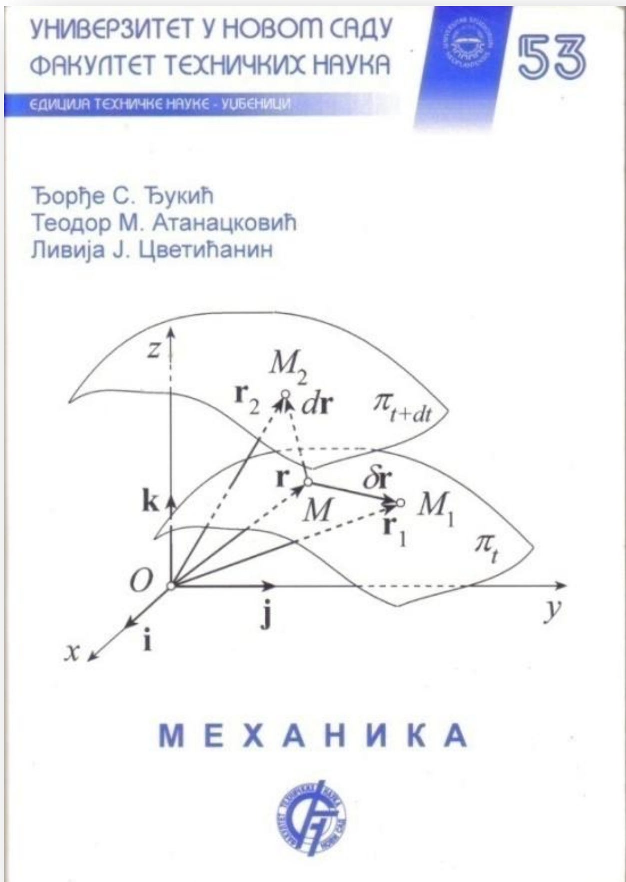
# Mehanika 2 (Kinematika)
# Vežbe 8

Miodrag Zuković

Novi Sad, 2023.

# Literatura



УНИВЕРЗИТЕТ У НОВОМ САДУ
ФАКУЛТЕТ ТЕХНИЧКИХ НАУКА
ЕДИЦИЈА ТЕХНИЧКЕ НАУКЕ - УЏБЕНИЦИ
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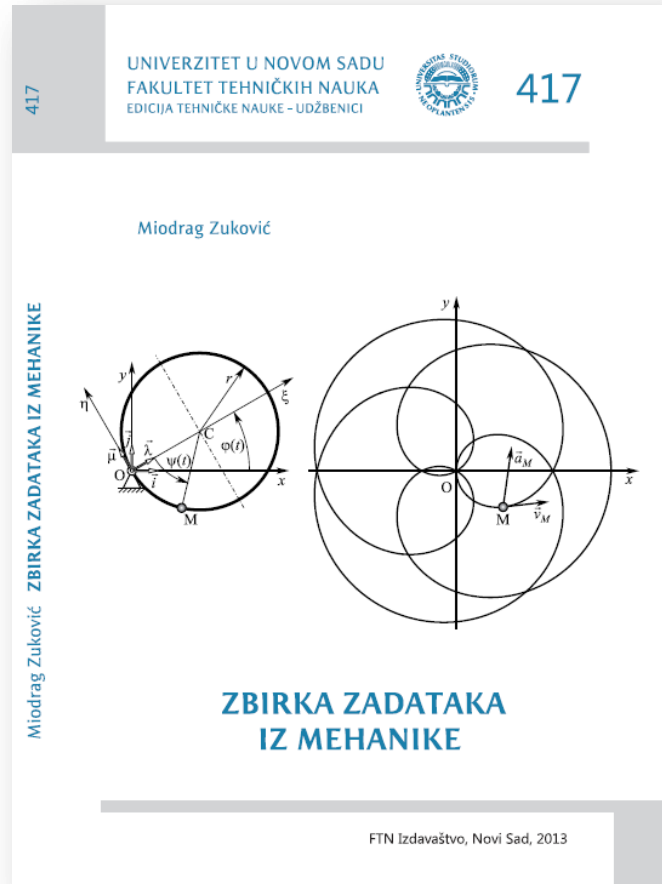
Теодор М. Атанацковић
Ливија Ј. Цветићанин
МЕХАНИКА

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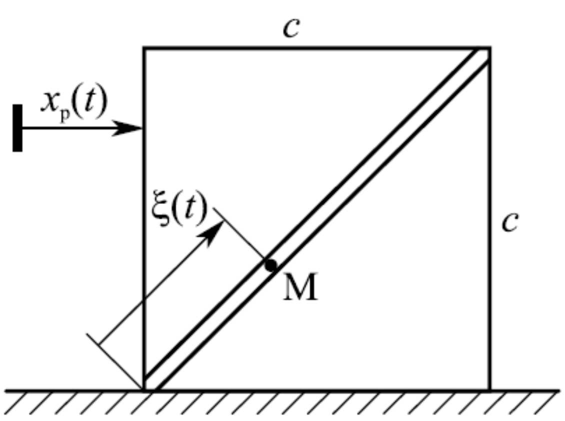
IZ MEHANIKE
FTN Izdavaštvo, Novi Sad, 2013

# Šta ćemo naučiti?

25. **Složeno kretanje tačke (osnovni pojmovi)**
26. **Brzine i ubrzanja tačaka tela pri složenom kretanju tačke**

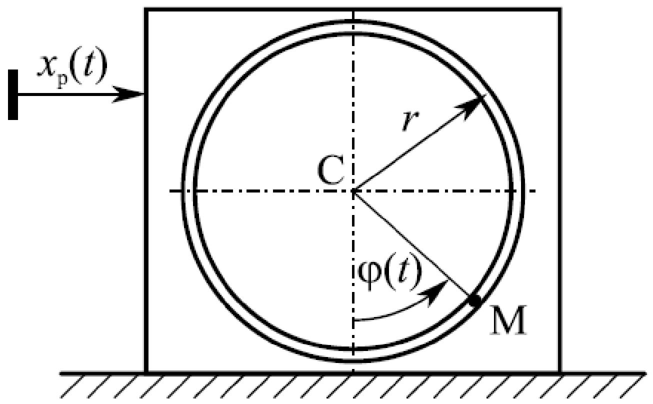
# Zadatak 1

**Zadatak 2.26** Kvadratna ploča, stranica dužine $c$, kreće se translatorno pravolinijski po zakonu $x_p(t) = a_p \frac{t^2}{2}[\mathrm{m}]$, gde je $a_p = const > 0$, u prikazanom smeru. Duž žleba u ploči, urezanog u pravcu dijagonale, kreće se tačka M po zakonu $\xi(t) = \sqrt{2}\frac{c}{T}t[\mathrm{m}]$, gde su $c$ i $T$ pozitivne realne konstante. Odrediti apsolutnu brzinu i apsolutno ubrzanje tačke M u trenutku $t_1 = T[\mathrm{s}]$.



# Zadatak 2

**Zadatak 2.27** Pravougaona ploča kreće se translatorno pravolinijski po zakonu $x_p(t) = v_p t$, gde je $v_p = const > 0$, u prikazanom smeru. Po kružnom žlebu u ploči, radijusa $r$, kreće se tačka M po zakonu $\varphi(t) = \pi \frac{t^2}{2}$. Odrediti apsolutnu brzinu i apsolutno ubrzanje tačke M u trenutku $t_1 = 1\text{s}$.



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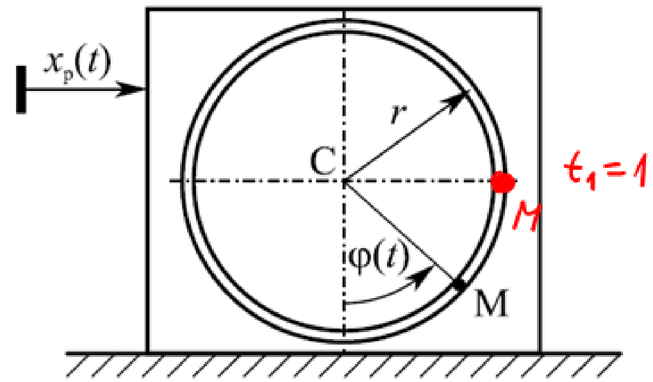



ПР. КР. → ПЛОЧА → ТРАНСЛ. → $\omega_p = 0$, $\varepsilon_p = 0$ , $\vec{a}_{cor} = 2\vec{\omega}_p \times \vec{v}_r = 0$

РЕЛ. КР. → $\mathcal{K}\{C, r\}_\varphi$

$x_p(t) = v_p t$

$\dot{x}_p(t) = v_p = const$

$\ddot{x}_p(t) = 0$

$t_1 = 1$ $\quad \dot{x}_p(1) = v_p$

$\ddot{x}_p(1) = 0$

$\varphi(t) = \pi \frac{t^2}{2}$

$\dot{\varphi}(t) = \pi \cdot t$

$\ddot{\varphi}(t) = \pi = const$

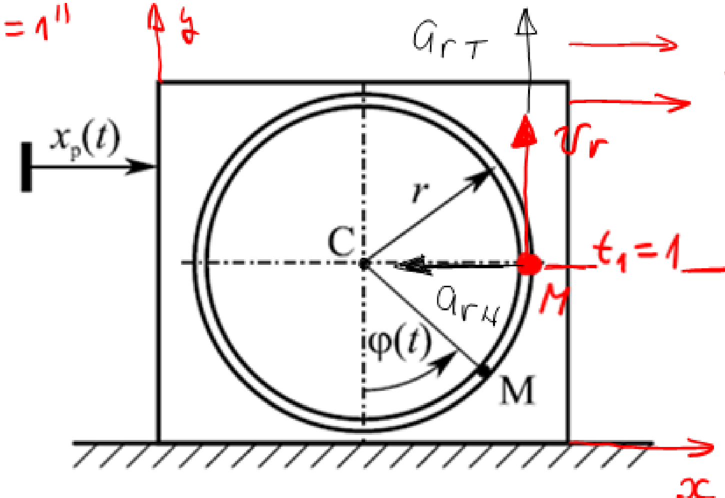
$t_1 = 1$

$\varphi(1) = \pi \cdot \frac{1^2}{2} = \frac{\pi}{2}$

$\dot{\varphi}(1) = \pi$

$\ddot{\varphi}(1) = \pi$

$„t_1 = 1"$




$$a_p = 0, \quad v_p = v_p \quad \Bigg\} \quad \varphi(1) = \frac{\pi}{2}, \quad \dot\varphi(1) = \ddot\varphi(1) = \pi$$

„M"

$$\vec v_M = \vec v_p + \vec v_r \; ; \quad v_r = r\,\dot\varphi(1) = r\pi$$

$$v_{Mx} = v_p = v_p$$

$$v_{My} = v_r = r\pi$$

$$v = \sqrt{v_{Mx}^2 + v_{My}^2} = \sqrt{v_p^2 + (r\pi)^2}$$

$$\vec a_M = \vec a_p + \vec a_r + \cancel{\vec a_{cor}}^{\,0}$$

$$\vec a_M = \cancel{\vec a_p}_{\,0} + \vec a_{rT} + \vec a_{rN}$$

$$a_{rT} = r\,\ddot\varphi_1 = r\pi$$

$$a_{rN} = r\,\dot\varphi^2(1) = r\pi^2 = \frac{v_r^2}{r} = \frac{r^2\pi^2}{\cancel{r}}$$

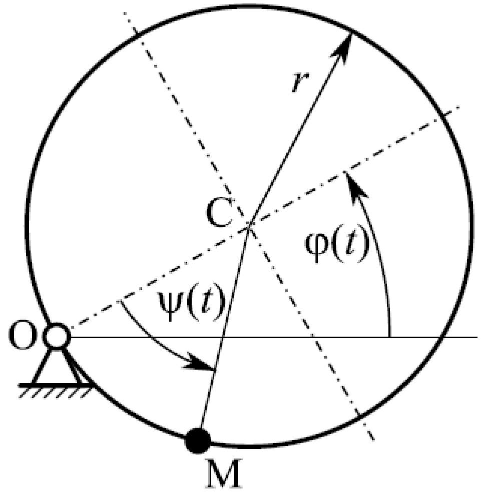
$$a_{Mx} = -a_{rN} = -r\pi^2$$

$$a_{My} = a_{rT} = r\pi$$

$$a_M = \sqrt{a_{Mx}^2 + a_{My}^2} = \sqrt{r^2\pi^4 + r^2\pi^2} = r\pi\sqrt{\pi^2 + 1}$$

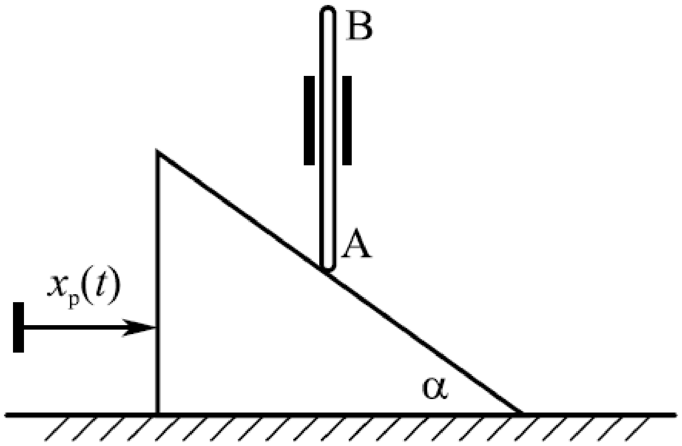
# Zadatak 3

**Zadatak 2.29** Kružna žica, radijusa $r$, obrće se oko nepokretne ose konstantnom po zakonu $\varphi(t) = \alpha t$. Po žici se kreće tačka M po zakonu $\psi(t) = \beta t$. Odrediti trajektoriju kretanja tačke M, a zatim i apsolutnu brzinu i ubrzanje.



# Zadatak 4

**Zadatak 2.30** Klin, nagibnog ugla $\alpha$, kreće se translatorno pravolinijski po poznatom zakonu $x_p(t)$. Na njegovu strmu stranu oslanja se štap $AB$, koji može da se kreće duž vertikalne vođice. Odrediti apsolutnu brzinu i ubrzanje štapa.



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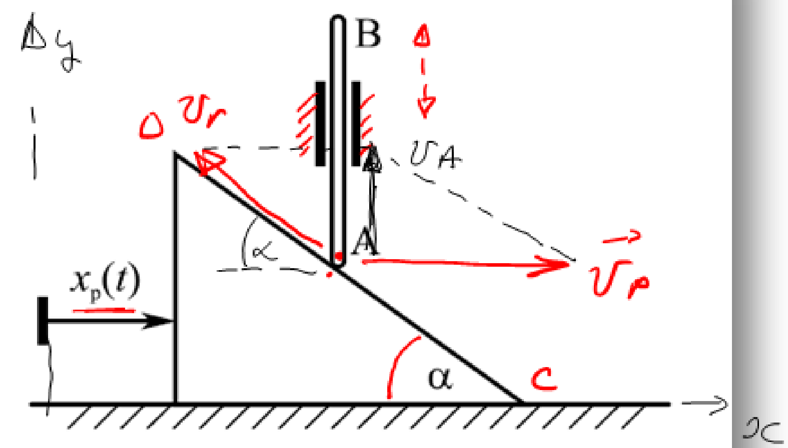



$\overline{AB}$ – ТРАНСЛ. $\longrightarrow$ $\vec{v}_A$, $\vec{a}_A$ „A"

ПР. КР. – ПЛОЧА – ТРАНСЛ.

РЕЛ. КР. – ПРАВОЛ. – $\overline{CO}$

АПС. КР. – ПРАВОЛ. – $\overline{AB}$

$$\vec{v}_A = \vec{v}_p + \vec{v}_r \quad ; \quad v_p = \dot{x}_p$$

$$x: \quad 0 = v_p - v_r \cos\alpha \qquad (1)$$

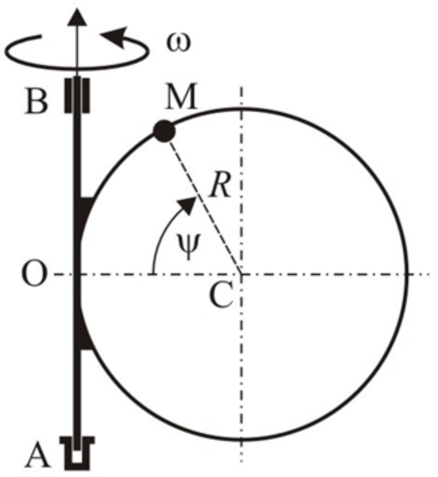
$$y: \quad v_A = v_r \sin\alpha \qquad (2)$$

$$(1) \rightarrow \quad v_r = \frac{v_p}{\cos\alpha}$$

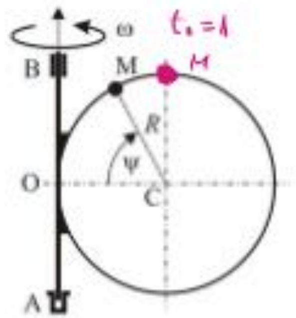
$$(2) \rightarrow \quad v_A = \frac{v_p}{\cos\alpha} \cdot \sin\alpha = v_p \operatorname{tg}\alpha$$

# Zadatak 5

3. Obruč, radijusa $R$=1 m, obrće se oko vertikalne ose AB konstantnom ugaonom brzinom $\omega$=1 $s^{-1}$=*const*. Po obruču se kreće prsten M po zakonu $\psi(t)=\dfrac{\pi}{2}+t^2-1\,[\mathrm{rad}]$. Odrediti apsolutnu brzinu i apsolutno ubrzanje prstena u trenutku $t_1$=1 s.



3. Obruč, radijusa $R=1$ m, obrće se oko vertikalne ose AB konstantnom ugaonom brzinom $\omega=1\ s^{-1}=const$. Po obruču se kreće prsten M po zakonu $\psi(t)=\frac{\pi}{2}+t^2-1$ [rad]. Odrediti apsolutnu brzinu i apsolutno ubrzanje prstena u trenutku $t_1=1$ s.




$$\omega_p=\omega=1=const$$

$$\varepsilon_p=\dot{\omega}_p=0$$

$$\psi(t)=\frac{\pi}{2}+t^2-1$$

$$\dot{\psi}(t)=\omega_r(t)=2t$$

$$\ddot{\psi}(t)=\varepsilon_r(t)=2=const$$

$t_1=1$

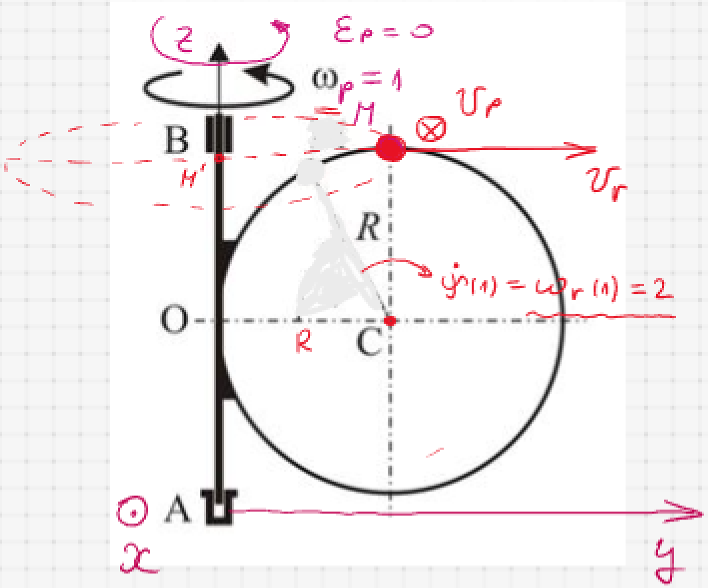
$\omega_p=1$, $\varepsilon_p=0$

$$\psi(1)=\frac{\pi}{2}+1^2-1=\frac{\pi}{2}$$

$$\dot{\psi}(1)=\omega_r(1)=2$$

$$\ddot{\psi}(1)=\varepsilon_r(1)=2$$

„$t_1 = 1$"




$$\vec{v}_M = \vec{v}_p + \vec{v}_r$$

$$v_p = \overline{MM'}\,\omega_p = R\,\omega_p$$

$$v_p = 1 \cdot 1 = 1 \ \frac{m}{s}$$

$$v_r = R \cdot \omega_r = 1 \cdot 2 = 2 \ \frac{m}{s}$$

$$v_{Mx} = -v_p = -1$$

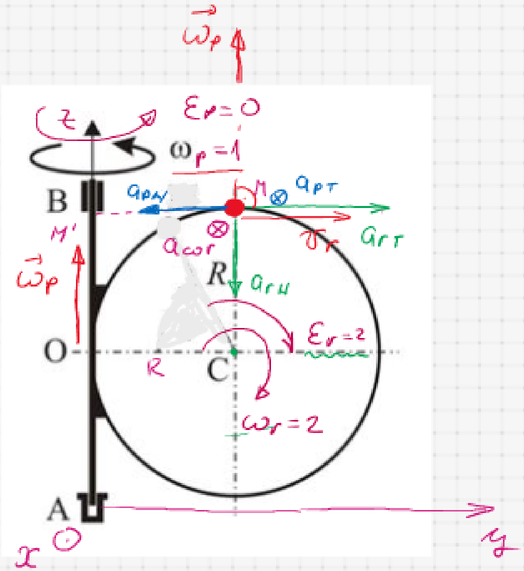
$$v_{My} = v_r = 2$$

$$v_{Mz} = 0$$

$$v = \sqrt{v_{Mx}^2 + v_{My}^2 + v_{Mz}^2} = \sqrt{(-1)^2 + 2^2 \ 0^2}$$

„$t_1 = 1$" $\quad v = \sqrt{5}$

„$t_1 = 1''$"

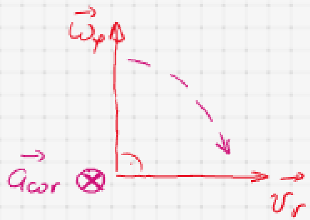



$$\vec{a}_M = \vec{a}_p + \vec{a}_r + \vec{a}_{cor}$$

$$* \quad \boxed{\vec{a}_M = \vec{a}_{pT} + \vec{a}_{pN} + \vec{a}_{rT} + \vec{a}_{rN} + \vec{a}_{cor}}$$

$$a_{pT} = \overline{MM'}\,\varepsilon_p = R\,\varepsilon_p = 0$$

$$a_{pN} = \overline{MM'}\,\omega_p^2 = R\,\omega_p^2 = 1\cdot 1^2 = 1 \quad \frac{m}{s^2}$$




$$a_{rT} = R\,\varepsilon_r = 1\cdot 2 = 2 \quad \frac{m}{s^2}$$

$$a_{rN} = R\,\omega_r^2 = 1\cdot 2^2 = 4 \quad \frac{m}{s^2}$$

$$= \frac{v_r^2}{R} = \frac{2^2}{1} = 4$$

$$\vec{a}_{cor} = 2\,\vec{\omega}_p \times \vec{v}_r$$

$$a_{cor} = 2\,\omega_p \cdot v_r \; \cancel{\sin\left(\frac{\pi}{2}\right)}^{\,1}$$

$$a_{cor} = 2\cdot 1\cdot 2\cdot 1 = 4 \quad \frac{m}{s^2}$$

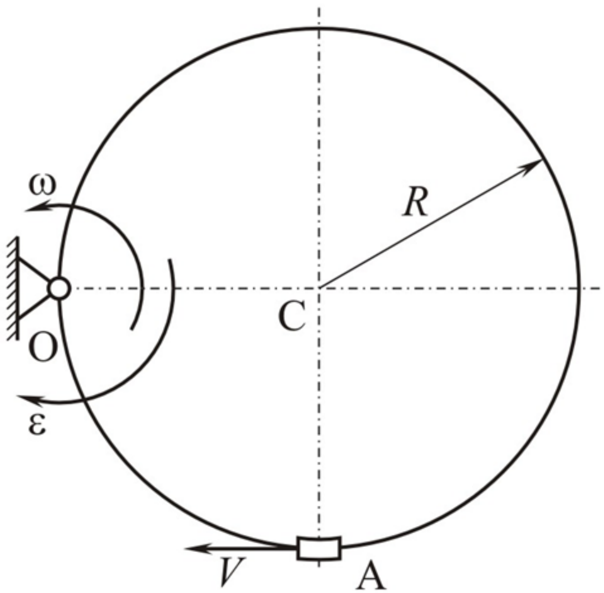
$$* \quad a_{Mx} = -a_{pT} - a_{cor} = -0 - 4 = -4$$

$$a_{My} = -a_{pN} + a_{rT} = -1 + 2 = 1$$

$$a_{Mz} = -a_{rN} = -4$$

$$a_M = \sqrt{a_{Mx}^2 + a_{My}^2 + a_{Mz}^2} = \sqrt{16+1+16} = \sqrt{33}$$

# Zadatak 6




3. Obruč poluprečnika $R$=1 m zglobno je vezan u tački O za podlogu i obrće se u ravni crteža. Po obruču se relativnom brzinom konstantnog intenziteta $V$ kreće klizač A. U prikazanom položaju obruč ima ugaonu brzinu $\omega = 1\ s^{-1}$ i ugaono ubrzanje $\varepsilon = 1\ s^{-2}$, datih smerova. Odrediti apsolutnu brzinu i apsolutno ubrzanje klizača u zadatom položaju.

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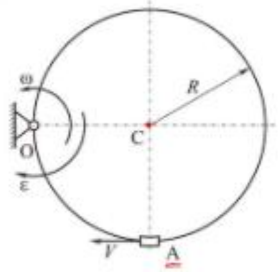
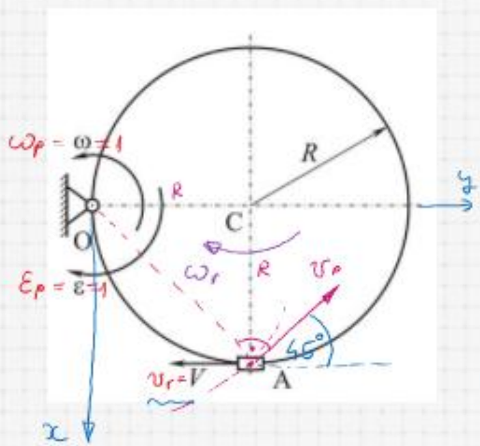



$\overline{OA} = \sqrt{R^2+R^2} = R\sqrt{2}$




„A"

$\vec{v} = \vec{v}_p + \vec{v}_r$ *

$v_r = V = const$

$v_p = \overline{OA}\,\omega_p = R\sqrt{2}\,\omega_p = 1\cdot\sqrt{2}\cdot 1 = \sqrt{2}\ \frac{m}{s}$

$TP_{np} = \mathcal{K}\{O, \overline{OA}\}_{\omega_p}$

* $v_x = -v_p \sin 45^\circ$

$v_y = v_p \cos 45^\circ - v_r$

$v_x = -\sqrt{2}\cdot\frac{\sqrt{2}}{2} = -1$

$v_y = \cancel{\sqrt{2}}\cdot\frac{\cancel{\sqrt{2}}}{\cancel{2}} - V = 1 - V$

$v_r = R\omega_r$

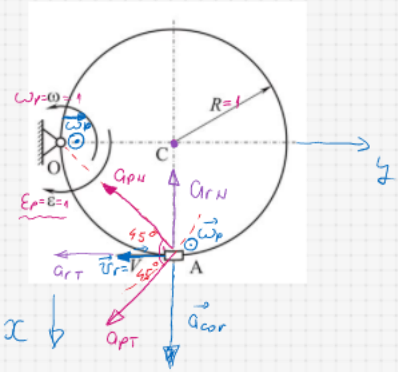
$\omega_r = \frac{v_r}{R}$

$\omega_r = \frac{V}{R} = const$

$\varepsilon_r = 0$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{1+(1-V)^2}$

„A"




$\vec{a} = \vec{a}_p + \vec{a}_r + \vec{a}_{cor}$

$\vec{a} = \vec{a}_{pT} + \vec{a}_{pN} + \vec{a}_{rT} + \vec{a}_{rN} + \vec{a}_{cor}$

$a_{pT} = \overline{OA} \cdot \varepsilon_p = R\sqrt{2}\,\varepsilon_p = \sqrt{2} \quad \frac{m}{s^2}$

$a_{pN} = \overline{OA} \cdot \omega_p^2 = R\sqrt{2}\,\omega_p^2 = \sqrt{2} \quad \frac{m}{s^2}$

$a_{rT} = \dot{v}_r = \frac{d}{dt}(V) = 0 = R\,\varepsilon_r$

$a_{rN} = \frac{v_r^2}{R} = \frac{V^2}{R} = V^2 \quad \frac{m}{s^2}$

$= R\,\omega_r^2 = \cancel{R} \cdot \frac{V^2}{R^{\cancel{2}}}$

$\vec{a}_{cor} = 2\vec{\omega}_p \times \vec{v}_r$

$a_{cor} = 2\,\omega_p\,v_r\,\cancel{\sin(90^\circ)}^{\,1}$

$a_{cor} = 2 \cdot 1 \cdot V = 2V$

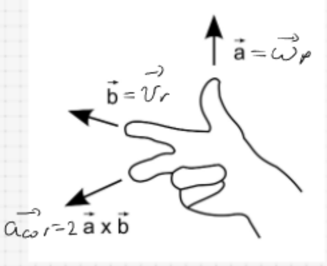
$a_x = a_{pT}\sin 45^\circ - a_{pN}\sin 45 - a_{rN} + a_{cor}$

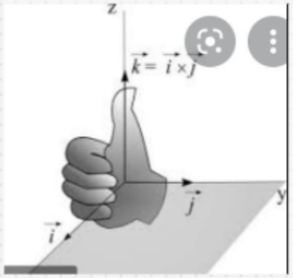
$a_y = -a_{pT}\cos 45^\circ - a_{pN}\cos 45^\circ - a_{rT}$

$a_x =$

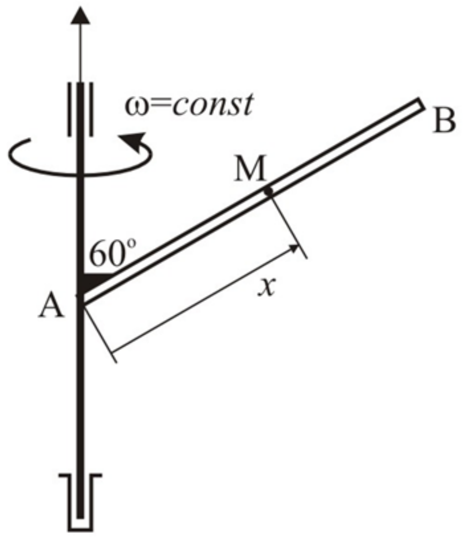
$a_y =$

$a = \sqrt{a_x^2 + a_y^2} = \dots$

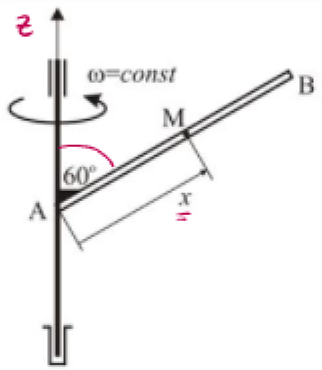


# Zadatak 7




3. Cev AB se obrće oko nepokretne ose konstantnom ugaonom brzinom $\omega=2s^{-1}=const$. Sa osom obrtanja cev gradi ugao 60°. U cevi se po zakonu $x(t)=t^2+t$ [m] kreće tačka M. Odrediti apsolutnu brzinu i ubrzanje tačke u trenutku $t_1=1$s.




3. Cev AB se obrće oko nepokretne ose konstantnom ugaonom brzinom $\omega=2s^{-1}=const$. Sa osom obrtanja cev gradi ugao 60°. U cevi se po zakonu $x(t)=t^2+t$ [m] kreće tačka M. Odrediti apsolutnu brzinu i ubrzanje tačke u trenutku $t_1=1$s.

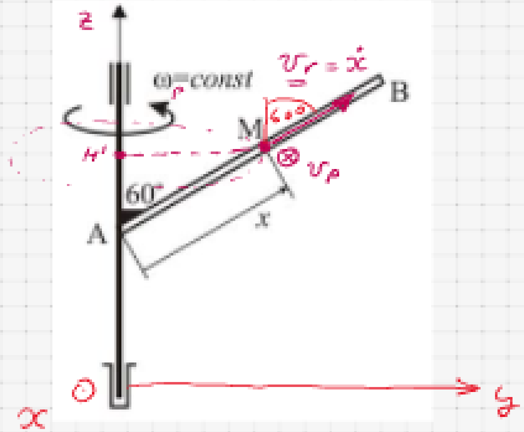
ПРЕН. КР. – ЦЕВ – обртање око осе z – $\omega_p = \omega = 2 = const$

$\varepsilon_p = 0$

РЕЛ. КР. – у односу на цев – праволин. кр. – $x(t) = t^2 + t$

$\dot{x}(t) = 2t + 1$

$\ddot{x}(t) = 2 = const$




$\vec{v}_M = \boxed{\vec{v} = \vec{v}_p + \vec{v}_r}$

*

$v_r = \dot{x} = 2t + 1$

$v_p = \overline{MM'}\,\omega_p$

$v_p = (x \sin 60^\circ)\,\omega$

$v_p = (t^2 + t)\sin 60^\circ\,\omega$

$t_1 = 1\,s$

$v_r(1) = \dot{x}(1) = 2 \cdot 1 + 1 = 3\ \frac{m}{s}$

$v_p(1) = (1^2 + 1)\,\frac{\sqrt{3}}{\cancel{2}} \cdot \cancel{2} = 2\sqrt{3}\ \frac{m}{s}$

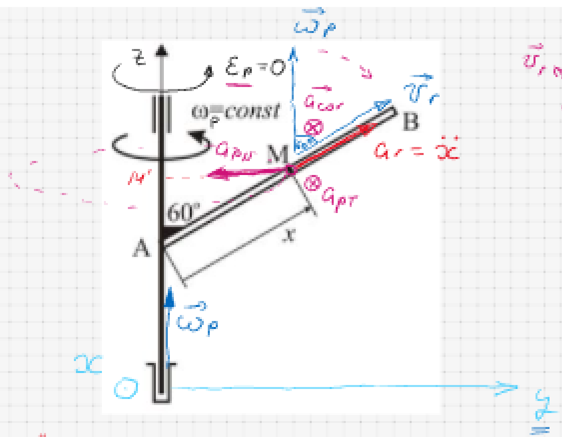
$* \quad v_x = -v_p = -2\sqrt{3}$

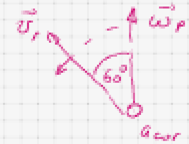
$v_y = v_r \sin 60^\circ = 3 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

$v_z = v_r \cos 60^\circ = 3 \cdot \frac{1}{2} = \frac{3}{2}$

$v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(-2\sqrt{3})^2 + \left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$

$= \sqrt{12 + \frac{9 \cdot 3}{4} + \frac{9}{4}} = \dots$




„M"

$$\vec a = \vec a_p + \vec a_r + \vec a_{cor}$$

$$\vec a = \underbrace{\cancel{\vec a_{pT}}}_{0} + \vec a_{pN} + \vec a_r + \vec a_{cor}$$

; $a_r = \ddot x = 2 = const$

$a_{pN(1)} = \overline{MM'}_{(1)}\,\omega_p^2 = \sqrt{3}\cdot 2^2 = 4\sqrt{3}\ \frac{m}{s^2}$

$\overline{MM'} = x\sin 60^\circ$

$\overline{MM'}(1) = x(1)\cdot\frac{\sqrt{3}}{2} = \cancel{2}\,\frac{\sqrt{3}}{\cancel{2}} = \sqrt{3}\ \ m$

$a_{pT} = \overline{MM'}\,\varepsilon_p = 0$

$\vec a_{cor} = 2\,\vec\omega_p \times \vec v_r$

$a_{cor} = 2\,\omega_p v_r \sin(60^\circ)$

$a_{cor}(1) = 2\ \omega_p(1)\cdot v_r(1)\cdot\frac{\sqrt{3}}{2}$

$= \cancel{2}\cdot 2\cdot 3\,\frac{\sqrt{3}}{\cancel{2}} = 6\sqrt{3}$

$a_x = -a_{cor}$

$a_y = -a_{pN} + a_r\sin 60^\circ$

$a_z = a_r\cos 60^\circ$

$t_1 = 1$

$a_x = -6\sqrt{3}$

$a_y = -4\sqrt{3} + \cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}} = -3\sqrt{3}$

$a_z = 2\cdot\frac{1}{2} = 1$

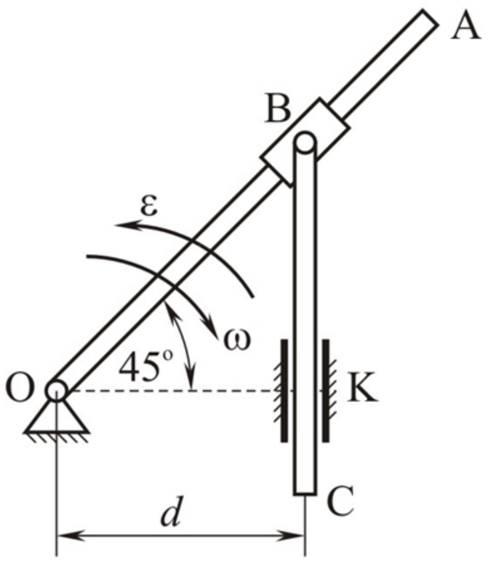
$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$= \sqrt{(-6\sqrt{3})^2 + (-3\sqrt{3})^2 + 1^2}$$

$$= \sqrt{108 + 27 + 1} = \sqrt{136}$$

# Zadatak 8

3. Na slici je prikazan kulisni mehanizam. Pri klaćenju krivaje OA, oko ose O upravne na ravan crteža, pomera se klizač B duž krivaje OA i dovodi u kretanje štap BC, koji može da se kreće u vertikalnoj vodici K. Rastojanje OK=$d$. Ukoliko je poznato da krivaja OA u dalom položaju ima ugaonu brzinu $\omega$ i ugaono ubrzanje $\varepsilon$ (smerovi su dati na slici), odrediti brzinu i ubrzanje tačke C.



# Šta smo naučili?

25. **Složeno kretanje tačke (osnovni pojmovi)**
26. **Brzine i ubrzanja tačaka tela pri složenom kretanju tačke**

# Mehanika 2 (Kinematika)
# Vežbe 8

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Novi Sad, 2023.