Kinematika
Kinematika krutog tela – 2. deo

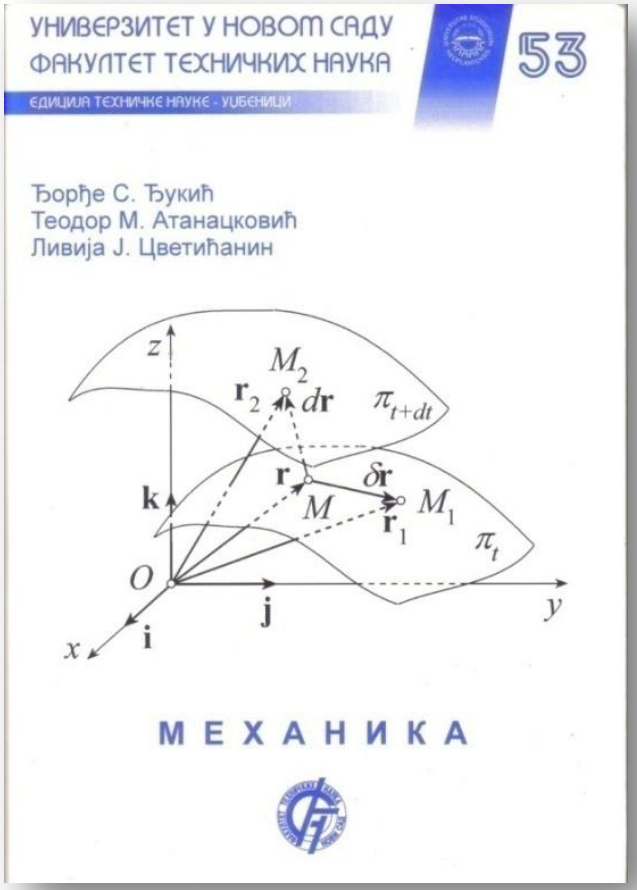
Kinematika i dinamika

Miodrag Zuković

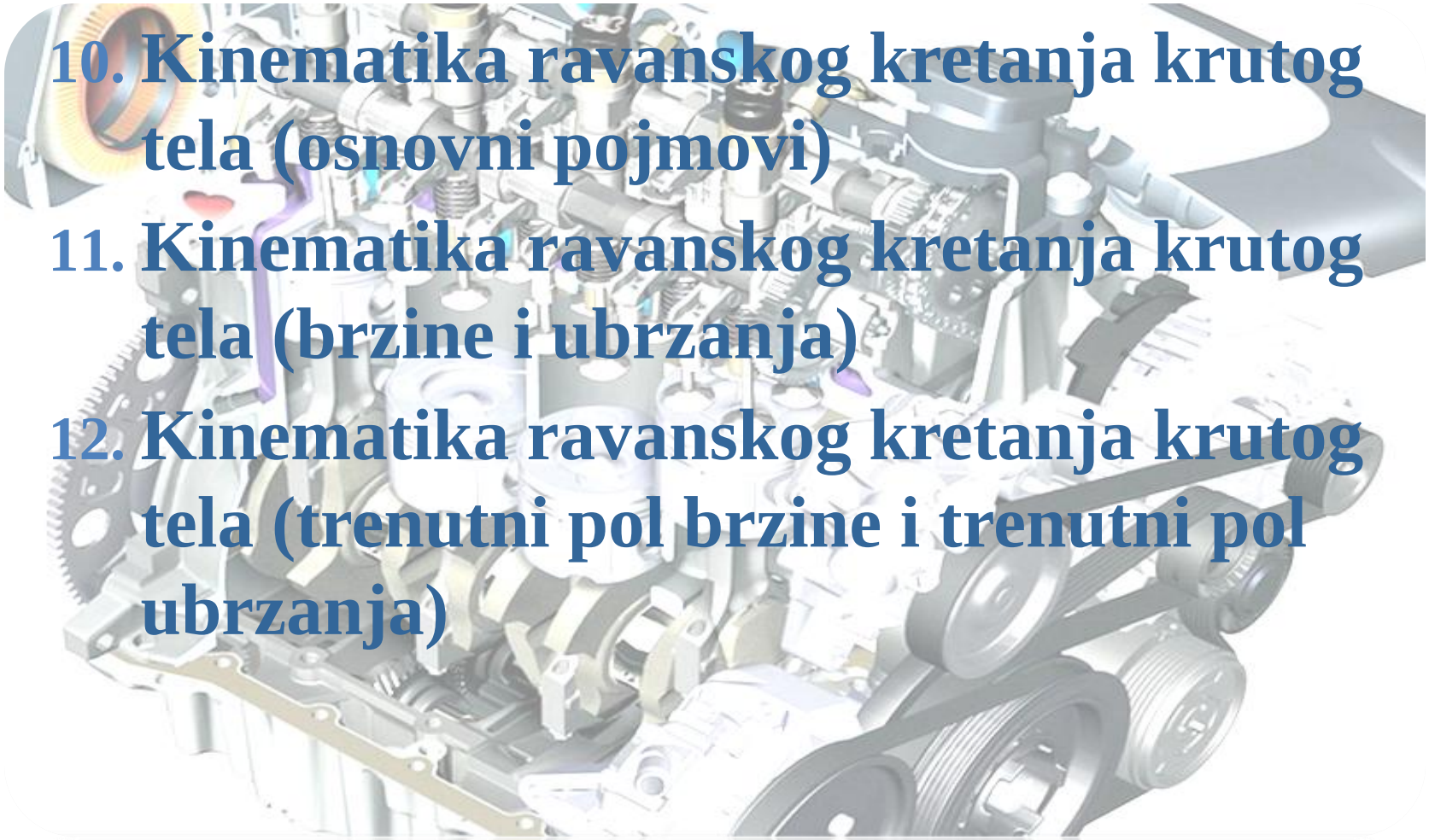
Novi Sad, 2021.

Literatura

- Đorđe S. Đukić, Teodor M. Atanacković, Livija J. Cvetićanin: Mehanika, Fakultet tehničkih nauka u Novom Sadu, Novi Sad, 2003.



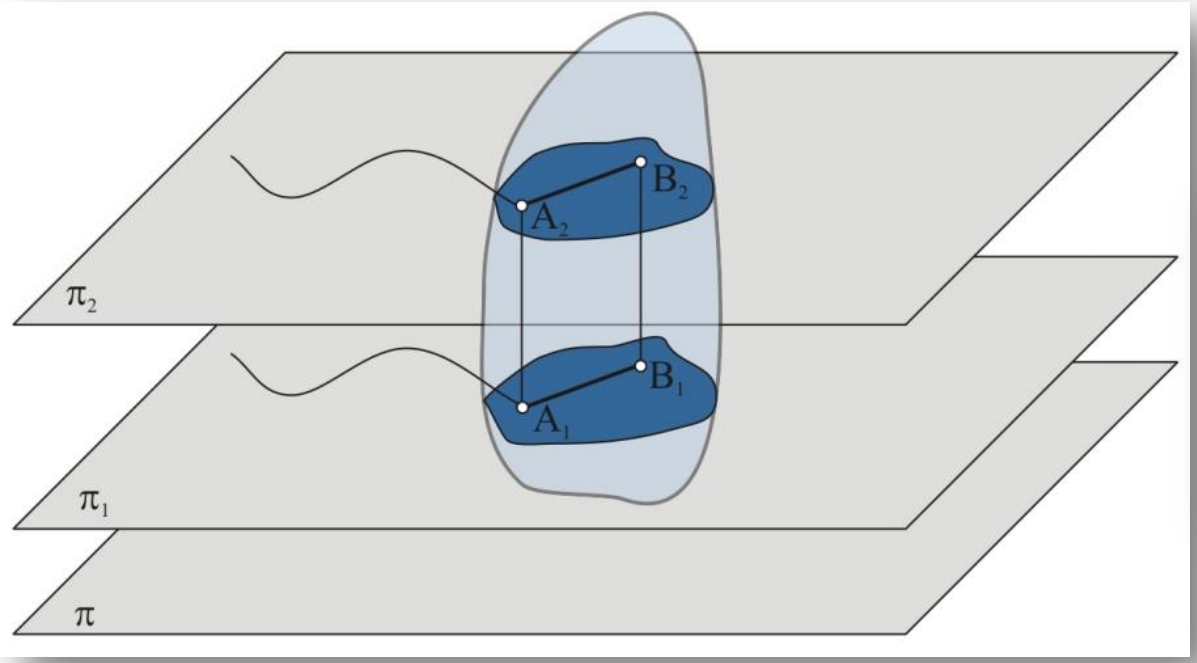
Šta ćemo naučiti?

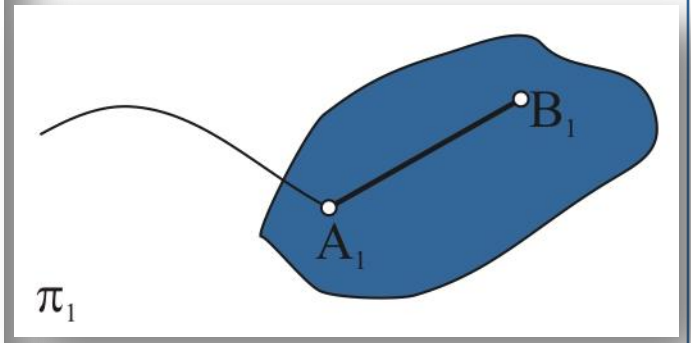


10. **Kinematika ravanskog kretanja krutog tela (osnovni pojmovi)**
11. **Kinematika ravanskog kretanja krutog tela (brzine i ubrzanja)**
12. **Kinematika ravanskog kretanja krutog tela (trenutni pol brzine i trenutni pol ubrzanja)**

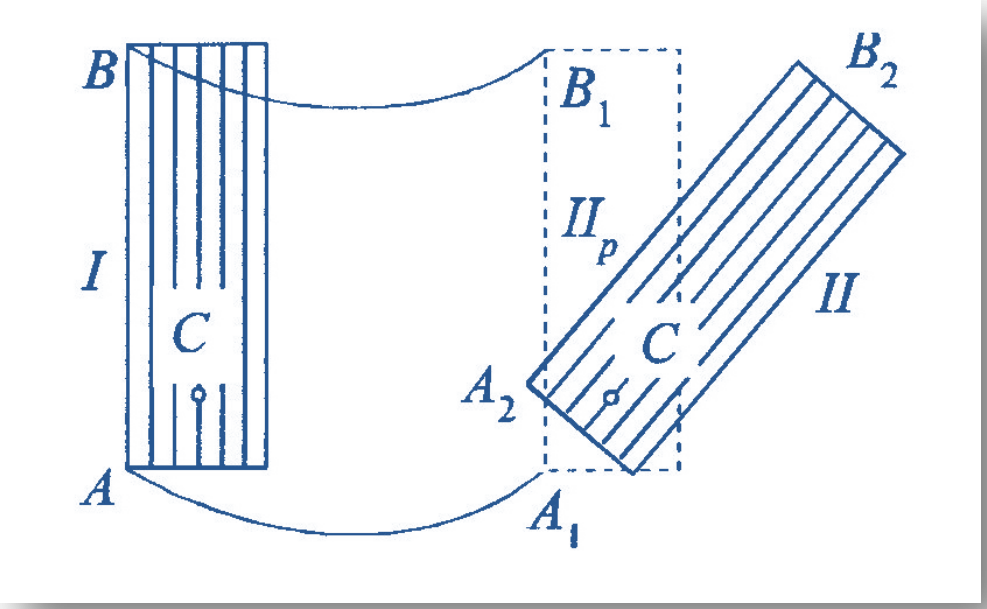
10. Kinematika ravanskog kretanja krutog tela (osnovni pojmovi)

- Pri ravanskom kretanju sve tačke tela kreću se u ravnima koje su paralelne nekoj nepokretnoj ravni (π).



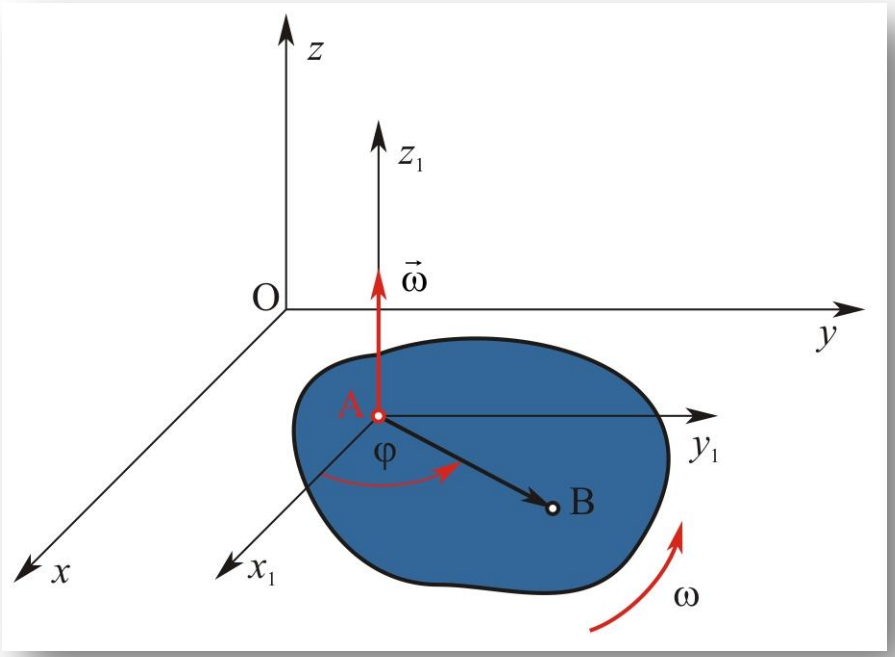


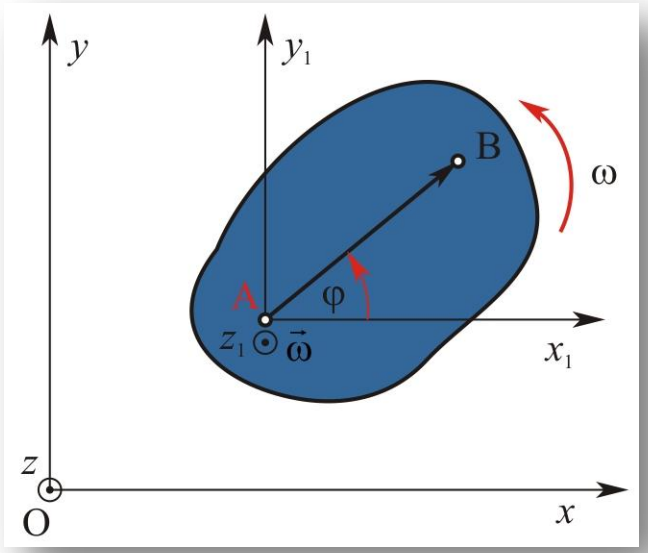
- Ravansko kretanje ima tri stepena slobode.
- Pri ravanskom kretanju tela ono se može dovesti iz jednog položaja u drugi jednom translacijom (dva stepena slobode) i jednim obrtanjem oko ose (jedan stepen slobode).



Parametarske jednačine ravanskog kretanja tela:

$$x_A = x_A(t), y_A = y_A(t), \varphi = \varphi(t)$$





Ugaona brzina i ugaono ubrzanje

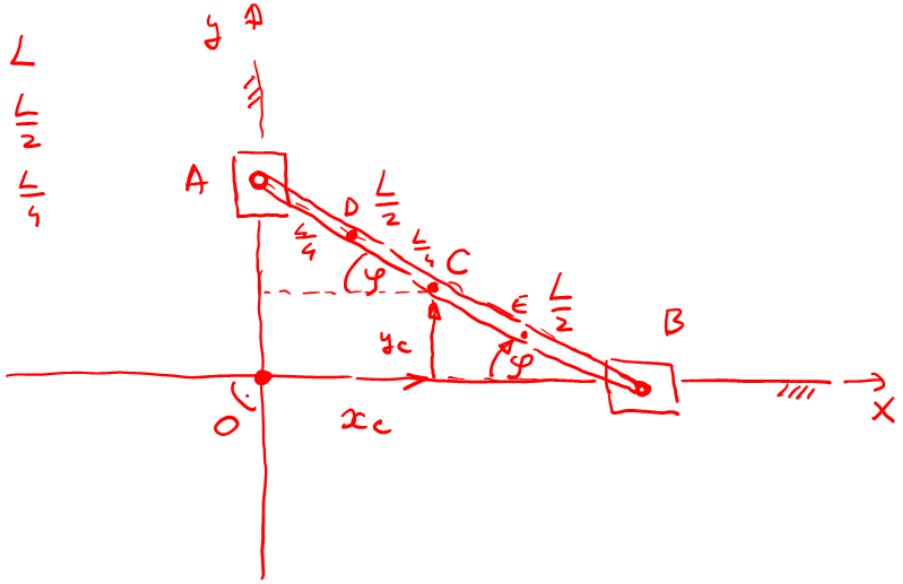
$$\vec{\omega} = \omega\vec{k}, \omega = \dot{\varphi}$$

$$\vec{\varepsilon} = \dot{\vec{\omega}} = \varepsilon\vec{k}, \varepsilon = \dot{\omega} = \ddot{\varphi}$$

$\overline{AB} = L$

$\overline{AC} = \frac{L}{2}$

$\overline{AD} = \frac{L}{4}$



ТРАЈ. $C = ?$

$$x_C = \frac{L}{2}\cos\varphi \,/^2$$
$$y_C = \frac{L}{2}\sin\varphi \,/^2$$
$\xrightarrow{\text{ЕЛ.}}$ φ

$$(x-p)^2 + (y-q)^2 = r^2$$

$$x_C^2 + y_C^2 = \left(\frac{L}{2}\right)^2$$ КРУЖНИЦА

ТРАЈ $D = ?$

$$x_D = \frac{L}{4}\cdot\cos\varphi \rightarrow \frac{x_D}{\frac{L}{4}} = \cos\varphi\,/^2$$

$$y_D = \frac{3L}{4}\cdot\sin\varphi \rightarrow \frac{y_D}{\frac{3L}{4}} = \sin\varphi\,/^2$$

$$\frac{x_D^2}{\left(\frac{L}{4}\right)^2} + \frac{y_D^2}{\left(\frac{3L}{4}\right)^2} = 1$$

ЕЛИПСА

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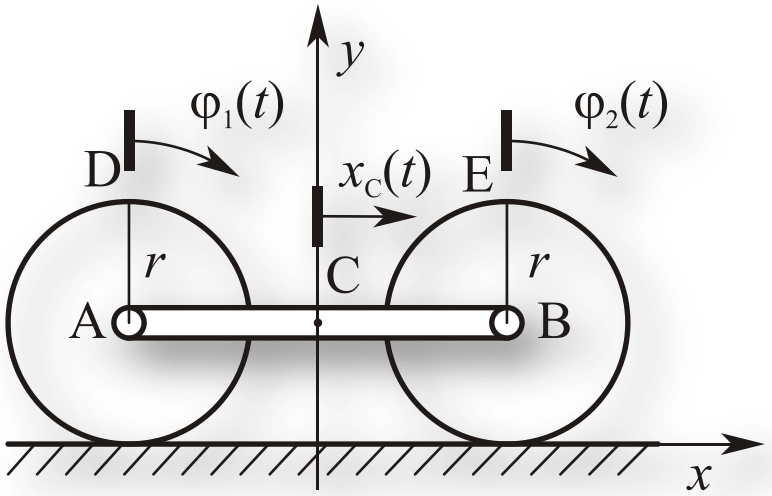
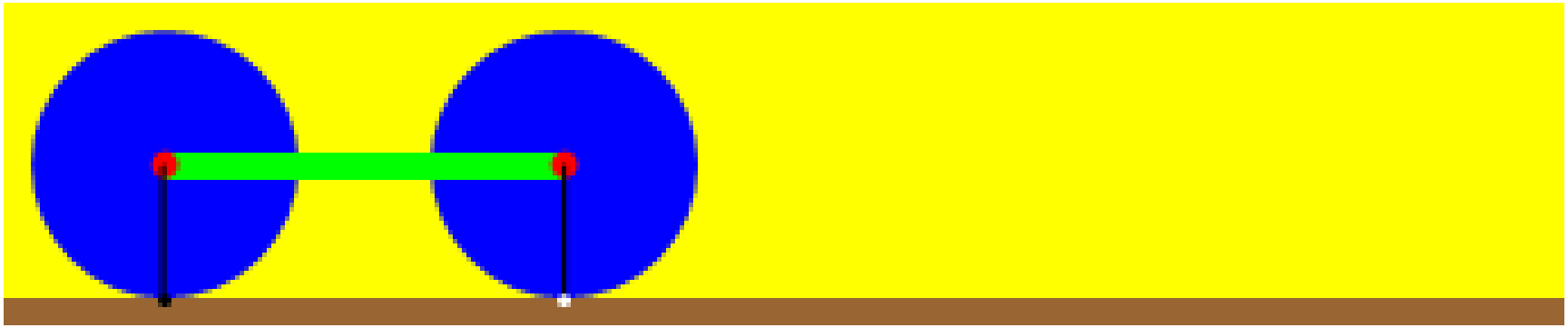
Primer

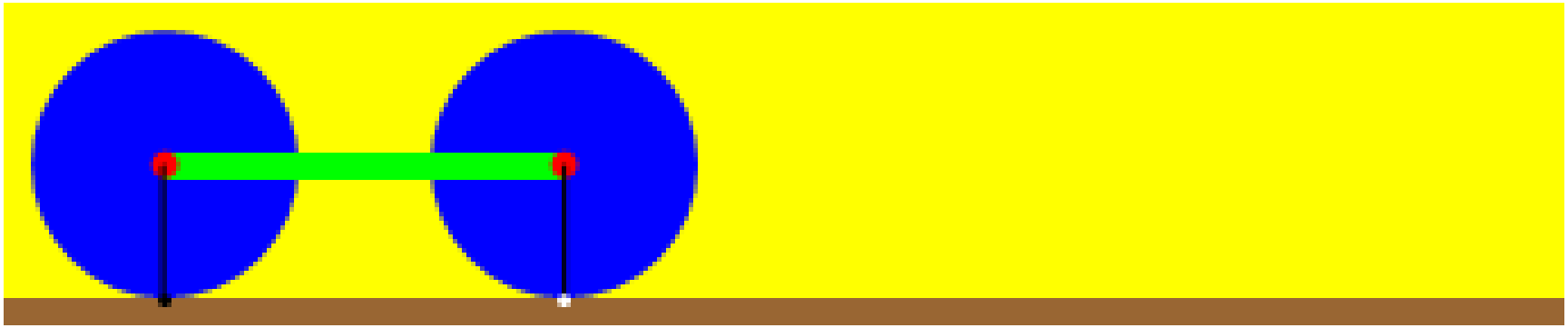
- Date su jednačine kretanja prikazanog sistema krutih tela:

$$x_c(t) = v_0 t, \quad \varphi_1(t) = k\frac{v_0}{r}t, \quad \varphi_2(t) = \frac{v_0}{r}t.$$

Odrediti trajektorije kretanja tačaka D i E na obodima diskova za k=1/2 i k=2.





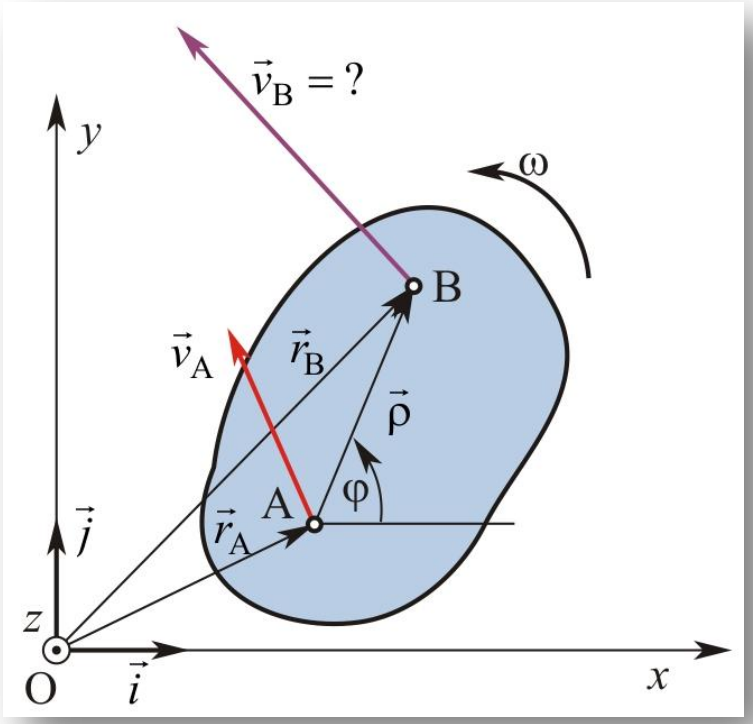


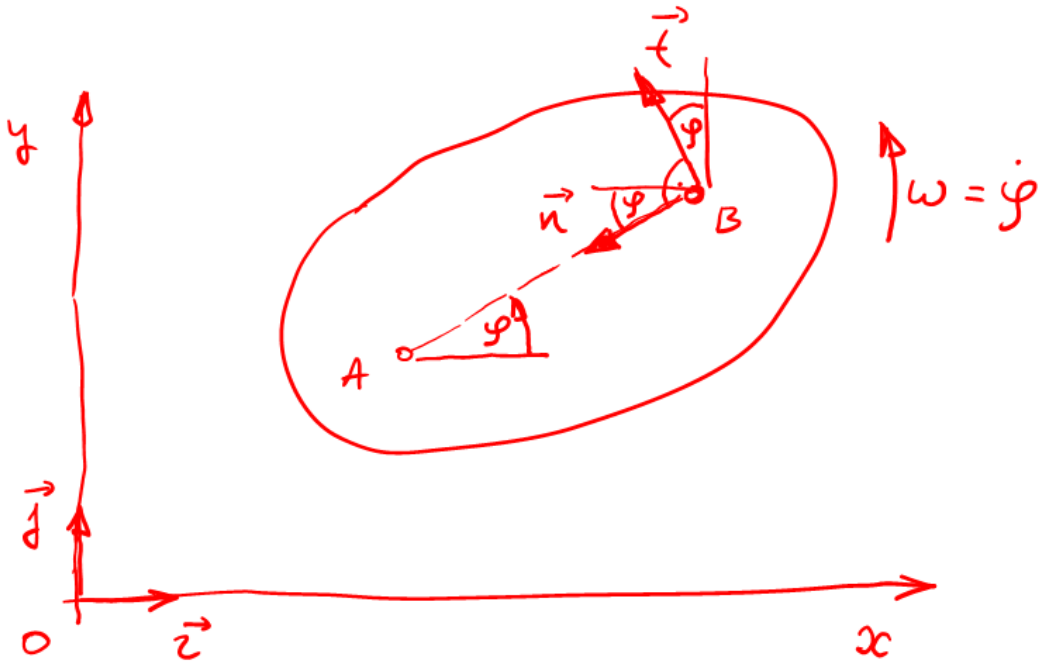
11. Kinematika ravanskog kretanja krutog tela (brzine i ubrzanja)

Kinematika ravanskog kretanja krutog tela – brzine

- Veza između brzina tačaka tela pri ravanskom kretanju

$$\vec{r}_B = \vec{r}_A + \vec{\rho}$$
$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{\rho}}$$
$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$$





$$|\vec{t}| = |\vec{n}| = 1$$

$$\vec{t} = -\sin\varphi\,\vec{i} + \cos\varphi\,\vec{j}$$

$$\vec{n} = -\cos\varphi\,\vec{i} - \sin\varphi\,\vec{j}$$

Kinematika ravanskog kretanja krutog tela – brzine

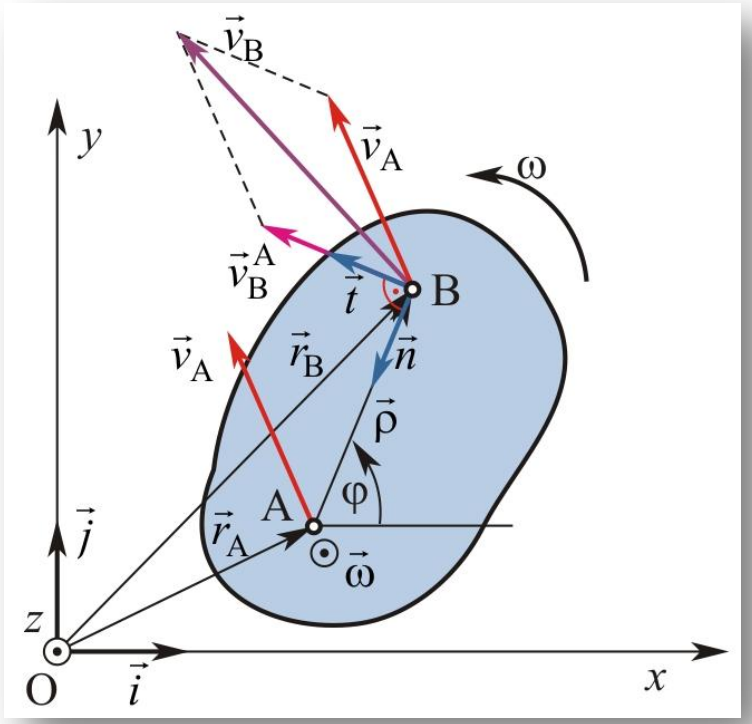
- Veza između brzina tačaka tela pri ravanskom kretanju

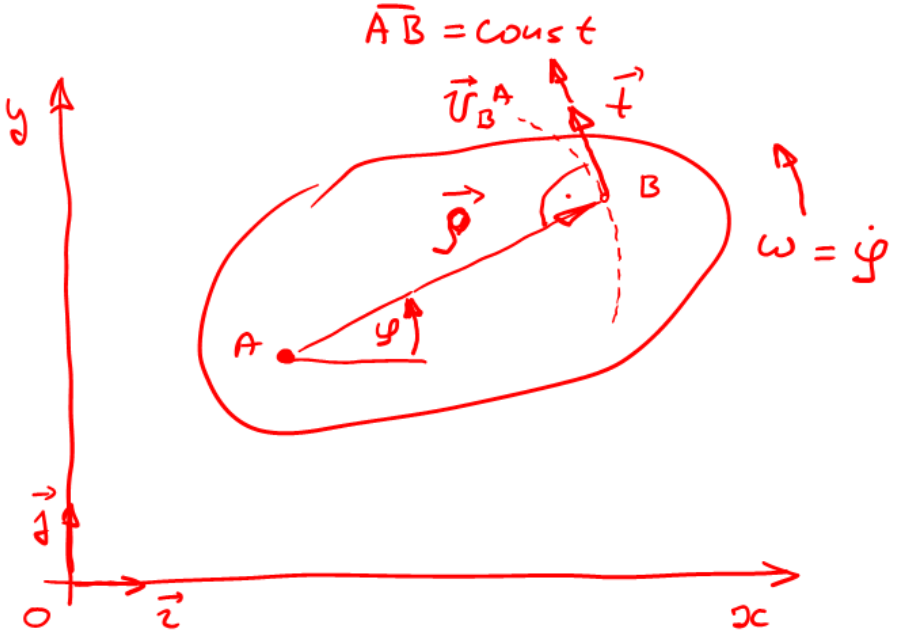
$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$$

$$\vec{\rho} = \overline{AB}\cos\varphi\vec{i} + \overline{AB}\sin\varphi\vec{j} + 0\vec{k}$$

$$\vec{v}_B^A = \dot{\vec{\rho}} = -\overline{AB}\dot{\varphi}\sin\varphi\vec{i} + \overline{AB}\dot{\varphi}\cos\varphi\vec{j} = \overline{AB}\dot{\varphi}\vec{t}$$

$$v_B^A = \overline{AB}\,\dot{\varphi}$$





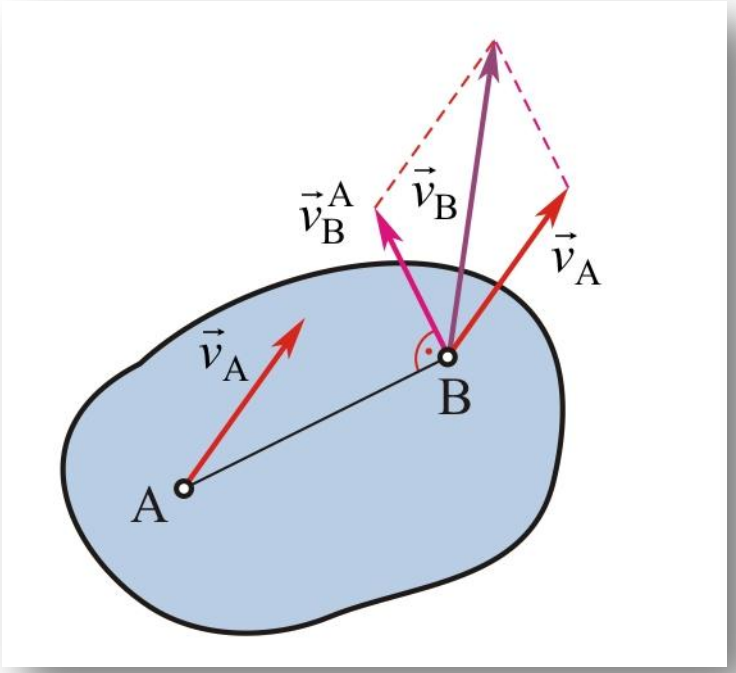
$$\vec{\rho} = \overline{AB}\cos\varphi\,\vec{i} + \overline{AB}\sin\varphi\,\vec{j}$$

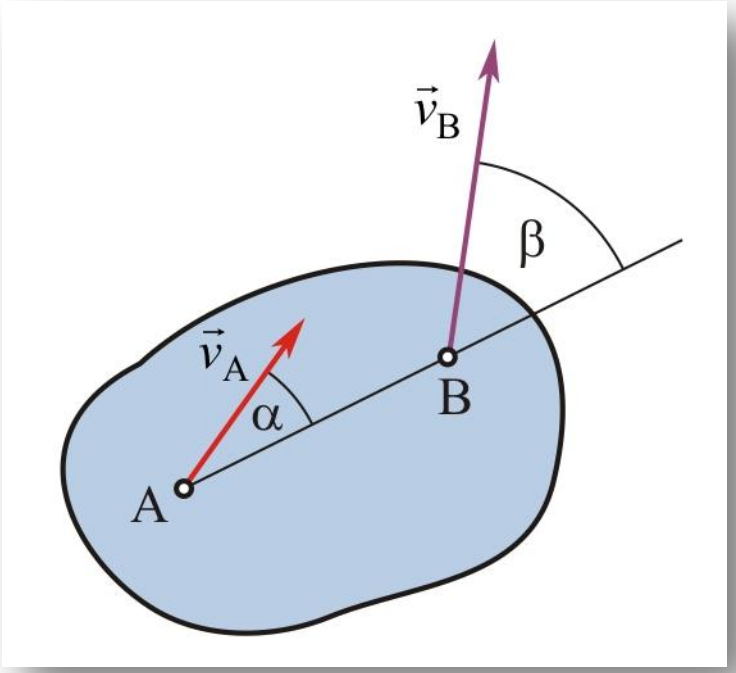
$$\dot{\vec{\rho}} = -\overline{AB}\sin\varphi\cdot\dot{\varphi}\,\vec{i} + \overline{AB}\cos\varphi\cdot\dot{\varphi}\,\vec{j}$$

$$\dot{\vec{\rho}} = \overline{AB}\,\dot{\varphi}\,(\underbrace{-\sin\varphi\,\vec{i} + \cos\varphi\,\vec{j}}_{\vec{\epsilon}})$$

$$\dot{\vec{\rho}} = \overline{AB}\,\dot{\varphi}\,\vec{\epsilon} = \vec{v}_B^A \;;\quad v_B^A = \overline{AB}\cdot\omega$$

Projekcije brzina dve tačke na spojnu pravu





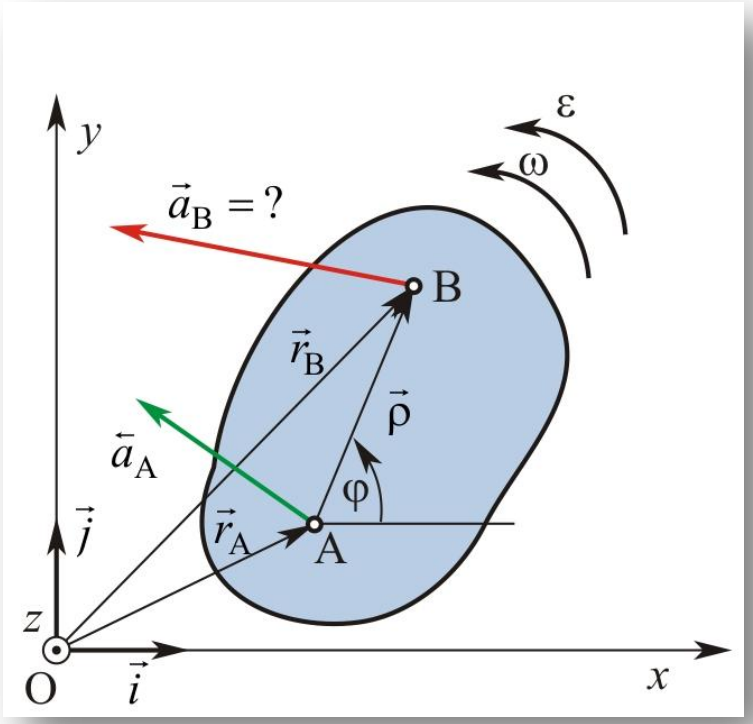
$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$$

$$v_B \cos\beta = v_A \cos\alpha$$

Kinematika ravanskog kretanja krutog tela – ubrzanja

- Veza između ubrzanja tačaka tela pri ravanskom kretanju

$$\vec{r}_B = \vec{r}_A + \vec{\rho}$$
$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{\rho}}$$
$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$$
$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{\rho}}$$
$$\vec{a}_B = \vec{a}_A + \vec{a}_B^A$$



$$\vec{v}_B^A = \dot{\vec{\rho}} = -\overline{AB}\dot{\varphi}\sin\varphi\vec{i} + \overline{AB}\dot{\varphi}\cos\varphi\vec{j} = \overline{AB}\dot{\varphi}\vec{t}$$

$$\vec{a}_B^A = \ddot{\vec{\rho}} = -\overline{AB}\ddot{\varphi}\sin\varphi\vec{i} - \overline{AB}\dot{\varphi}\cos\varphi\cdot\dot{\varphi}\vec{i} + \overline{AB}\ddot{\varphi}\cos\varphi\vec{j} - \overline{AB}\dot{\varphi}\cdot\sin\varphi\,\dot{\varphi}\vec{j}$$

$$= \overline{AB}\ddot{\varphi}(-\sin\varphi\vec{i} + \cos\varphi\vec{j}) + \overline{AB}\dot{\varphi}^2(-\cos\varphi\vec{i} - \sin\varphi\vec{j})$$

$$\vec{\epsilon} \qquad\qquad \vec{n}$$

$$\vec{a}_B^A = \overline{AB}\ddot{\varphi}\vec{t} + \overline{AB}\dot{\varphi}^2\vec{n}$$

$$\vec{a}_B^A = \vec{a}_{BT}^A + \vec{a}_{BN}^A;$$

$$a_{BT}^A = \overline{AB}\ddot{\varphi} = \overline{AB}\varepsilon$$

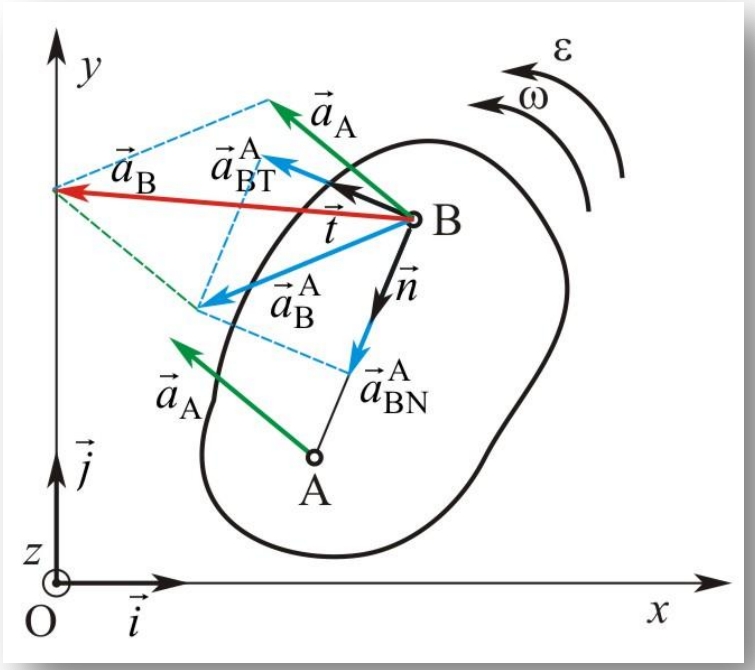
$$a_{BN}^A = \overline{AB}\dot{\varphi}^2 = \overline{AB}\omega^2$$

Kinematika ravanskog kretanja krutog tela – ubrzanja

- Veza između ubrzanja tačaka tela pri ravanskom kretanju

$$\vec{r}_B = \vec{r}_A + \vec{\rho}$$
$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{\rho}}$$
$$\vec{a}_B = \vec{a}_A + \vec{a}_B^A$$

$$\vec{\rho} = \overline{AB}\cos\varphi\vec{i} + \overline{AB}\sin\varphi\vec{j} + 0\vec{k}$$



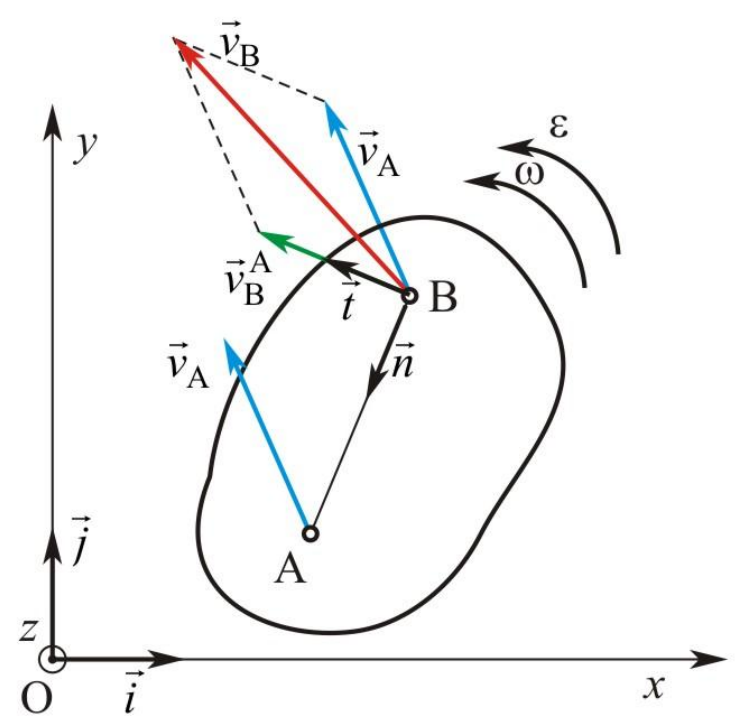
$$\vec{a}_B^A = \ddot{\vec{\rho}} = -\overline{AB}\ddot{\varphi}\sin\varphi\vec{i} - \overline{AB}\dot{\varphi}^2\cos\varphi\vec{i} + \overline{AB}\ddot{\varphi}\cos\varphi\vec{j} - \overline{AB}\dot{\varphi}^2\sin\varphi\vec{j}$$
$$\vec{a}_B^A = \overline{AB}\ddot{\varphi}\vec{t} + \overline{AB}\dot{\varphi}^2\vec{n} = \vec{a}_{BT}^A + \vec{a}_{BN}^A$$

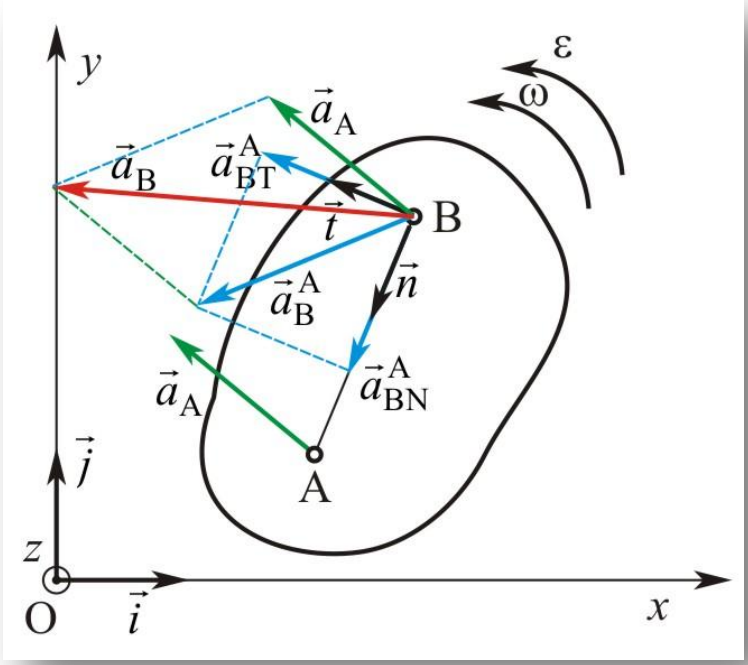
Kinematika ravanskog kretanja krutog tela – brzine i ubrzanja

- Veza između brzina i ubrzanja tačaka tela pri ravanskom kretanju

$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$$

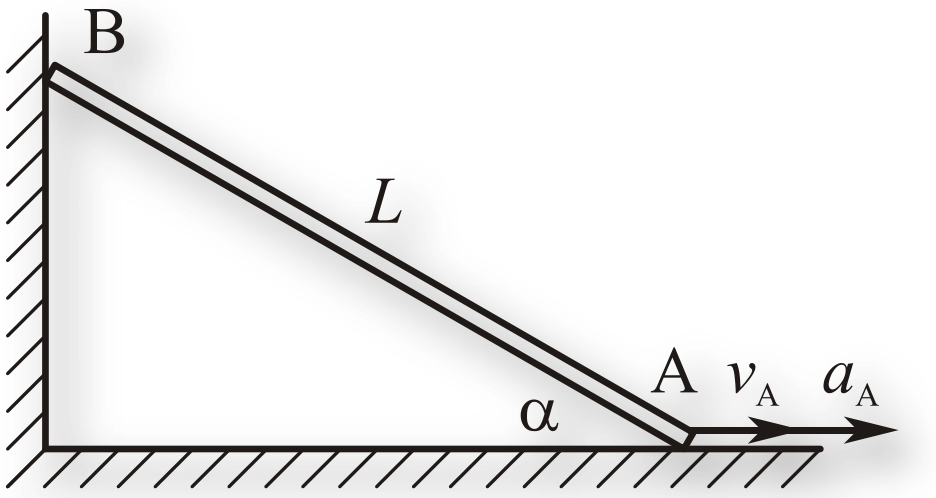
$$\vec{a}_B = \vec{a}_A + \vec{a}_{BT}^A + \vec{a}_{BN}^A$$





Primer

Štap AB, dužine L, oslanja se krajem A o horizontalni pod, a krajem B o vertikalni zid. U položaju u kome štap gradi ugao α sa horizontalom brzina i ubrzanje tačke A iznose v_A i a_A. Odrediti brzinu i ubrzanje tačke B i ugaonu brzinu i ugaono ubrzanje štapa u datom položaju.



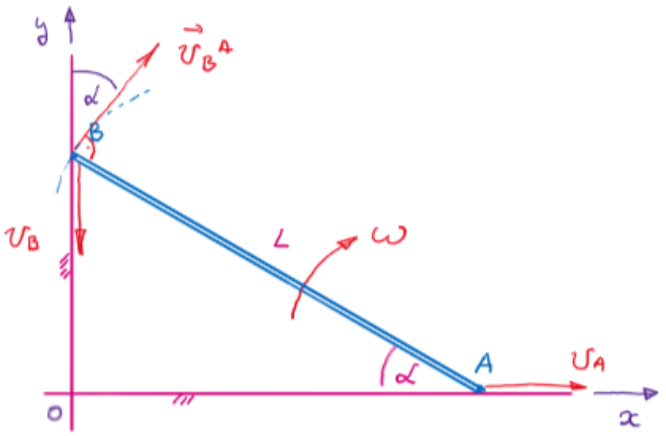
v_A, a_A, L, α

$\vec{v}_B = ?$

$\vec{a}_B = ?$

$\omega = ?$

$\varepsilon = ?$



$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$; $\vec{v}_B^A \perp \bar{BA}$ (смер према ω)

$v_B^A = \bar{AB}\,\omega$

x: $\quad 0 = v_A + v_B^A \sin\alpha \quad (1) \qquad v_B = ?$

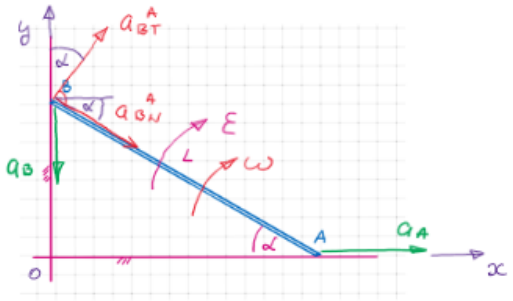
y: $\quad -v_B = 0 + v_B^A \cos\alpha \quad (2) \qquad v_B^A = ?$

(1) $\quad v_B^A = -\dfrac{v_A}{\sin\alpha}$

(2) $\quad v_B = -v_B^A \cos\alpha = -\left(-\dfrac{v_A}{\sin\alpha}\right)\cos\alpha$

$$v_B = v_A \operatorname{ctg}\alpha$$

$$v_B^A = \bar{AB}\,\omega \rightarrow \omega = \frac{v_B^A}{\bar{AB}} = \frac{-\frac{v_A}{\sin\alpha}}{L} = -\frac{v_A}{L\sin\alpha}$$



$\vec{a}_B = \vec{a}_A + \vec{a}_B^{\,A}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{BT}^{\,A} + \vec{a}_{BN}^{\,A}$; $\vec{a}_{BT}^{\,A} \perp \overline{AB}$ (смер према ε)

$a_{BT}^{A} = \overline{AB}\,\varepsilon = ?$

$\vec{a}_{BN}^{\,A}$ $B \to A$

$a_{BN}^{A} = \overline{AB}\,\omega^2 = L \dfrac{v_A^2}{L^2 \sin^2\alpha}$

$x: \quad 0 = a_A + a_{BT}^{A} \sin\alpha + a_{BN}^{A} \cos\alpha \qquad (3)$

$a_B = ?$

$y: \quad -a_B = 0 + a_{BT}^{A} \cos\alpha - a_{BN}^{A} \sin\alpha \qquad (4)$

$a_{BT}^{A} = ?$

(3) $a_{BT}^{A} = -\dfrac{1}{\sin\alpha}\left(a_A + a_{BN}^{A}\cos\alpha\right)$

$= -\dfrac{1}{\sin\alpha}\left(a_A + \dfrac{v_A^2}{L\sin^2\alpha}\cdot\cos\alpha\right)$

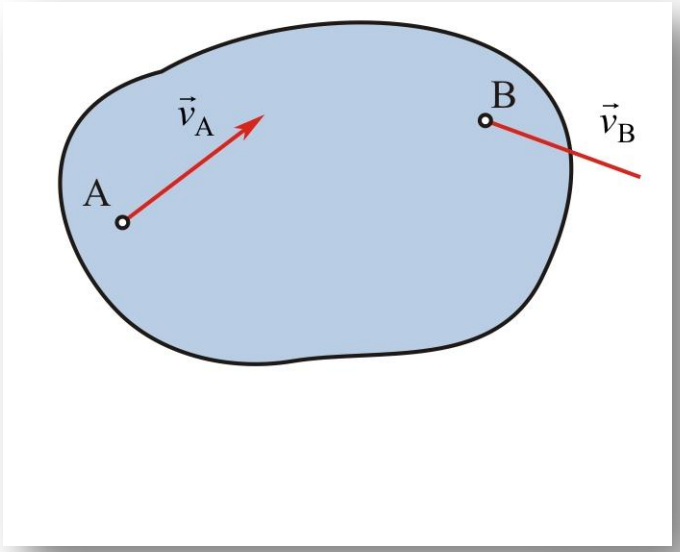
$a_{BT}^{A} = \overline{AB}\cdot\varepsilon \quad \to \quad \varepsilon = \dfrac{a_{BT}^{A}}{\overline{AB}} = -\dfrac{1}{L\sin\alpha}\left(a_A + \dfrac{v_A^2}{L\sin^2\alpha}\cos\alpha\right)$

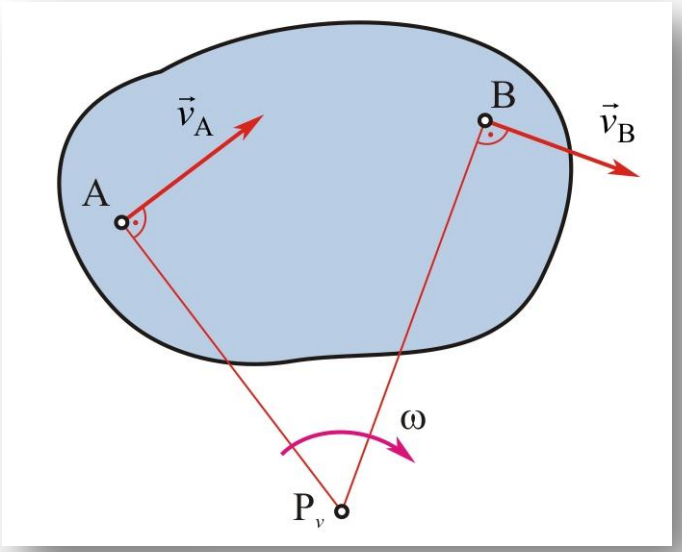
(4) $a_B = -a_{BT}^{A}\cos\alpha + a_{BN}^{A}\sin\alpha$

$a_B = -\left(-\dfrac{1}{\sin\alpha}\left(a_A + \dfrac{v_A^2}{L\sin^2\alpha}\cos\alpha\right)\right)\cdot\cos\alpha + \dfrac{v_A^2}{L\sin^2\alpha}\cdot\sin\alpha$

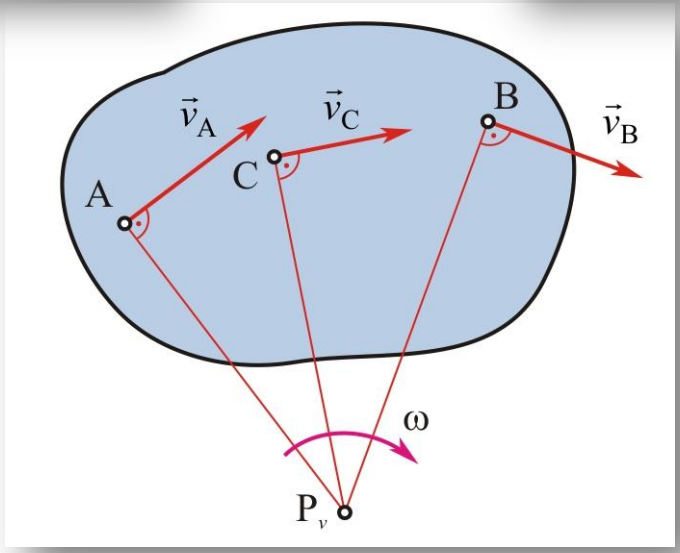
12. Kinematika ravanskog kretanja krutog tela (trenutni pol brzine i trenutni pol ubrzanja)

Trenutni pol brzine

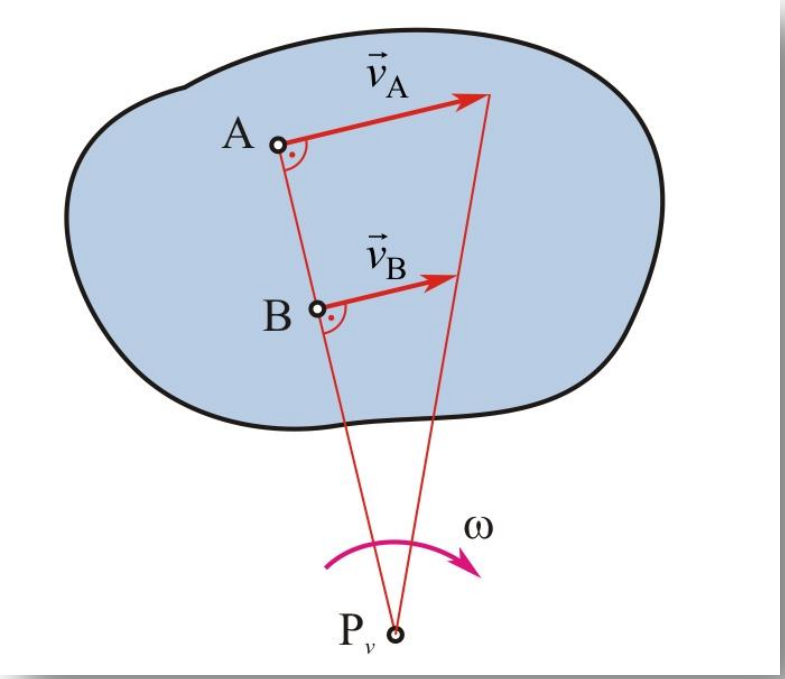


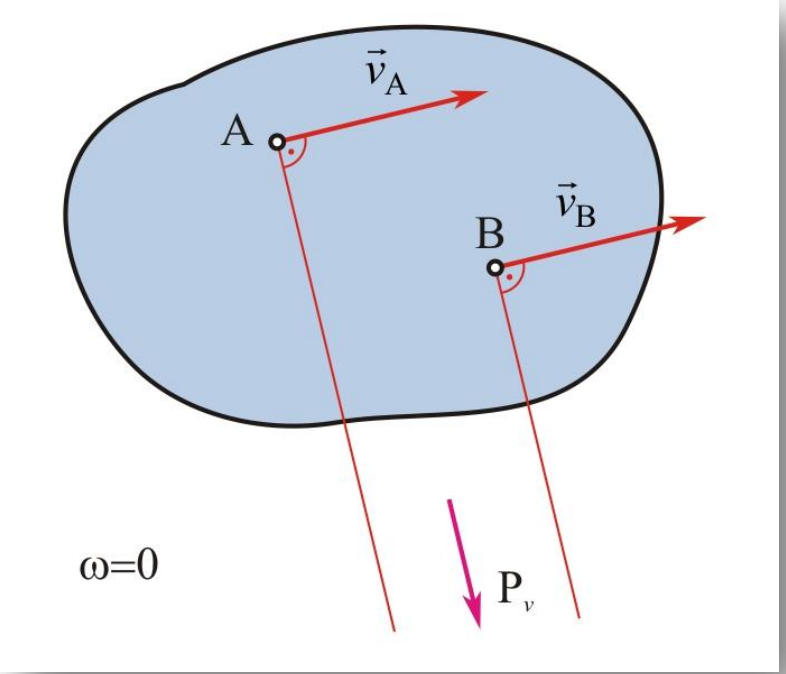


$$v_A = \overline{P_v A}\,\omega$$



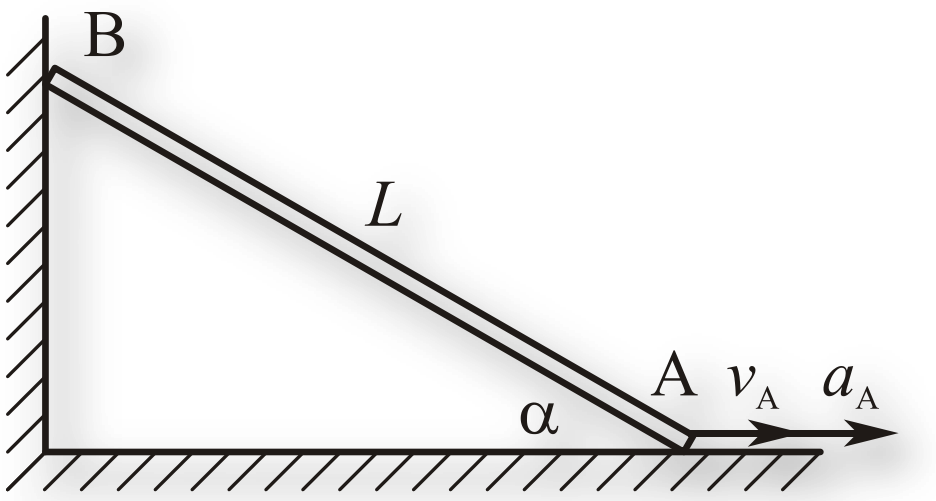
Trenutni pol brzine





Primer

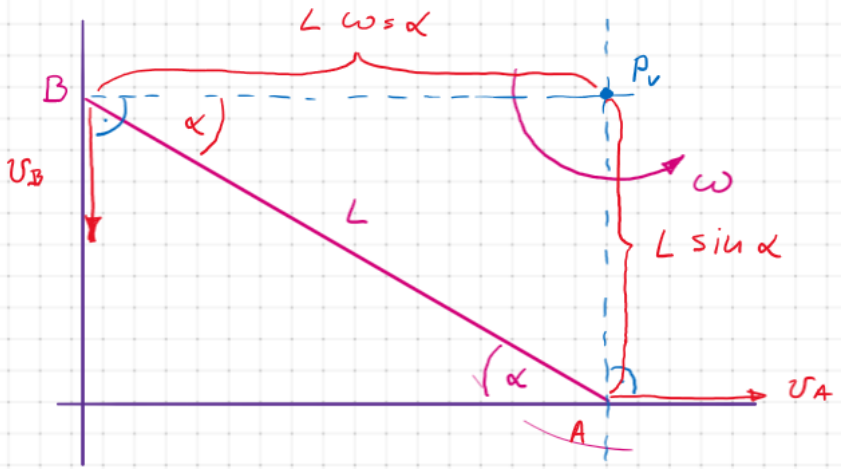
Štap AB, dužine L, oslanja se krajem A o horizontalni pod, a krajem B o vertikalni zid. U položaju u kome štap gradi ugao α sa horizontalom brzina i ubrzanje tačke A iznose v_A i a_A. Odrediti brzinu tačke B i ugaonu brzinu štapa u datom položaju.



v_A, L, α

$\vec{v}_B = ?$

$\omega = ?$

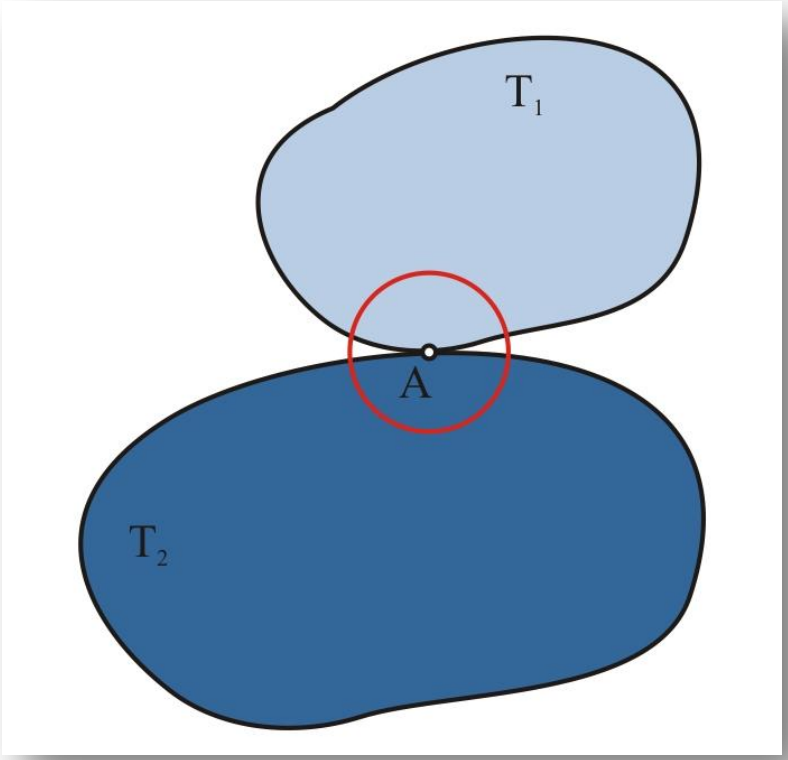


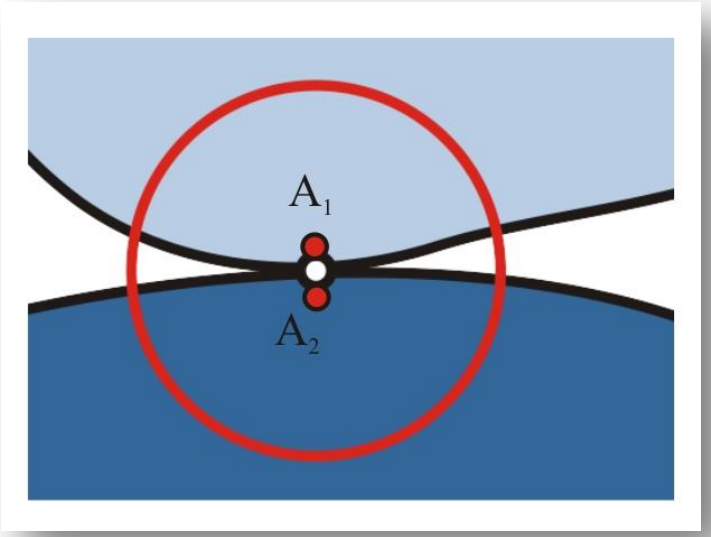
$$v_A = \overline{P_v A} \cdot \omega \rightarrow \omega = \frac{v_A}{\overline{P_v A}} = \frac{v_A}{L \sin\alpha}$$

$$v_B = \overline{P_v B}\,\omega = (L\cos\alpha)\frac{v_A}{L\sin\alpha}$$

$$v_B = v_A \operatorname{ctg}\alpha$$

Kotrljanje sa i bez klizanja

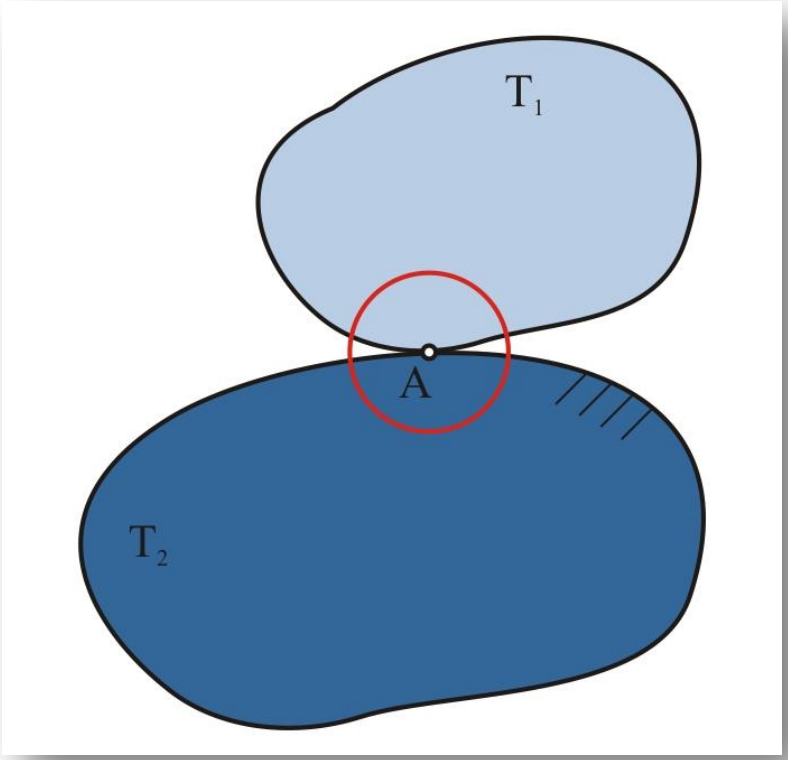


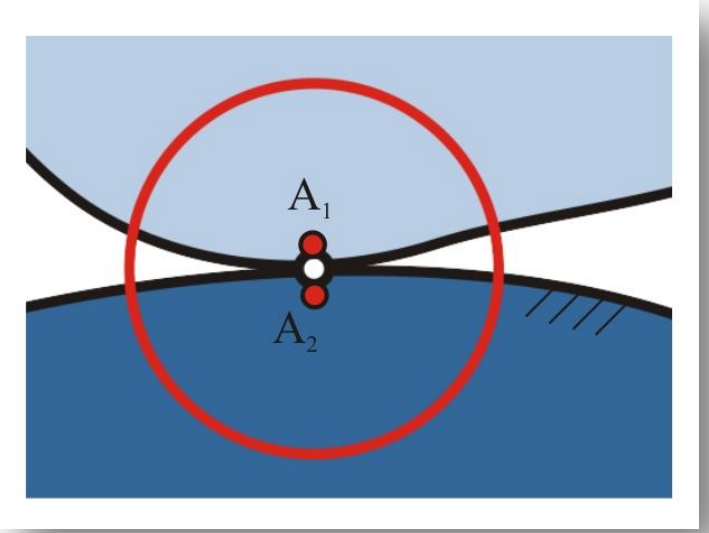


KBK $\vec{v}_{A_1} = \vec{v}_{A_2}$

KSK $\vec{v}_{A_1} \neq \vec{v}_{A_2}$

Kotrljanje sa i bez klizanja po nepokretnoj podlozi

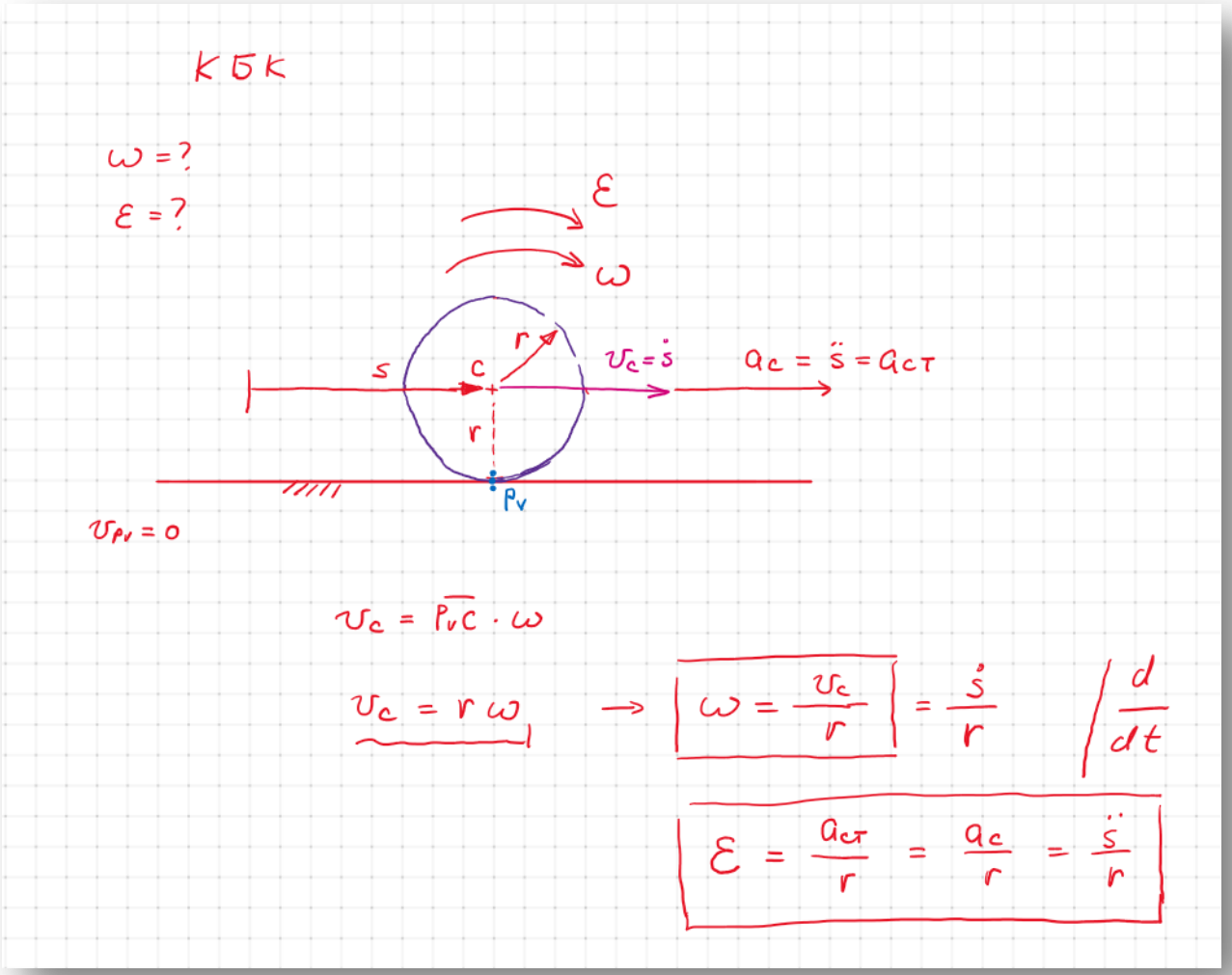




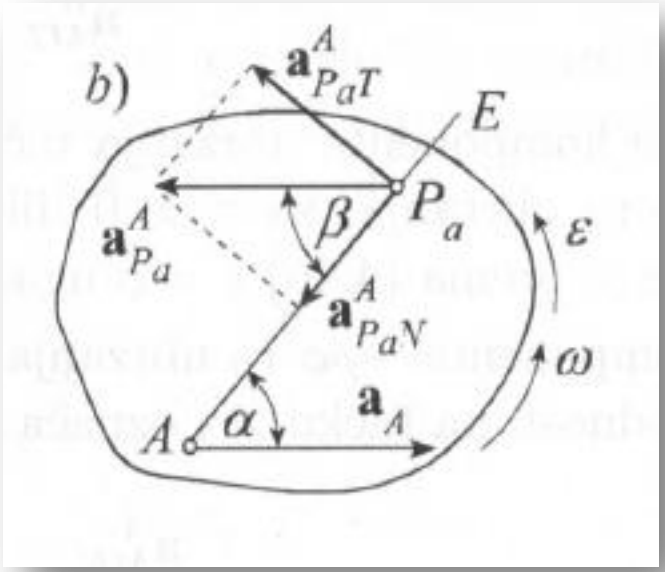
KBK $\vec{v}_{A_1} = \vec{v}_{A_2} = 0$

KSK $\vec{v}_{A_1} \neq \vec{v}_{A_2} = 0$

Primer


КБК
ω = ?
ε = ?
ε
ω
r
s
C
$v_c = \dot{s}$
$a_c = \ddot{s} = a_{cT}$
r
P_v
$v_{P_v} = 0$
$v_c = \overline{P_v C} \cdot \omega$
$v_c = r\omega$
$\omega = \frac{v_c}{r} = \frac{\dot{s}}{r}$
$/\frac{d}{dt}$
$\varepsilon = \frac{a_{cT}}{r} = \frac{a_c}{r} = \frac{\ddot{s}}{r}$

Trenutni pol ubrzanja



1. Odredi se ugao α prema relaciji $\operatorname{tg}\alpha = |\varepsilon|/\omega^2$. Iz tačke A pod uglom α prema vektoru $\mathbf{a}_A$, a u smeru ugaonog ubrzanja ε ravanskog kretanja, povuče se pravac AE (Slika 4.27*b*).
2. Izračuna se dužina

$$AP_a = \frac{a_A}{\sqrt{\varepsilon^2+\omega^4}},$$

i nanese od tačke A na pravac AE čime se definiše položaj tačke P_a. Tako dobijena tačka P_a je pol ubrzanja.

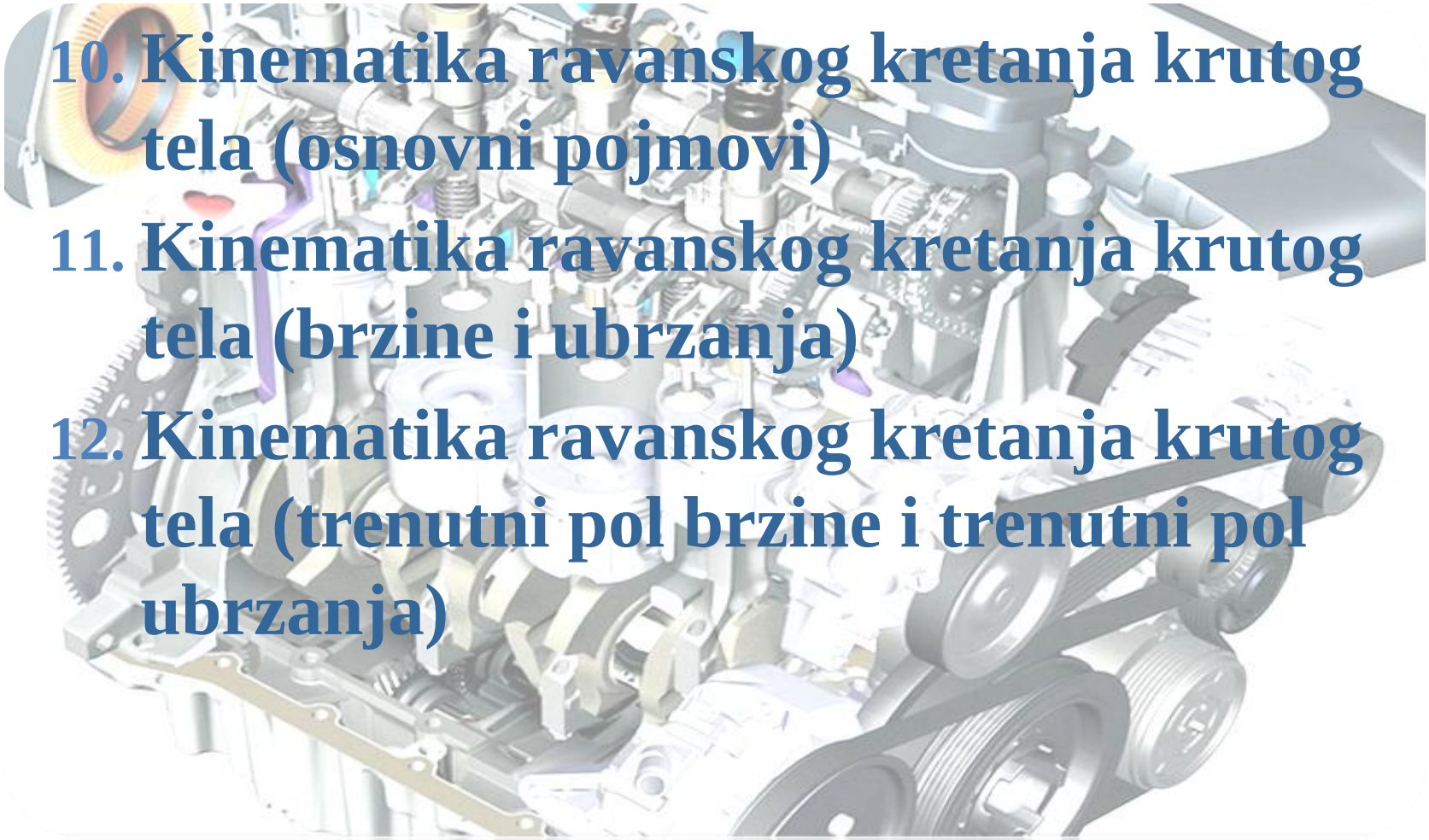
$$\mathbf{a}_{P_a} = \mathbf{a}_A + \mathbf{a}^A_{P_a}.$$

$$\operatorname{tg}\beta = \frac{a^A_{P_aT}}{a^A_{P_aN}} = \frac{\varepsilon}{\omega^2}.$$

$$a^A_{P_a} = AP_a\sqrt{\varepsilon^2+\omega^4} = a_A.$$

$$\mathbf{a}^A_{P_a} = -\mathbf{a}_A$$

Šta smo naučili?



10. **Kinematika ravanskog kretanja krutog tela (osnovni pojmovi)**
11. **Kinematika ravanskog kretanja krutog tela (brzine i ubrzanja)**
12. **Kinematika ravanskog kretanja krutog tela (trenutni pol brzine i trenutni pol ubrzanja)**

Kinematika
Kinematika krutog tela – 2. deo

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.