Kinematika
Kinematika složenog kretanja tačke

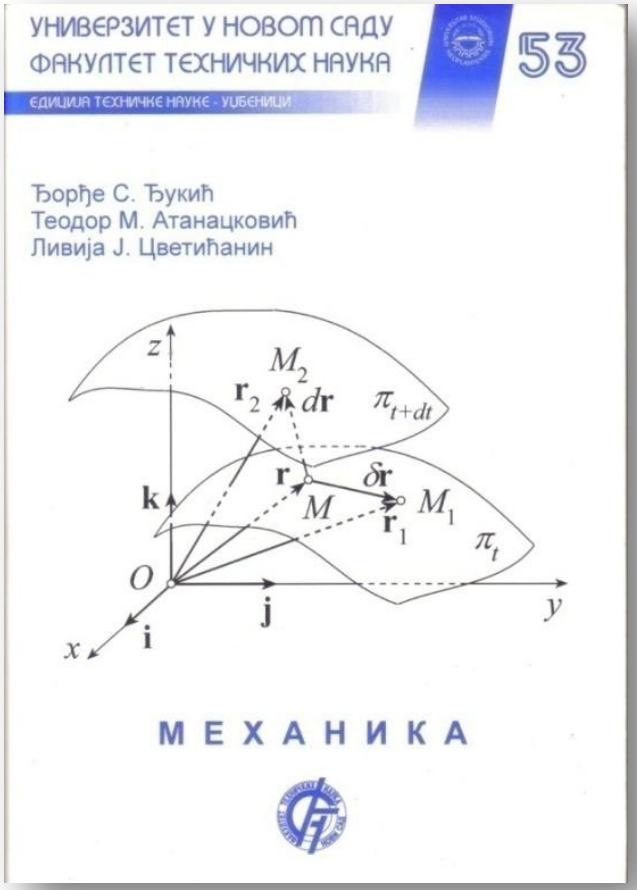
Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.

Literatura

- Đorđe S. Đukić, Teodor M. Atanacković, Livija J. Cvetićanin: Mehanika, Fakultet tehničkih nauka u Novom Sadu, Novi Sad, 2003.

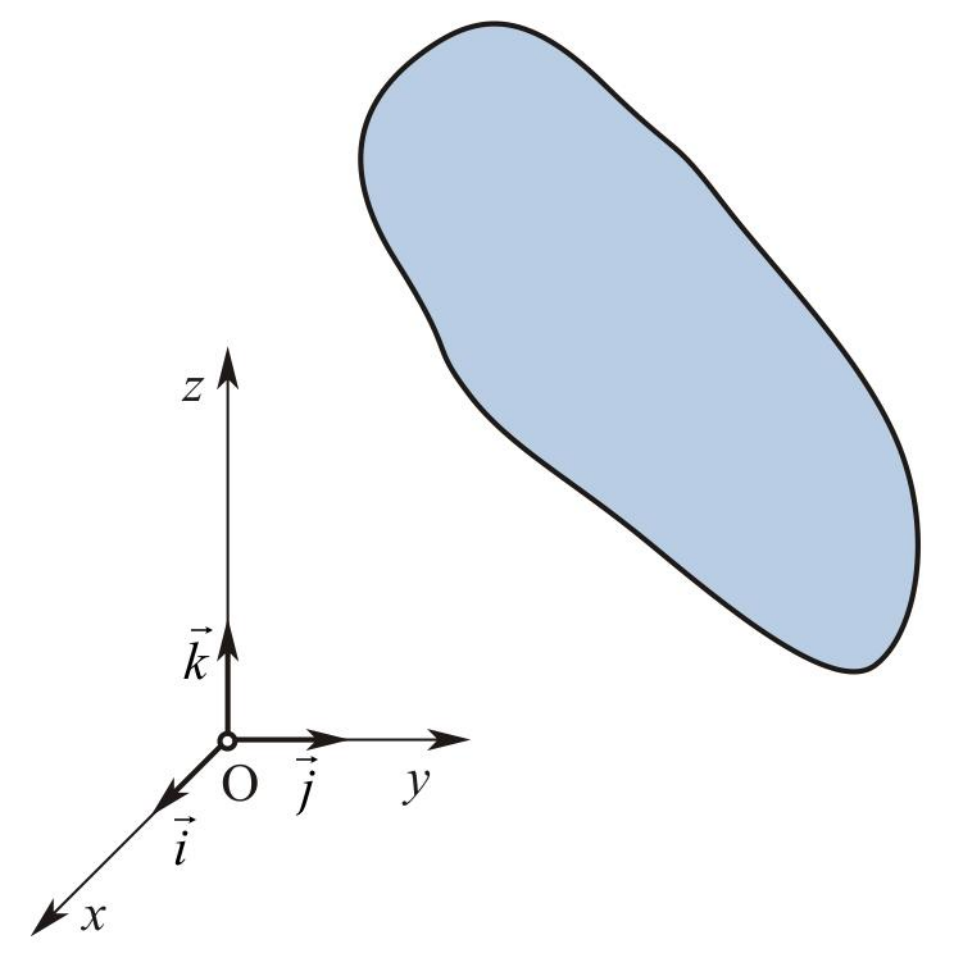


Šta ćemo naučiti?

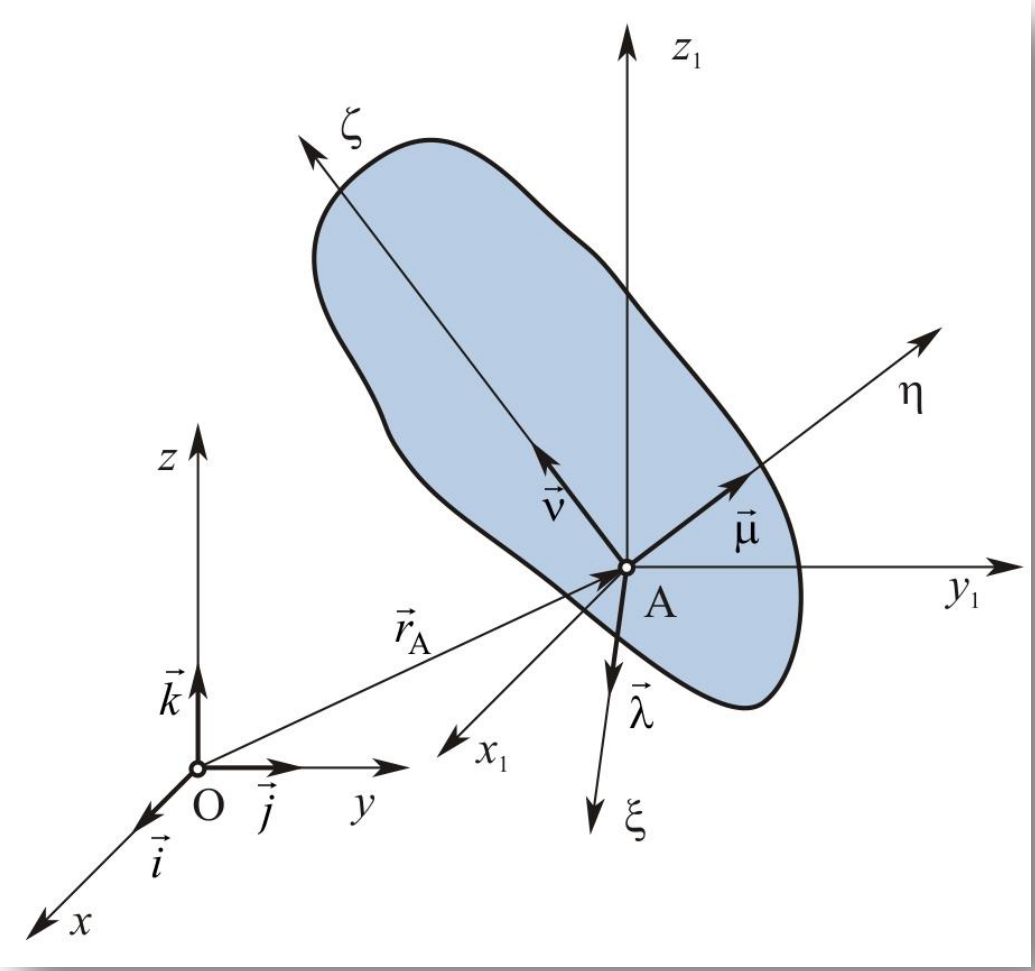
13. **Osnove kinematike opšteg kretanja krutog tela**
14. **Složeno kretanje tačke**
15. **Mehanizmi. Klipni mehanizam. Zglobni četvorougao. Kulisni mehanizam**

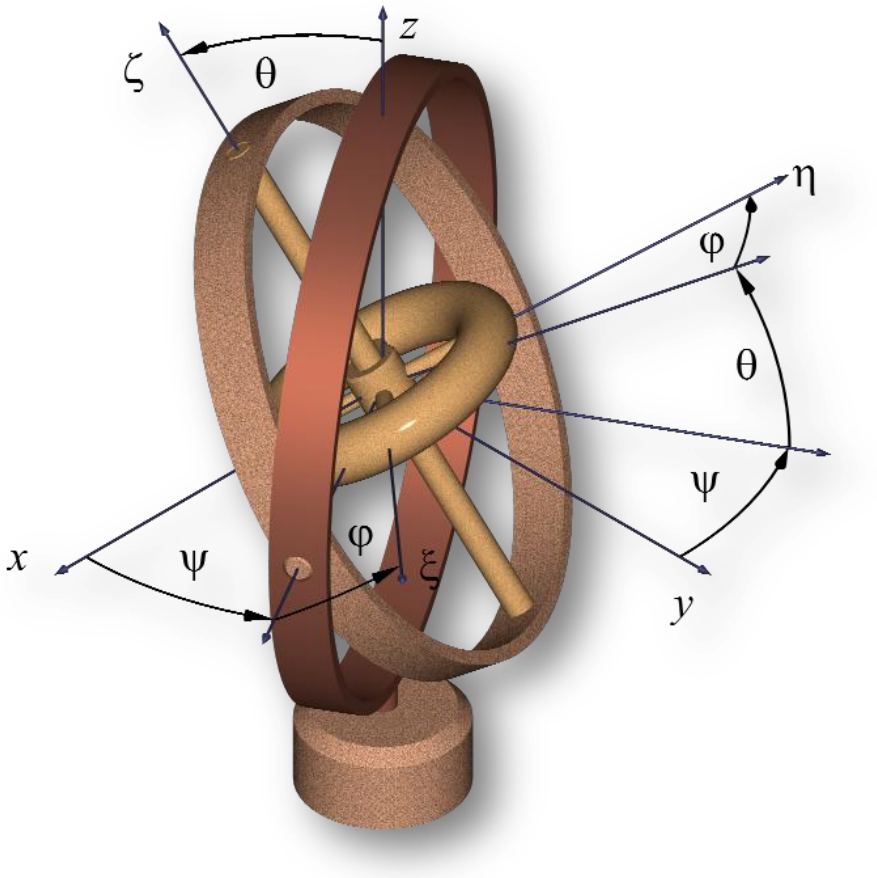
13. Osnove kinematike opšteg kretanja krutog tela

Opšte kretanje krutog tela



Opšte kretanje krutog tela



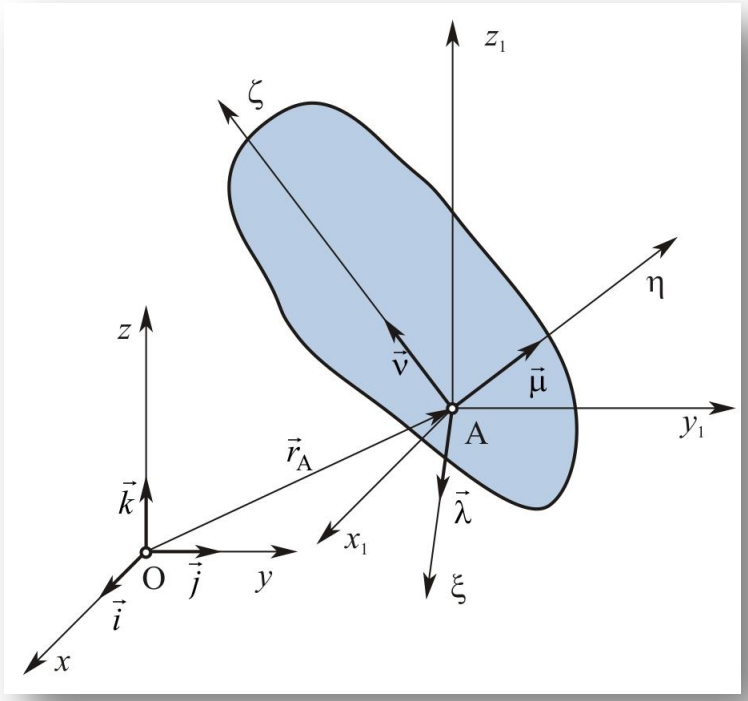


Ojlerovi uglovi:

Uglovi **precesije, nutacije i sopstvene rotacije** :

$$\psi = \psi(t);\ \theta = \theta(t);\ \varphi = \varphi(t)$$

Opšte kretanje krutog tela



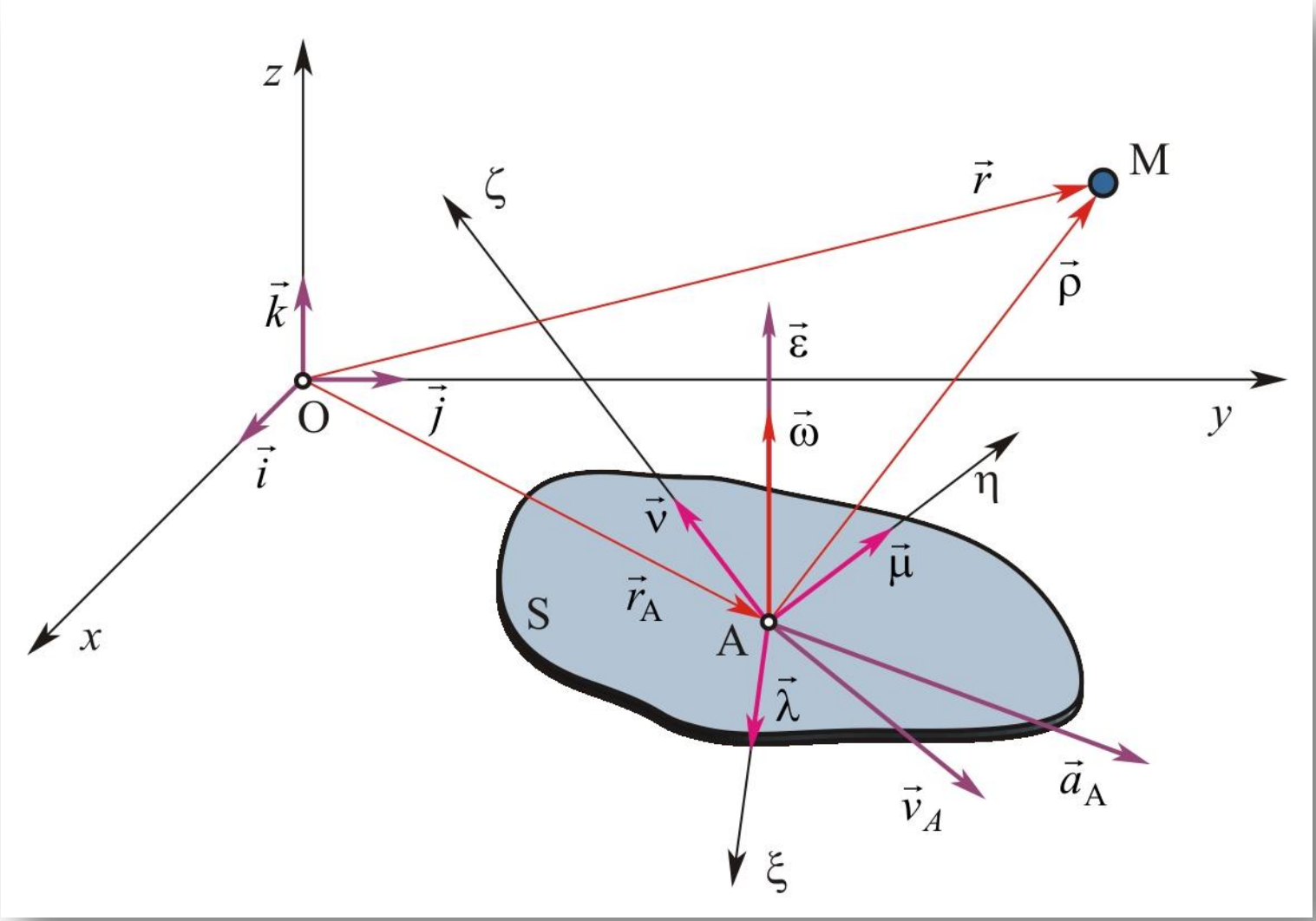
Opšte kretanje krutog tela ima 6 stepeni slobodekretanja

Parametarske jednačine ravanskog kretanja tela:

$$x_A = x_A(t), y_A = y_A(t), z_A = z_A(t)$$
$$\psi = \psi(t), \theta = \theta(t), \varphi = \varphi(t)$$

14. Složeno kretanje tačke

Složeno kretanje tačke

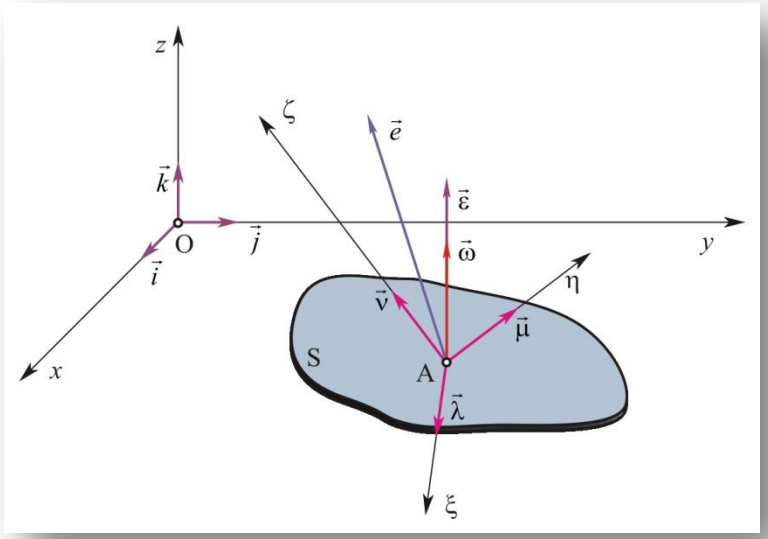


Složeno kretanje tačke

$$\vec{e} = e_\xi\ \vec{\lambda} + e_\eta\ \vec{\mu} + e_\zeta\ \vec{\nu}$$

$$\dot{\vec{e}} = \dot{e}_\xi\ \vec{\lambda} + \dot{e}_\eta\ \vec{\mu} + \dot{e}_\zeta\ \vec{\nu} + e_\xi\ \dot{\vec{\lambda}} + e_\eta\ \dot{\vec{\mu}} + e_\zeta\ \dot{\vec{\nu}}$$

$$\dot{\vec{\lambda}} = \vec{\omega}\times\vec{\lambda}\ ;\ \dot{\vec{\mu}} = \vec{\omega}\times\vec{\mu}\ ;\ \dot{\vec{\nu}} = \vec{\omega}\times\vec{\nu}$$



$$\dot{\vec{e}} = \dot{e}_\xi\ \vec{\lambda} + \dot{e}_\eta\ \vec{\mu} + \dot{e}_\zeta\ \vec{\nu} + \vec{\omega}\times e_\xi\vec{\lambda} + \vec{\omega}\times e_\eta\vec{\mu} + \vec{\omega}\times e_\zeta\vec{\nu}$$

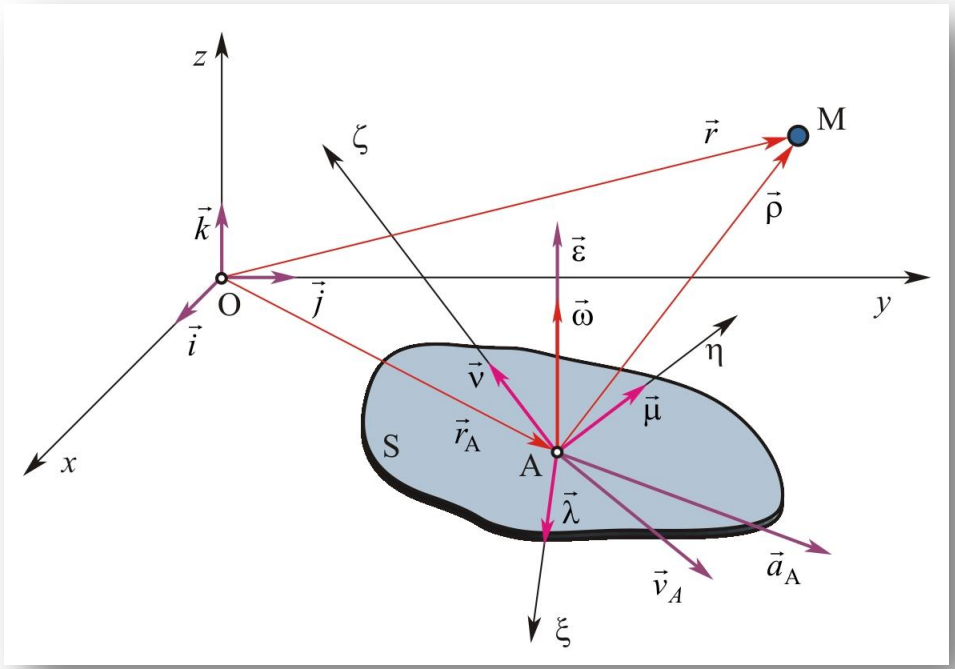
$$\dot{\vec{e}} = \dot{e}_\xi\ \vec{\lambda} + \dot{e}_\eta\ \vec{\mu} + \dot{e}_\zeta\ \vec{\nu} + \vec{\omega}\times\left(e_\xi\vec{\lambda} + e_\eta\vec{\mu} + e_\zeta\vec{\nu}\right)$$

$$\dot{\vec{e}} = \dot{e}_\xi\ \vec{\lambda} + \dot{e}_\eta\ \vec{\mu} + \dot{e}_\zeta\ \vec{\nu} + \vec{\omega}\times\vec{e}$$

$$\dot{\vec{e}} = \dot{\vec{e}}_r + \vec{\omega}\times\vec{e}$$

$$\dot{\vec{e}}_r = \dot{e}_\xi\ \vec{\lambda} + \dot{e}_\eta\ \vec{\mu} + \dot{e}_\zeta\ \vec{\nu}$$

Složeno kretanje tačke – položaj



$$\vec{r} = \vec{r}_A + \vec{\rho}$$

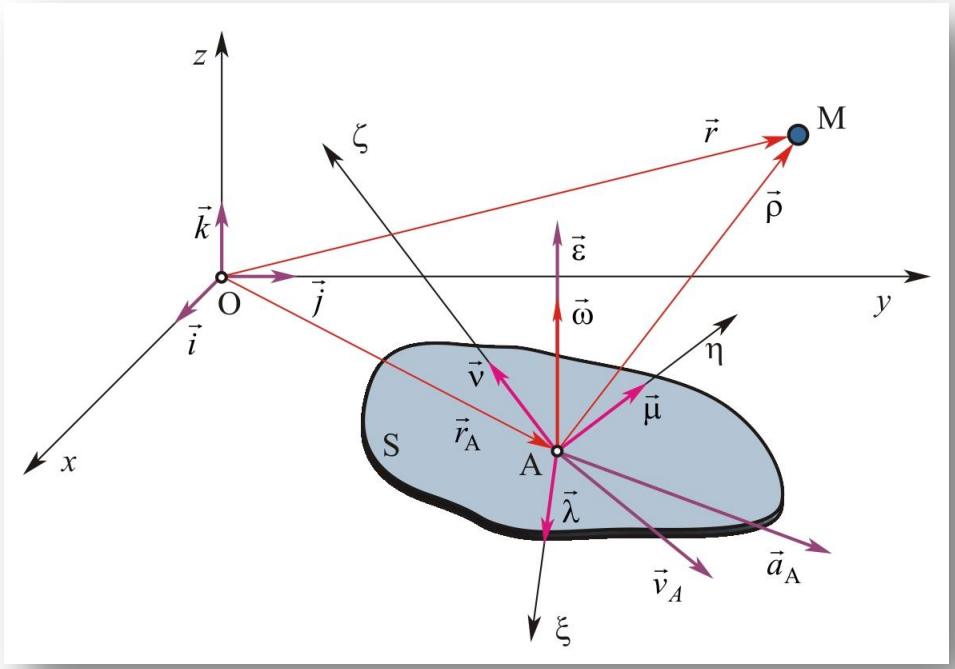
$$\vec{\rho} = \xi\,\vec{\lambda} + \eta\,\vec{\mu} + \zeta\,\vec{\nu}$$

Složeno kretanje tačke – apsolutna brzina

$$\vec{r} = \vec{r}_A + \vec{\rho}$$

$$\vec{\rho} = \xi\,\vec{\lambda} + \eta\,\vec{\mu} + \zeta\,\vec{\nu}$$

$$\vec{v} = \dot{\vec{r}} = \dot{\vec{r}}_A + \dot{\vec{\rho}}$$



$$\vec{v} = \vec{v}_A + \dot{\vec{\rho}}_r + \vec{\omega}\times\vec{\rho}$$

$$\vec{v}_r = \dot{\vec{\rho}}_r = \dot{\xi}\,\vec{\lambda} + \dot{\eta}\,\vec{\mu} + \dot{\zeta}\,\vec{\nu}$$

$$\vec{v}_p = \vec{v}_A + \vec{\omega}\times\vec{\rho}$$

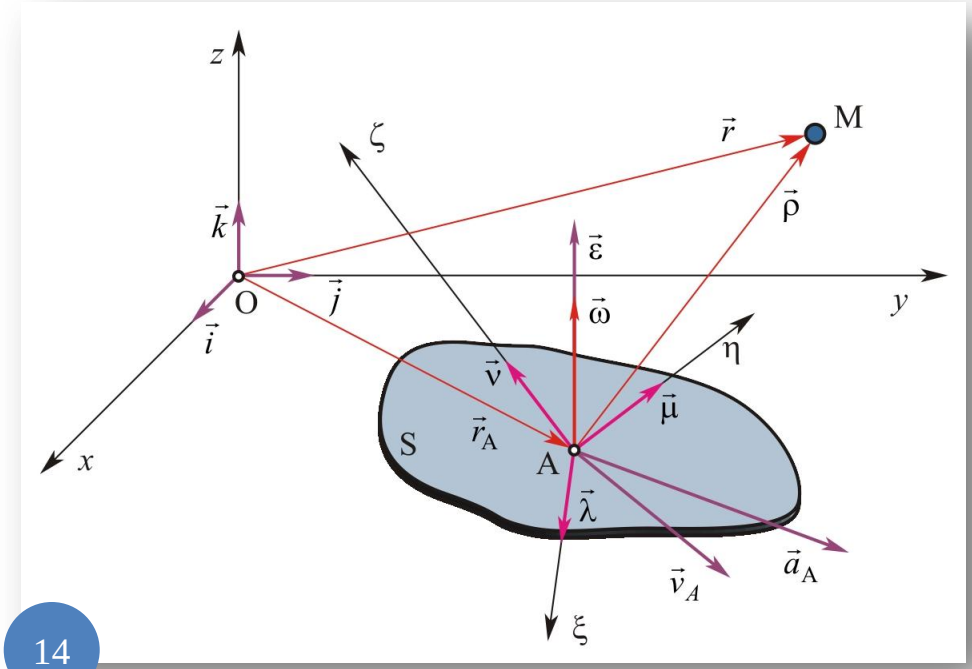
$$\vec{v} = \vec{v}_p + \vec{v}_r$$

Složeno kretanje tačke – apsolutno ubrzanje

$$\vec{r} = \vec{r}_A + \vec{\rho}$$

$$\vec{\rho} = \xi\,\vec{\lambda} + \eta\,\vec{\mu} + \zeta\,\vec{\nu}$$

$$\vec{v} = \vec{v}_p + \vec{v}_r$$



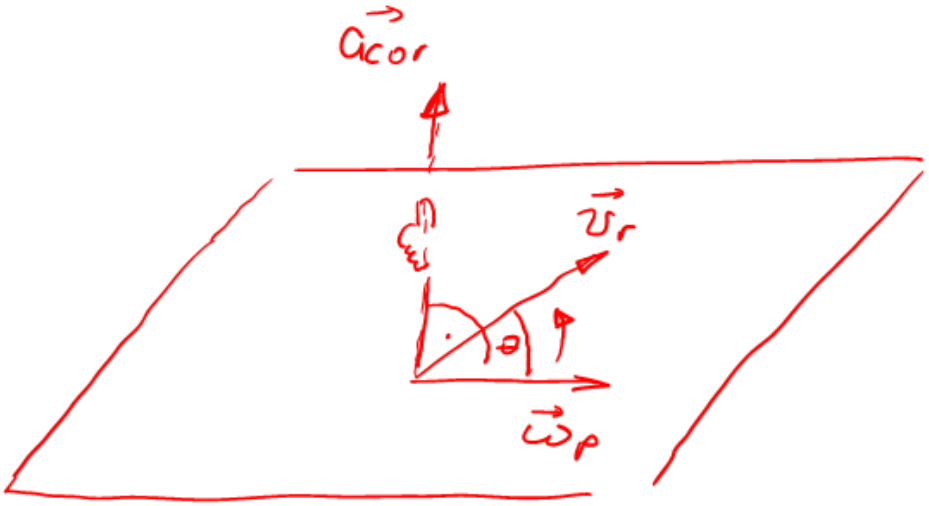
$$\vec{v} = \vec{v}_A + \vec{\omega}\times\vec{\rho} + \vec{v}_r$$

$$\vec{v}_r = \dot{\xi}\,\vec{\lambda} + \dot{\eta}\,\vec{\mu} + \dot{\zeta}\,\vec{\nu}$$

$$\vec{a} = \dot{\vec{v}} = \dot{\vec{v}}_A + \dot{\vec{\omega}}\times\vec{\rho} + \vec{\omega}\times\dot{\vec{\rho}} + \dot{\vec{v}}_r$$

$$\vec{a}_{cor} = \vec{a}_c = 2\vec{\omega} \times \vec{v}_r = 2\vec{\omega}_p \times \vec{v}_r$$

$$a_{cor} = 2\,\omega_p\, v_r \sin\alpha$$



$\vec{a}_{cor} = 0$

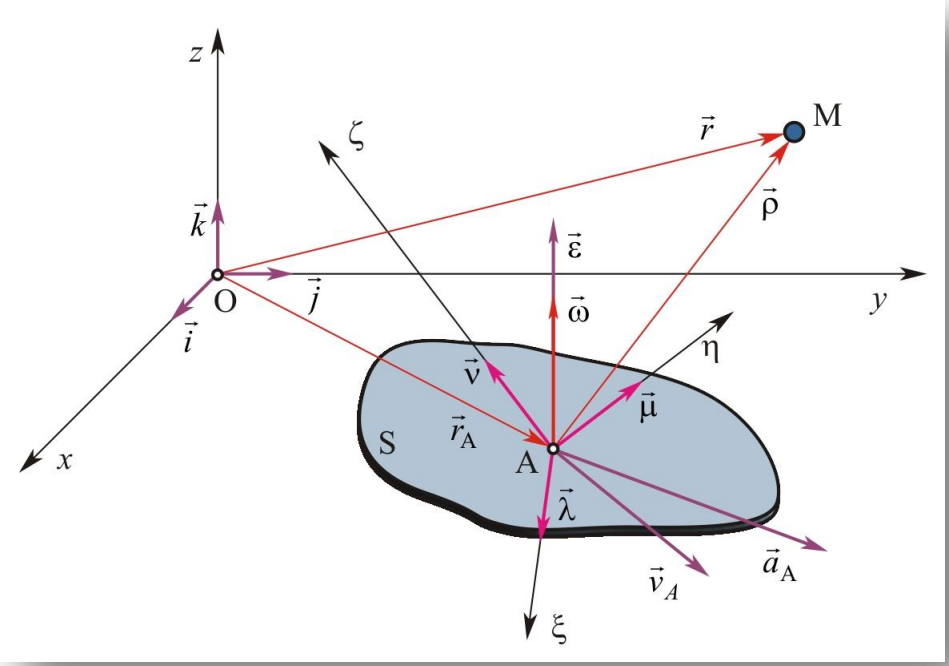
1° $\vec{\omega}_p = 0$

ТРАНСЛ. КР.

2° $\vec{v}_r = 0$

3° $\vec{\omega}_p \parallel \vec{v}_r$

Složeno kretanje tačke – apsolutno ubrzanje



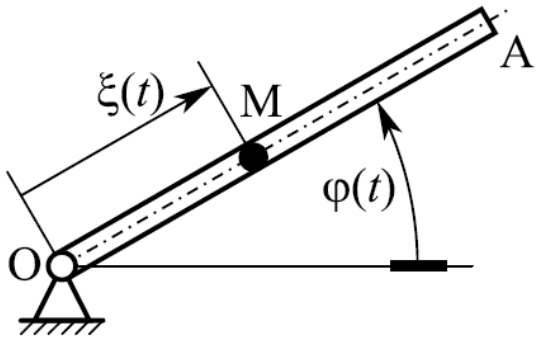
$$\vec{a} = \vec{a}_p + \vec{a}_r + \vec{a}_c$$

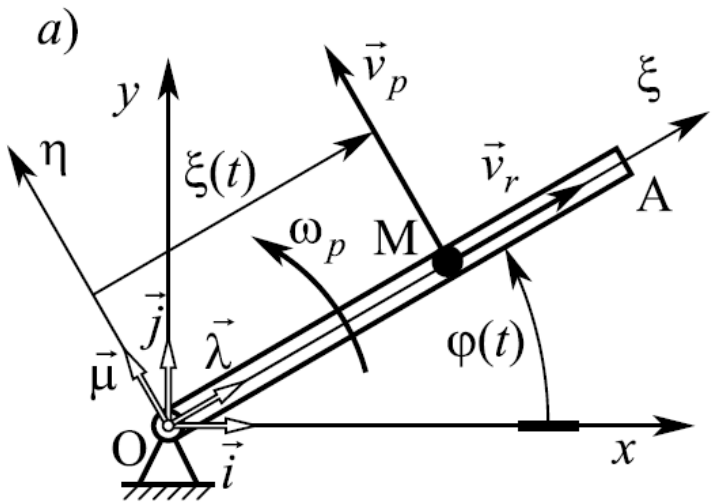
$$\vec{a}_p = \vec{a}_A + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho})$$
$$\vec{a}_r = \ddot{\xi}\,\vec{\lambda} + \ddot{\eta}\,\vec{\mu} + \ddot{\zeta}\,\vec{\nu}$$
$$\vec{a}_c = 2\vec{\omega}\times\vec{v}_r$$

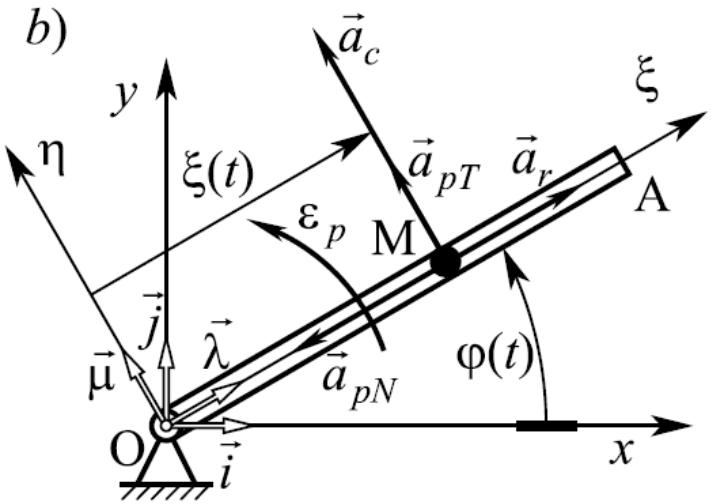
$$\vec{a} = \dot{\vec{v}}_A + \dot{\vec{\omega}}\times\vec{\rho} + \vec{\omega}\times\left(\dot{\vec{\rho}}_r + \vec{\omega}\times\vec{\rho}\right) + \ddot{\xi}\,\vec{\lambda} + \ddot{\eta}\,\vec{\mu} + \ddot{\zeta}\,\vec{\nu} + \vec{\omega}\times\vec{v}_r$$
$$\vec{a} = \vec{a}_A + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times\vec{v}_r + \vec{\omega}\times(\vec{\omega}\times\vec{\rho}) + \ddot{\xi}\,\vec{\lambda} + \ddot{\eta}\,\vec{\mu} + \ddot{\zeta}\,\vec{\nu} + \vec{\omega}\times\vec{v}_r$$
$$\vec{a} = \vec{a}_A + \vec{\varepsilon}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho}) + \ddot{\xi}\,\vec{\lambda} + \ddot{\eta}\,\vec{\mu} + \ddot{\zeta}\,\vec{\nu} + 2\vec{\omega}\times\vec{v}_r$$

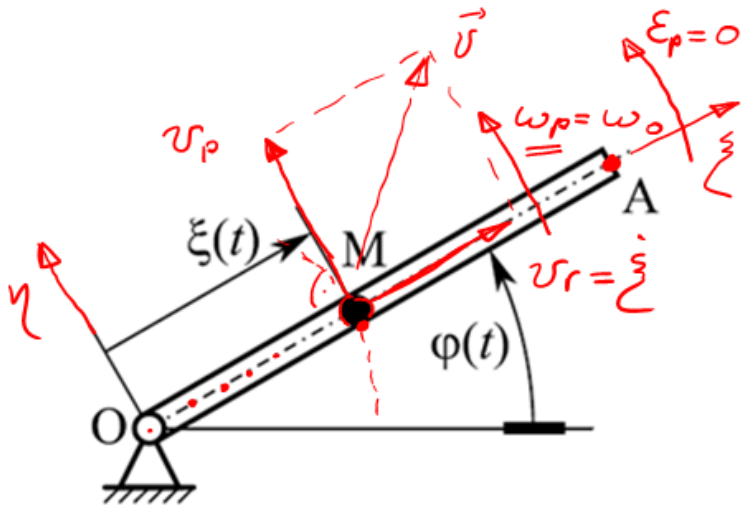
Primer

Zadatak 2.28 Pravolinijska cev OA, dužine L, se obrće oko nepokretne ose po zakonu $\varphi(t) = \omega_0 t[\text{rad}]$. U cevi se kreće kuglica M po zakonu $\xi(t) = \frac{L}{T} t[\text{m}]$. Odrediti apsolutnu brzinu i apsolutno ubrzanje kuglice u trenutku T.




a)
y
η
ξ(t)
$\vec{v}_p$
ξ
$\vec{v}_r$
A
ω_p
M
$\vec{j}$
$\vec{\lambda}$
$\vec{\mu}$
φ(t)
O
$\vec{i}$
x


b)
$\vec{a}_c$
y
ξ
η
ξ(t)
$\vec{a}_{pT}$
$\vec{a}_r$
A
ε_p
M
$\vec{j}$
$\vec{\lambda}$
$\vec{\mu}$
$\vec{a}_{pN}$
φ(t)
O
$\vec{i}$
x



$\varphi(t) = \omega_0 t$, $\xi(t) = \frac{L}{T} t$

$\dot{\varphi}(t) = \omega_p(t) = \omega_0$, $\dot{\xi}(t) = \frac{L}{T} = const$

$= const$

$\ddot{\varphi}(t) = \varepsilon_p(t) = 0$, $\ddot{\xi}(t) = 0$

ПРЕН. КРЕТ. → ЧЕВ → обртање око неп. осе (O) → φ, $\omega_p = \dot{\varphi}$, $\varepsilon_p = \ddot{\varphi}$

РЕЛ. КРЕТ. → у односу на ЧЕВ → ПРАВОЛ. → ξ, $v_r = \dot{\xi}$, $a_r = \ddot{\xi}$

$\varphi(t) = \omega_0 t$

$\xi(t) = \frac{L}{T} t$

$\vec{v}_M(T) = \vec{v}(T) = ?$

$\vec{a}_M(T) = \vec{a}(T) = ?$

$\vec{v} = \vec{v}_p + \vec{v}_r$

$v_r = \dot{\xi} = \frac{L}{T} = const$

$v_p = \overline{OM}\,\omega_p = \xi \cdot \omega_p = \xi(t) \cdot \omega_0$

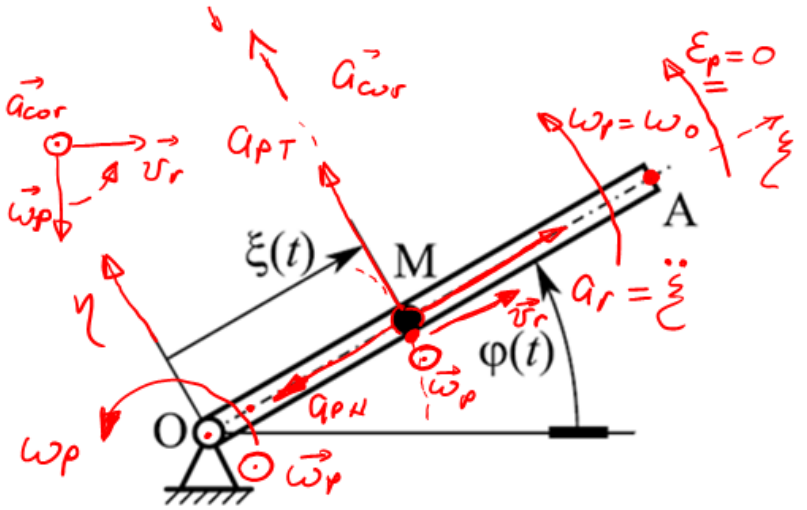
$v_p(T) = \xi(T) \cdot \omega_0 = \left(\frac{L}{T} \cdot T\right) \omega_0 = L \cdot \omega_0$

„T"

ξ : $v_\xi = v_r = \frac{L}{T}$

η : $v_\eta = v_p = L\omega_0$

$\rightarrow v_{(T)} = \sqrt{v_\xi^2 + v_\eta^2} = \sqrt{\left|\frac{L}{T}\right|^2 + |L\omega_0|^2}$



$\varphi(t) = \omega_0 t$, $\xi(t) = \frac{L}{T} t$

$\dot{\varphi}(t) = \omega_\rho(t) = \omega_0$, $\dot{\xi}(t) = \frac{L}{T} = const$

$\ddot{\varphi}(t) = \varepsilon_\rho(t) = 0$, $\ddot{\xi}(t) = 0$

ПРЕН. КРЕТ. → ЧЕВ → обртање око нег. осе (O) → φ, $\omega_\rho = \dot{\varphi}$, $\varepsilon_\rho = \ddot{\varphi}$

РЕЛ. КРЕТ. → У ОДНОСУ НА ЧЕВ → ПРАВОЛ. → ξ, $v_r = \dot{\xi}$, $a_r = \ddot{\xi}$

$\vec{a}_M = \vec{a} = \vec{a}_\rho + \vec{a}_r + \vec{a}_{cor}$

$\vec{a} = \vec{a}_{\rho T} + \vec{a}_{\rho N} + \vec{a}_r + \vec{a}_{cor}$

$a_r = \ddot{\xi} = 0$; $a_{\rho T} = \overline{OM}\,\varepsilon_\rho = 0$

$a_{\rho N} = \overline{OM}\,\omega_\rho^2 = \xi(t) \cdot \omega_0^2$, $a_{\rho N}(T) = \left(\frac{L}{T} \cdot T\right)\omega_0^2 = L\omega_0^2$

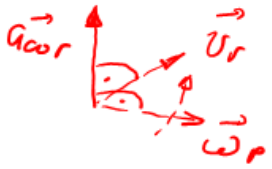
$\vec{a}_{cor} = 2\vec{\omega}_\rho \times \vec{v}_r$

$a_{cor} = 2\omega_\rho v_r \sin(90^\circ) = 2\omega_0 \cdot \frac{L}{T} = const$

$\xi: \; a_\xi = -a_{\rho N} + a_r = -L\omega_0^2$

$\eta: \; a_\eta = a_{\rho T} + a_{cor} = 2\omega_0 \frac{L}{T}$

$\Rightarrow a = \sqrt{a_\xi^2 + a_\eta^2}$



22

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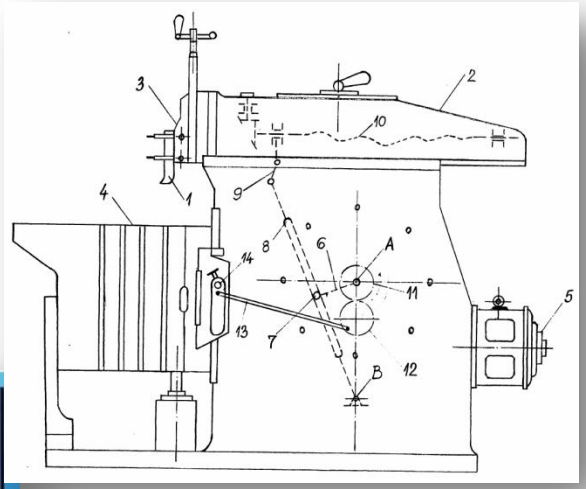
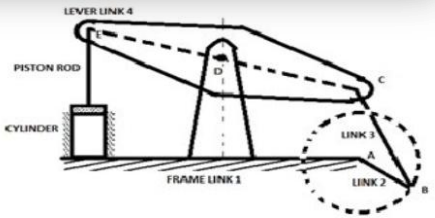
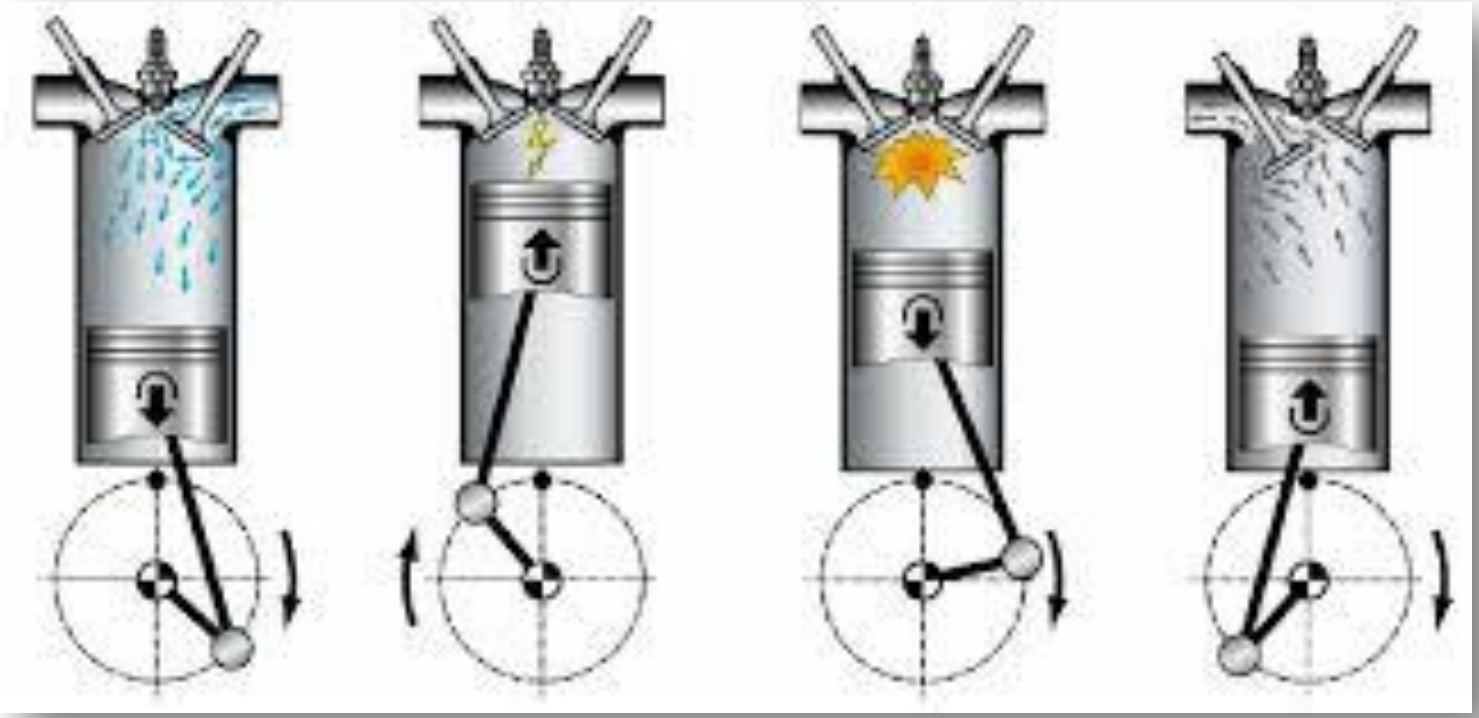
15. Mehanizmi. Klipni mehanizam. Zglobni četvorougao. Kulisni mehanizam

Mehanizmi


1
2
3
4
5
6
7
8
9
10
11
12
13
14
A
B


LEVER LINK 4
PISTON ROD
CYLINDER
FRAME LINK 1
LINK 3
LINK 2
A
B
C
D

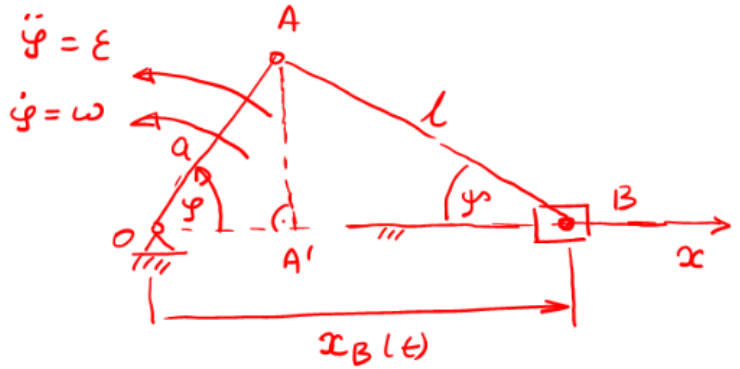
Klipni mehanizam



Klipni mehanizam

$\varphi(t) = ?$

$x_B(t) = ?$



$\overline{AA'} = a \sin\varphi = \ell \sin\psi$

$\sin\psi = \frac{a}{\ell} \sin\varphi$

$\cos\psi = \sqrt{1 - \sin^2\psi}$

$\cos\psi = \sqrt{1 - \left(\frac{a}{\ell}\right)^2 \sin^2\varphi}$

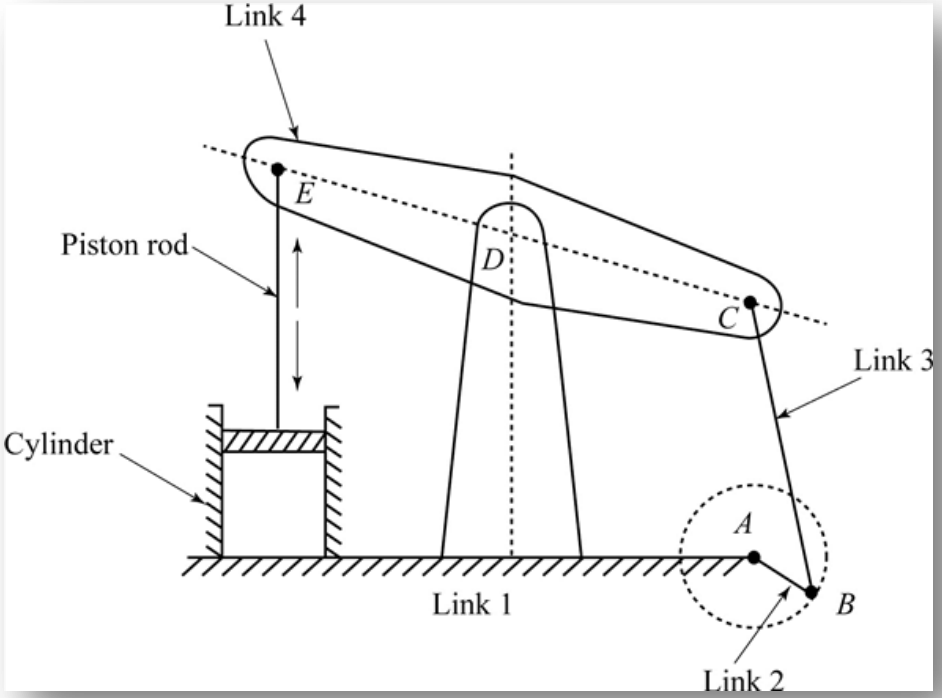
$x_B(t) = a\cos\varphi(t) + \ell\cos\psi(t)$

$x_B(t) = a\cos\varphi(t) + \ell \cdot \sqrt{1 - \left(\frac{a}{\ell}\right)^2 \sin^2\varphi(t)}$

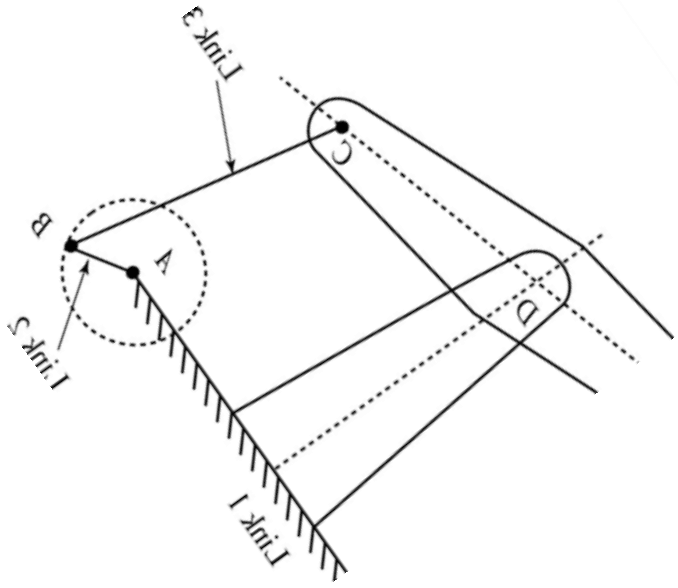
$v_B(t) = \dot{x}_B(t) = -a\dot{\varphi}_{(t)}\sin\varphi_{(t)} + \ell \frac{}{2\sqrt{}}$

$a_B(t) = \ddot{x}_B(t) = -a\ddot{\varphi}\sin\varphi - a\dot{\varphi}^2\cos\varphi +$

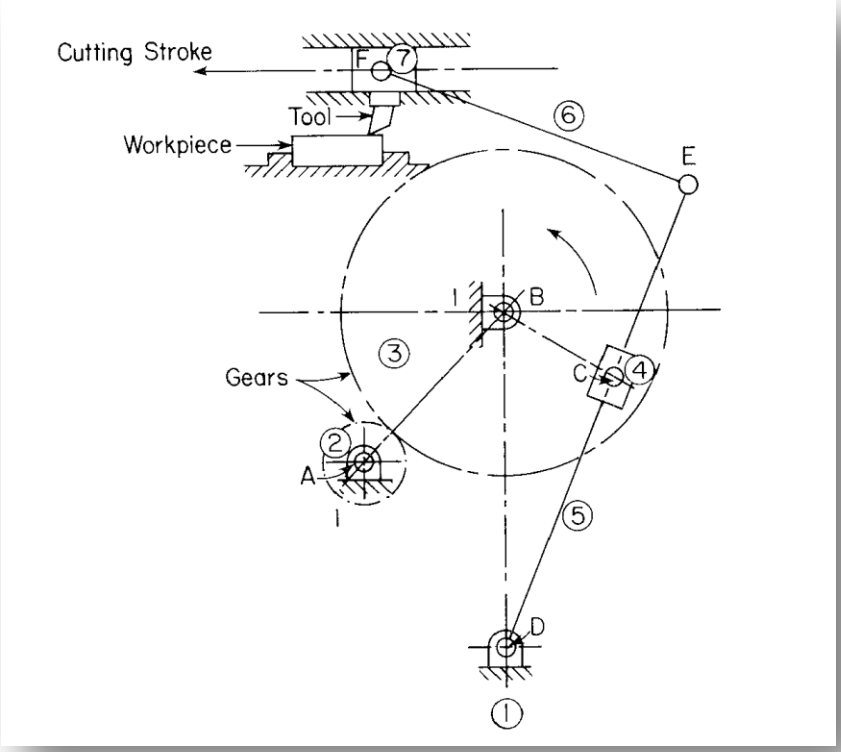
Zglobni četvorougao



Zglobni četvorougao



Kulisni mehanizam



Šta smo naučili?

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