Kinematika
Kinematika tačke – 2. deo

Kinematika i dinamika

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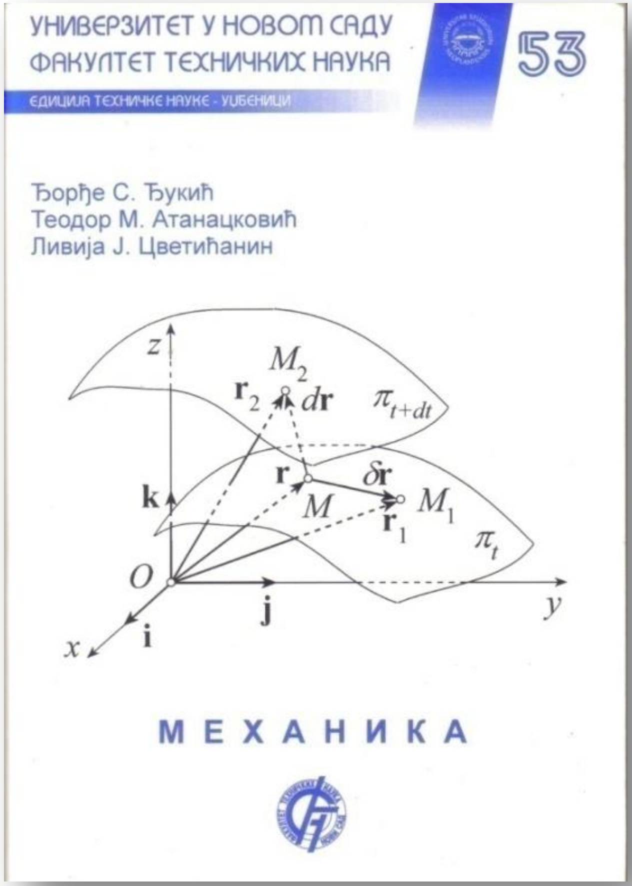
Literatura

- Đorđe S. Đukić, Teodor M. Atanacković, Livija J. Cvetićanin: Mehanika, Fakultet tehničkih nauka u Novom Sadu, Novi Sad, 2003.

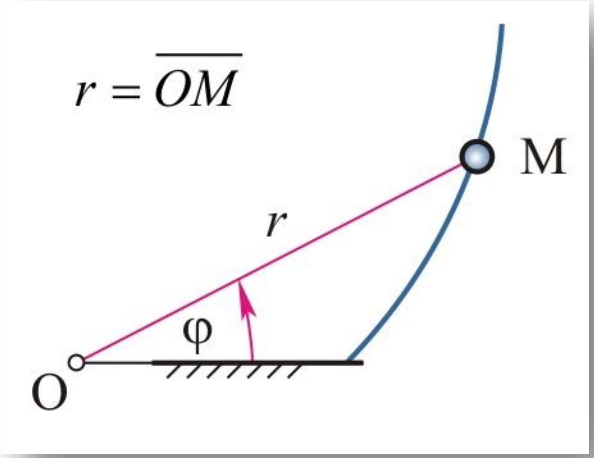


Šta ćemo naučiti?

5. **Kinematika tačke - polarni koordinatni sistem**
6. **Kinematika tačke - prirodni koordinatni sistem**

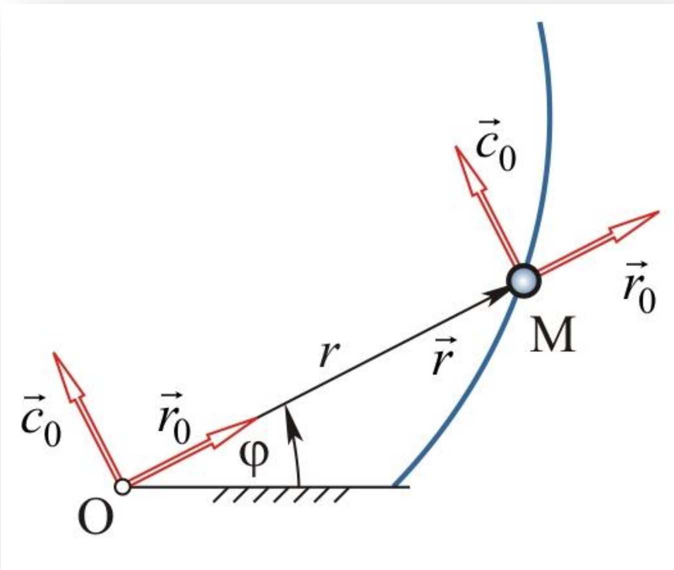
5. Kinematika tačke - polarni koordinatni sistem

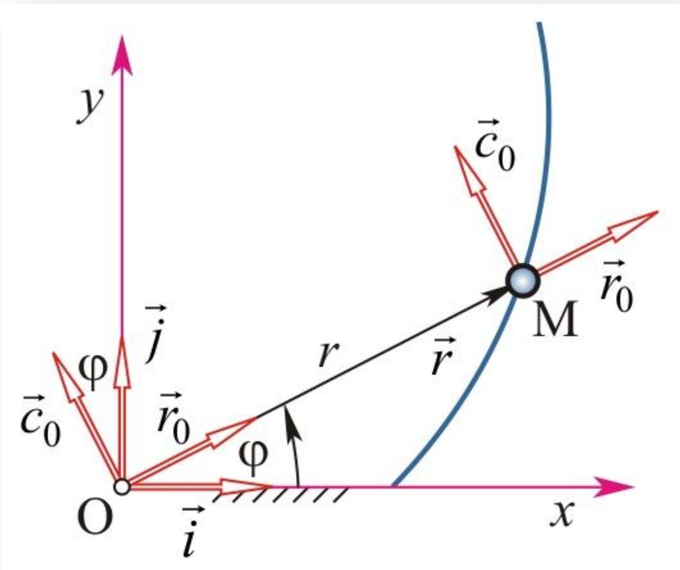
Polarne koordinate – parametarske jednačine kretanja



$$r(t)$$
$$\varphi(t)$$

Polarne koordinate – jedinični vektori





$\vec{r}_0$ - jedinični vektor radijalnog pravca

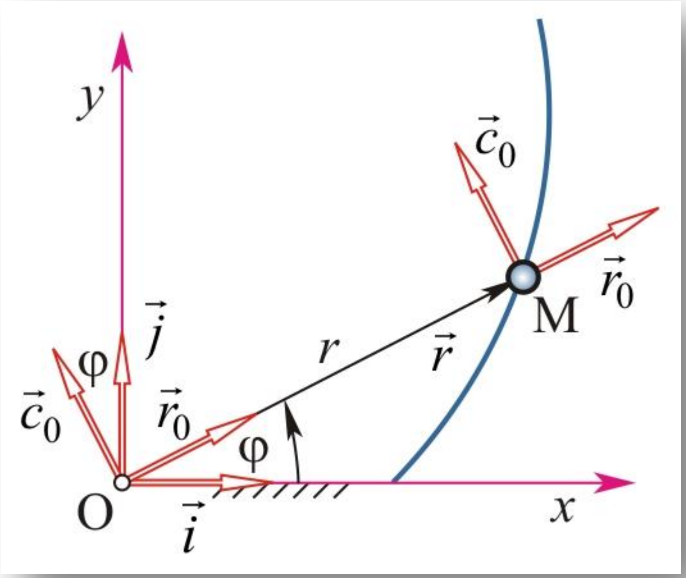
$\vec{c}_0$ - jedinični vektor cirkularnog pravca

Vektor položaja

$$\boxed{\vec{r} = r\,\vec{r}_0}$$

$$\vec{r}_0 = \cos\varphi\,\vec{i} + \sin\varphi\,\vec{j}$$
$$\vec{c}_0 = -\sin\varphi\,\vec{i} + \cos\varphi\,\vec{j}$$
$$\dot{\vec{r}}_0 = -\sin\varphi\,\dot{\varphi}\,\vec{i} + \cos\varphi\,\dot{\varphi}\,\vec{j} = \dot{\varphi}\,\vec{c}_0$$
$$\dot{\vec{c}}_0 = -\cos\varphi\,\dot{\varphi}\,\vec{i} - \sin\varphi\,\dot{\varphi}\,\vec{j} = -\dot{\varphi}\,\vec{r}_0$$

Polarne koordinate – veza sa Dekartovim koordinatama



$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

$$r = \sqrt{x^2 + y^2}$$
$$\tan\varphi = \frac{y}{x}$$

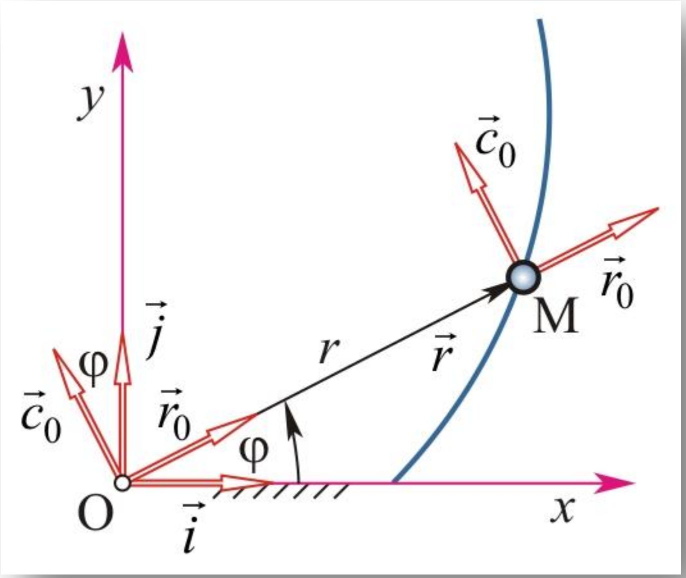
Polarne koordinate – brzina

Vektor položaja

$$\vec{r} = r\,\vec{r}_0$$

$$\dot{\vec{r}}_0 = \dot{\varphi}\,\vec{c}_0$$

$$\dot{\vec{c}}_0 = -\dot{\varphi}\,\vec{r}_0$$



Brzina

$$\vec{v} = \dot{\vec{r}} = \frac{d}{dt}(r\,\vec{r}_0) = \dot{r}\,\vec{r}_0 + r\,\dot{\vec{r}}_0$$

$$\vec{v} = \dot{r}\,\vec{r}_0 + r\,\dot{\varphi}\,\vec{c}_0$$

$$\vec{v} = \dot{r}\,\vec{r}_0 + r\,\dot{\varphi}\,\vec{c}_0$$

$$\vec{v} = v_r\,\vec{r}_0 + v_c\,\vec{c}_0$$

$$v_r = \dot{r}$$

$$v_c = r\,\dot{\varphi}$$

$$v = \sqrt{v_r^2 + v_c^2} = \sqrt{\dot{r}^2 + r^2\,\dot{\varphi}^2}$$

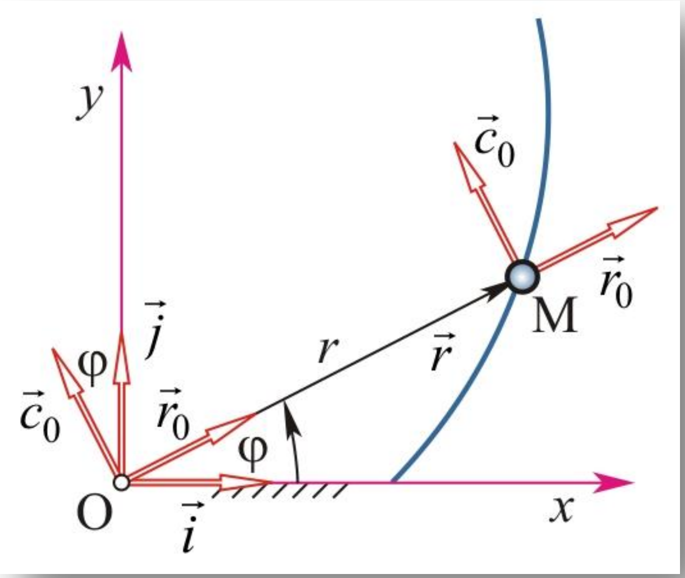
Polarne koordinate – ubrzanje

Brzina

$$\vec{v} = \dot{r}\,\vec{r}_0 + r\,\dot{\varphi}\,\vec{c}_0$$

$$\dot{\vec{r}}_0 = \dot{\varphi}\,\vec{c}_0$$

$$\dot{\vec{c}}_0 = -\dot{\varphi}\,\vec{r}_0$$



Ubrzanje

$$\vec{a} = \dot{\vec{v}} = \ddot{r}\,\vec{r}_0 + \dot{r}\,\dot{\vec{r}}_0 + \dot{r}\,\dot{\varphi}\,\vec{c}_0 + r\,\ddot{\varphi}\,\vec{c}_0 + r\,\dot{\varphi}\,\dot{\vec{c}}_0$$

$$\vec{a} = (\ddot{r} - r\,\dot{\varphi}^2)\,\vec{r}_0 + (r\,\ddot{\varphi} + 2\dot{r}\,\dot{\varphi})\,\vec{c}_0$$

$$\vec{a} = a_r\,\vec{r}_0 + a_c\,\vec{c}_0$$

$$a_r = \ddot{r} - r\,\dot{\varphi}^2$$

$$a_c = r\,\ddot{\varphi} + 2\dot{r}\,\dot{\varphi}$$

$$a = \sqrt{a_r^2 + a_c^2}$$

Primer

Kretanje tačke je opisano parametarskim jednačinama

$$\boxed{r(t) = t,\ \varphi(t) = \frac{\pi}{6}t}$$

a) Odrediti trajektoriju tačke,
b) odrediti brzinu i ubrzanje tačke u proizvoljnom trenutku vremena *t*,
c) odrediti položaj, brzinu i ubrzanje tačke u trenutku $t^*=3s$.

Kretanje tačke je opisano parametarskim jednačinama

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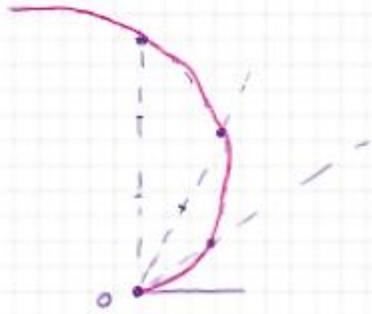


ТРАЈ. = ?

$$\left.\begin{matrix} r(t) \\ \varphi(t) \end{matrix}\right\} \xrightarrow[t]{\text{ЕЛИМ.}} \underline{r(\varphi)} \quad \text{ЛП}$$

$$r = t$$

$$\varphi = \frac{\pi}{6}t \longrightarrow \varphi = \frac{\pi}{6}r \rightarrow \boxed{r = \frac{6}{\pi}\varphi} \quad \text{ЛП}$$

АРХИМЕДОВА СПИРАЛА

$$\varphi = \frac{\pi}{6} \rightarrow r = 1$$

$$\varphi = \frac{\pi}{3} \rightarrow r = 2$$

$$\varphi = \frac{\pi}{2} \rightarrow r = 3$$

OK $\quad t \geq 0$

$$r(t) = t \rightarrow r \geq 0$$

$$\varphi(t) = \frac{\pi}{6}t \rightarrow \varphi \geq 0$$

$$r(t) = t, \quad \varphi(t) = \frac{\pi}{6}t \;\Big\}\; \rightarrow \; \dot r = 1, \quad \dot\varphi = \frac{\pi}{6} \; \rightarrow \; \ddot r = 0, \quad \ddot\varphi = 0$$

$$\left.\begin{aligned} v_r &= \dot r = 1 = \text{const} \\ v_c &= r\dot\varphi = \frac{\pi}{6}t \end{aligned}\right\} \rightarrow v = \sqrt{v_r^2 + v_c^2} = \sqrt{1^2 + \left(\frac{\pi}{6}t\right)^2}$$

$$\left.\begin{aligned} a_r &= \overset{0}{\cancel{\ddot r}} - r\dot\varphi^2 = -\frac{\pi^2}{36}t \\ a_c &= r\underset{0}{\cancel{\ddot\varphi}} + 2\dot r\dot\varphi = \frac{\pi}{3} \end{aligned}\right\} \rightarrow a = \sqrt{a_r^2 + a_c^2} = \sqrt{\frac{\pi^4}{36^2}t^2 + \frac{\pi^2}{9}}$$

$t^* = 3 \rightarrow$

$$v_r(3) = 1$$

$$v_c(3) = \frac{\pi}{6}\cdot 3 = \frac{\pi}{2}$$

$$v(3) = \sqrt{1 + \frac{\pi^2}{4}}$$

$$a_r(3) = -\frac{\pi^2}{36}\cdot 3 = -\frac{\pi^2}{12}$$

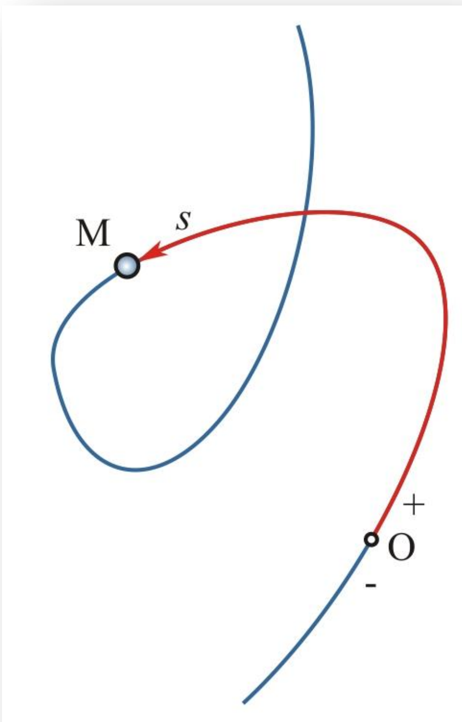
$$a_c(3) = \frac{\pi}{3}$$

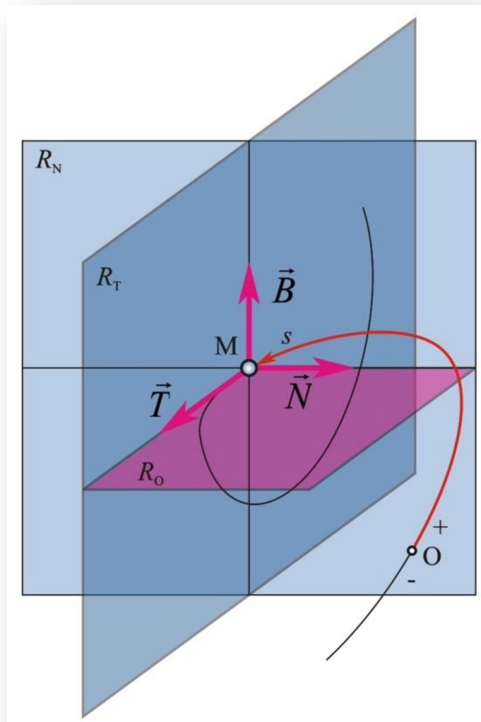
$$a(3) = \sqrt{a_r^2(3) + a_c^2(3)}$$

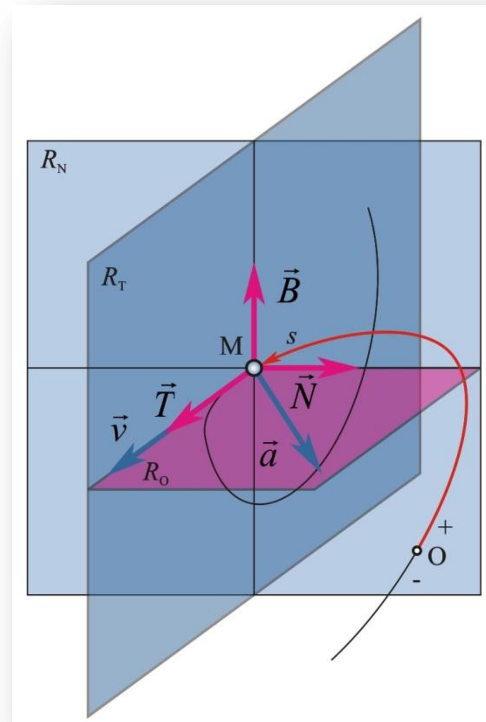
$$=$$

6. Kinematika tačke - prirodni koordinatni sistem

Prirodne koordinate







s – prirodna koordinata
$s(t)$ – zakon kretanja

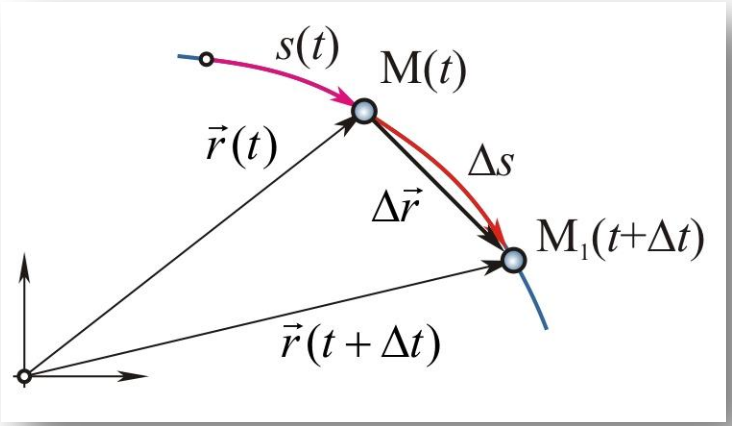
$$\vec{B} = \vec{T} \times \vec{N}$$

R_O – oskulatorna ravan
R_T – tangentna ravan
R_O – normalna ravan

Prirodni koordinatni sistem – brzina

$$\vec{v} = \dot{\vec{r}} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{ds}{dt}\frac{d\vec{r}}{ds}$$



$$\frac{d\vec{r}}{ds} = \left|\frac{d\vec{r}}{ds}\right|\vec{T} = \frac{|d\vec{r}|}{ds}\vec{T}$$

$$\frac{|d\vec{r}|}{ds} = \lim_{\Delta s\to 0}\frac{|\Delta\vec{r}|}{\Delta s} = 1$$

$$\frac{d\vec{r}}{ds} = \vec{T}$$

$$\vec{v} = \frac{ds}{dt}\vec{T} = \dot{s}\,\vec{T} = v_T\vec{T}$$

$$v_T = \dot{s}$$

$$|\vec{v}| = v = |\dot{s}|$$

$$v^2 = \dot{s}^2$$

Prirodni koordinatni sistem – ubrzanje

$$\vec{v} = \dot{s}\,\vec{T}$$

$$\vec{a} = \dot{\vec{v}}$$

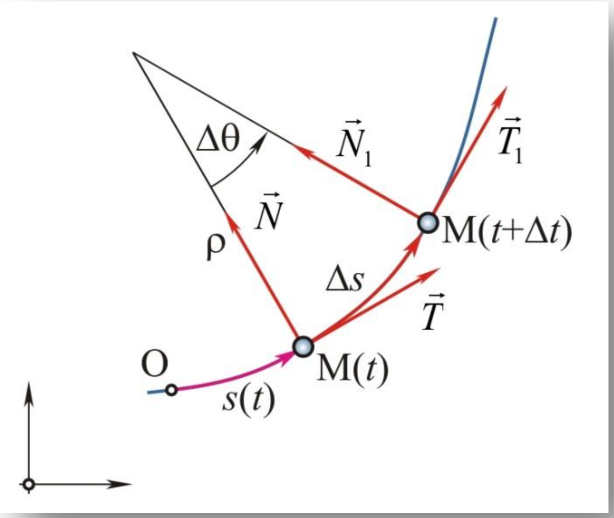
$$\vec{a} = \ddot{s}\,\vec{T} + \dot{s}\,\dot{\vec{T}}$$

$$\dot{\vec{T}} = \frac{d\vec{T}}{dt} = \frac{ds}{dt}\frac{d\theta}{ds}\frac{d\vec{T}}{d\theta}$$

$$\frac{ds}{dt} = \dot{s}$$

$$\frac{d\theta}{ds} = \chi = \frac{1}{R_k}$$

$$\frac{d\vec{T}}{d\theta} = ?$$



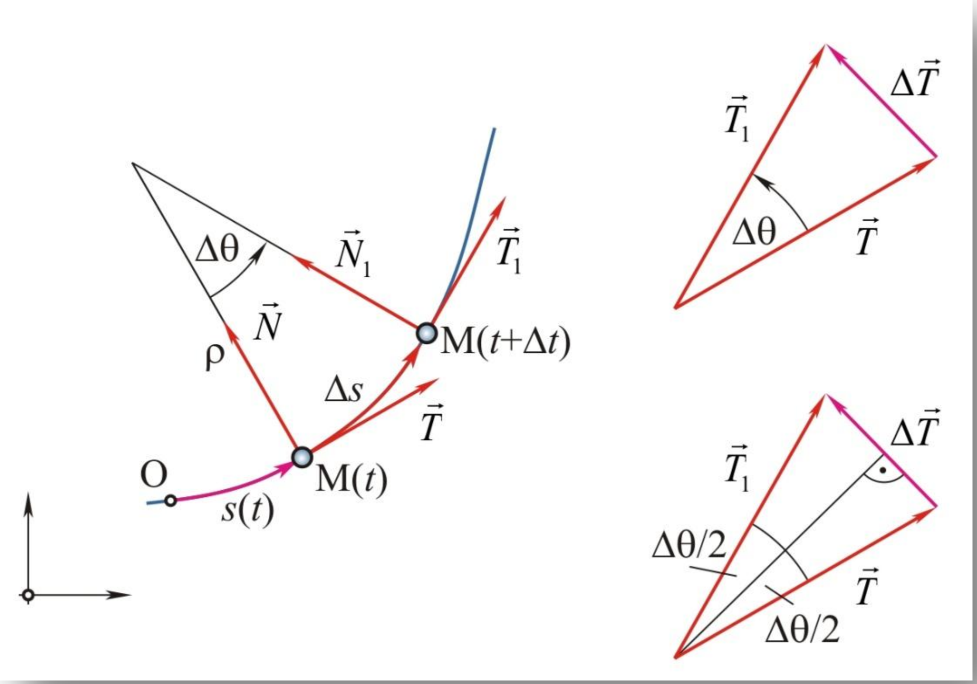
Prirodni koordinatni sistem – ubrzanje

$$\frac{d\vec{T}}{d\theta} = ?$$

$$\vec{T} \cdot \vec{T} = 1$$

$$\frac{d}{d\theta}(\vec{T} \cdot \vec{T}) = \frac{d\vec{T}}{d\theta} \cdot \vec{T} + \vec{T} \cdot \frac{d\vec{T}}{d\theta} = 0$$

$$\frac{d\vec{T}}{d\theta} \perp \vec{T} \rightarrow \frac{d\vec{T}}{d\theta} = \left|\frac{d\vec{T}}{d\theta}\right| \vec{N}$$

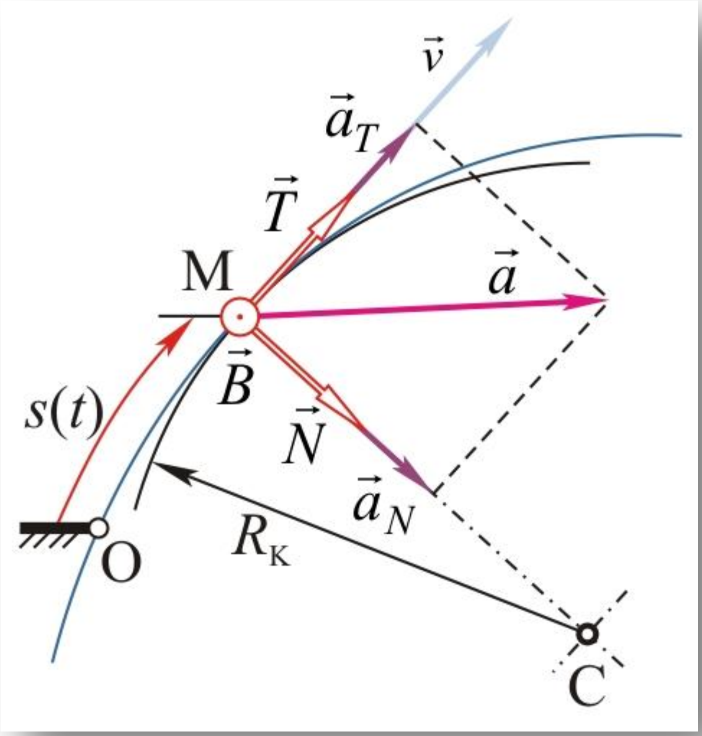


$$\left|\frac{d\vec{T}}{d\theta}\right| = \lim_{\Delta\theta \to 0} \frac{\left|\Delta\vec{T}\right|}{\Delta\theta} = \lim_{\Delta\theta \to 0} \frac{2\sin\dfrac{\Delta\theta}{2}}{\Delta\theta} = 1 \rightarrow \frac{d\vec{T}}{d\theta} = \vec{N}$$

Prirodni koordinatni sistem – ubrzanje

$$\vec{a} = \ddot{s}\,\vec{T} + \dot{s}\,\dot{\vec{T}}$$

$$\dot{\vec{T}} = \frac{ds}{dt}\frac{d\theta}{ds}\frac{d\vec{T}}{d\theta} = \dot{s}\,\frac{1}{R_k}\,\vec{N}$$



$$\vec{a} = \ddot{s}\,\vec{T} + \frac{\dot{s}^2}{R_k}\,\vec{N}$$

$$a_T = \ddot{s}$$

$$a_N = \frac{\dot{s}^2}{R_k} = \frac{v^2}{R_k}$$

$$\vec{a} = a_T\,\vec{T} + a_N\,\vec{N}$$

$$|\vec{a}| = a = \sqrt{a_T^2 + a_N^2}$$

Poluprečnik krivine trajektorije

$$\vec{a} = a_T \vec{T} + a_N \vec{N}$$

$$\dot{s}^2 = v^2 \quad / \frac{d}{dt}$$

$$2\dot{s}\ddot{s} = 2v\dot{v}$$

$$a_T = \ddot{s}$$

$$a_N = \frac{\dot{s}^2}{R_k} = \frac{v^2}{R_k}$$

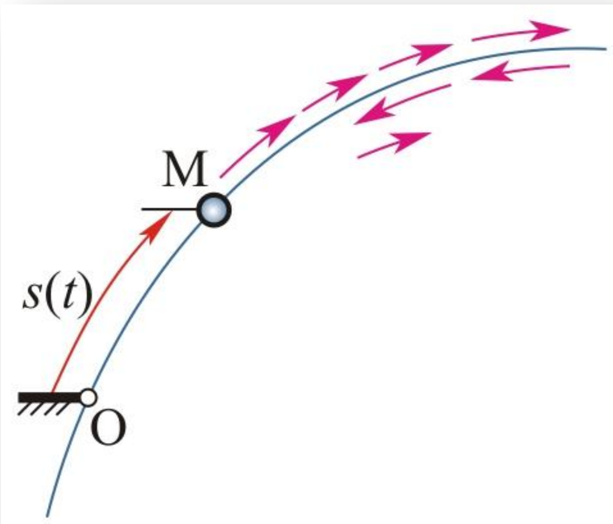
$$a_T^2 = \ddot{s}^2 = \frac{v^2}{\dot{s}^2}\dot{v}^2 = \dot{v}^2$$

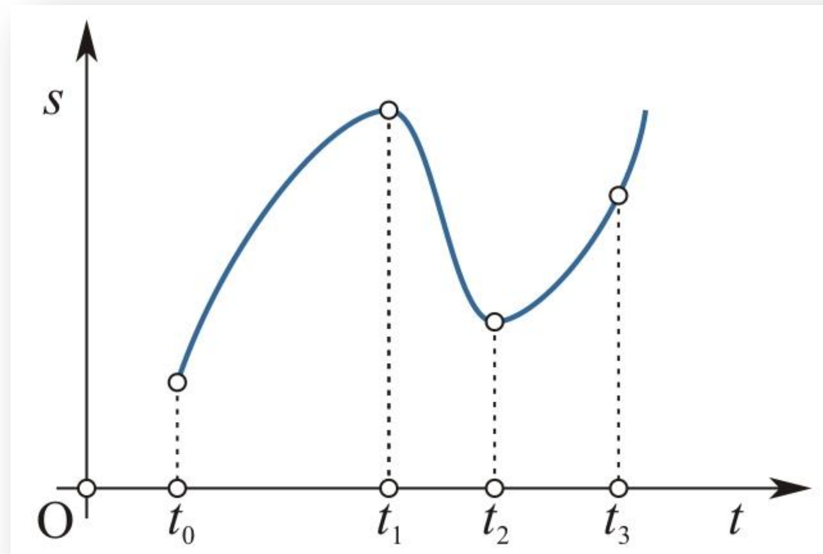
$$a = \sqrt{a_T^2 + a_N^2}$$

$$a_N = \sqrt{a^2 - a_T^2}$$

$$\boxed{R_k = \frac{v^2}{a_N}}$$

Pređeni put





$$s(t) = \pm \int_{t_0}^{t} v(t)dt$$

$$P[t_0, t_3] = P[t_0, t_1] + P[t_1, t_2] + P[t_2, t_3]$$

$$P[t_0, t_3] = |s(t_1) - s(t_0)| + |s(t_2) - s(t_1)| + |s(t_3) - s(t_2)|$$

Пример

Пример 4.4 *Материјална тачка врши кретање по кругу полупречника R сагласно следећим параметарсим једначинама:*

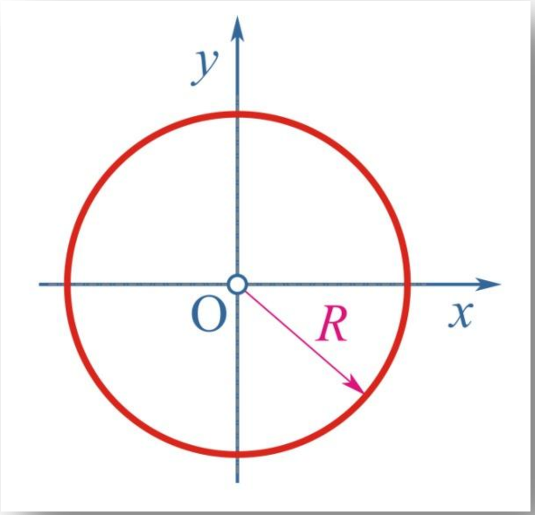
$$x(t) = R\cos\varphi(t); \quad y(t) = R\sin\varphi(t),$$

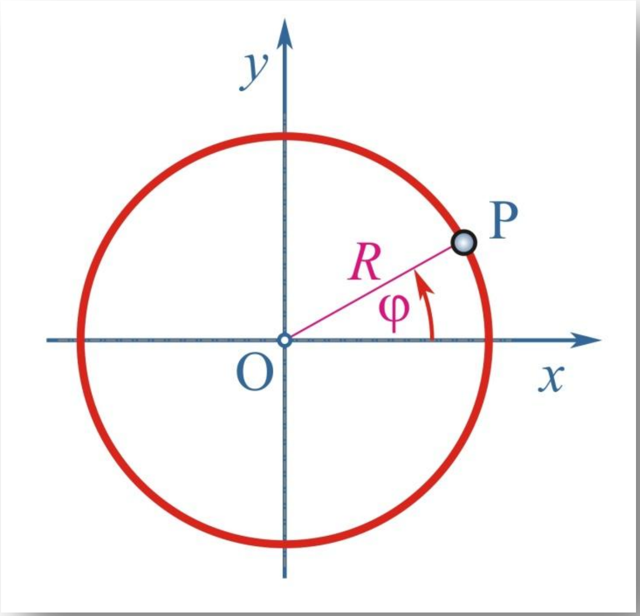
где је $\varphi(t)$ два пута диференцијабилна функција времена t. Одредити векторе брзине и убрзања у призвољном тренутку времена и одредити њихове пројекције на правац тангенте и правац нормале на круг.

Трајекторија

$$\left.\begin{aligned} x &= R\cos\varphi \\ y &= R\sin\varphi \end{aligned}\right\} \rightarrow \left.\begin{aligned} x^2 &= R^2\cos^2\varphi \\ y^2 &= R^2\sin^2\varphi \end{aligned}\right\} \rightarrow x^2 + y^2 = R^2\left(\cos^2\varphi + \sin^2\varphi\right)$$

$$\boxed{x^2 + y^2 = R^2}$$



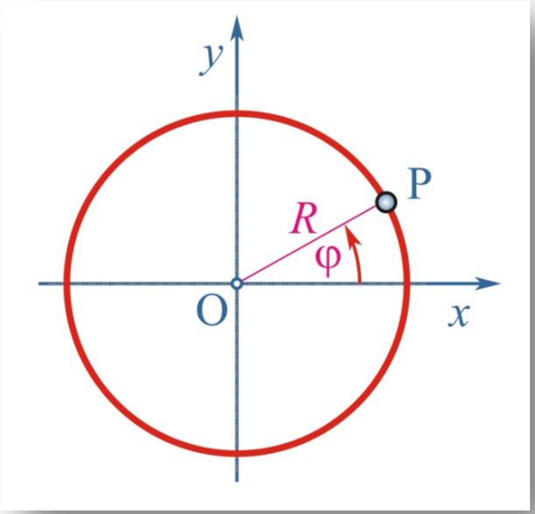


Брзина и убрзање

$$\left.\begin{aligned} x(t) &= R\cos\varphi(t) \\ y(t) &= R\sin\varphi(t) \end{aligned}\right\} \rightarrow \begin{aligned} \dot{x}(t) &= -R\sin\varphi(t)\cdot\dot{\varphi}(t) \\ \dot{y}(t) &= R\cos\varphi(t)\cdot\dot{\varphi}(t) \\ \ddot{x}(t) &= -R\cos\varphi(t)\cdot\dot{\varphi}^2(t) - R\sin\varphi(t)\cdot\ddot{\varphi}(t) \\ \ddot{y}(t) &= -R\sin\varphi(t)\cdot\dot{\varphi}^2(t) + R\cos\varphi(t)\cdot\ddot{\varphi}(t) \end{aligned}$$

$$\vec{v}(t) = \dot{x}(t)\vec{i} + \dot{y}(t)\vec{j}$$

$$\vec{a}(t) = \ddot{x}(t)\vec{i} + \ddot{y}(t)\vec{j}$$



Брзина

$$\vec{v}(t) = \left(-R\sin\varphi(t)\cdot\dot{\varphi}(t)\right)\vec{i} + \left(R\cos\varphi(t)\cdot\dot{\varphi}(t)\right)\vec{j}$$

$$\vec{v}(t) = \left(-\sin\varphi(t)\vec{i} + \cos\varphi(t)\vec{j}\right)R\dot{\varphi}(t)$$

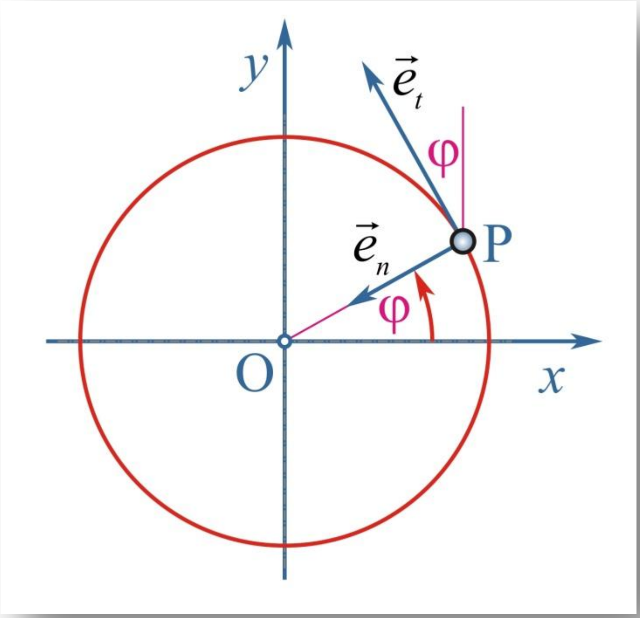
$$\vec{e}_t(t) = -\sin\varphi(t)\vec{i} + \cos\varphi(t)\vec{j}$$

$$\vec{e}_n(t) = -\cos\varphi(t)\vec{i} - \sin\varphi(t)\vec{j}$$

$$\vec{v}(t) = R\dot{\varphi}(t)\cdot\vec{e}_t(t)$$

$$\vec{v}(t) = v_t(t)\cdot\vec{e}_t(t)$$

$$v_t(t) = R\dot{\varphi}(t)$$



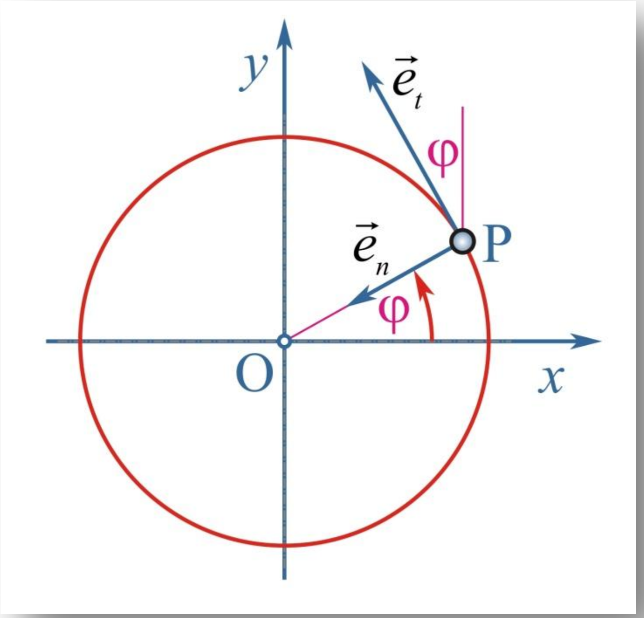
Убрзање

$$\vec{a}(t)=\left(-R\cos\varphi(t)\cdot\dot{\varphi}^2(t)-R\sin\varphi(t)\cdot\ddot{\varphi}(t)\right)\vec{i}+\left(-R\sin\varphi(t)\cdot\dot{\varphi}^2(t)+R\cos\varphi(t)\cdot\ddot{\varphi}(t)\right)\vec{j}$$

$$\vec{a}(t)=\left(-R\sin\varphi(t)\vec{i}+\cos\varphi(t)\vec{j}\right)R\ddot{\varphi}(t)+\left(-\cos\varphi(t)\vec{i}-\sin\varphi(t)\vec{j}\right)R\dot{\varphi}^2(t)$$

$$\vec{e}_t(t)=-\sin\varphi(t)\vec{i}+\cos\varphi(t)\vec{j}$$

$$\vec{e}_n(t)=-\cos\varphi(t)\vec{i}-\sin\varphi(t)\vec{j}$$



$$\vec{a}(t)=\left(R\ddot{\varphi}(t)\right)\cdot\vec{e}_t(t)+\left(R\dot{\varphi}^2(t)\right)\cdot\vec{e}_n(t)$$

$$\vec{a}(t)=a_t(t)\cdot\vec{e}_t(t)+a_n(t)\cdot\vec{e}_n(t)$$

$$a_t(t)=R\ddot{\varphi}(t)$$

$$a_n(t)=R\dot{\varphi}^2(t)$$

Брзина и убрзање

$$\vec{v}(t) = R\dot{\varphi}(t)\cdot\vec{e}_t(t)$$

$$\vec{v}(t) = v_t(t)\cdot\vec{e}_t(t)$$

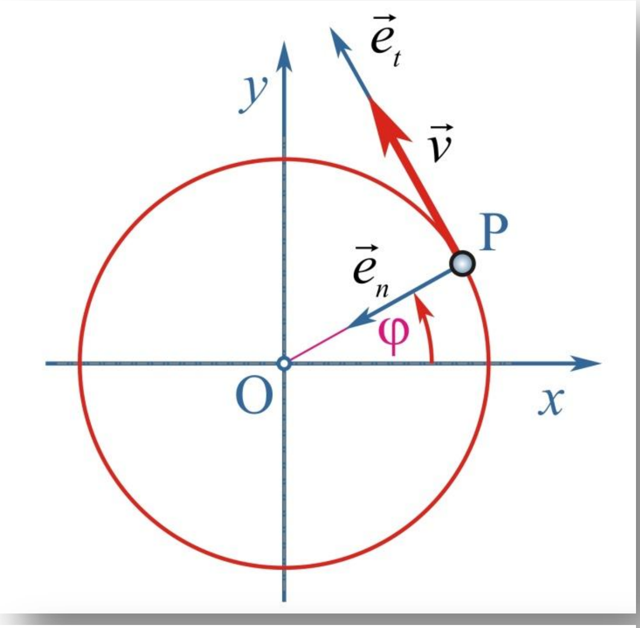
$$v_t(t) = R\dot{\varphi}(t)$$

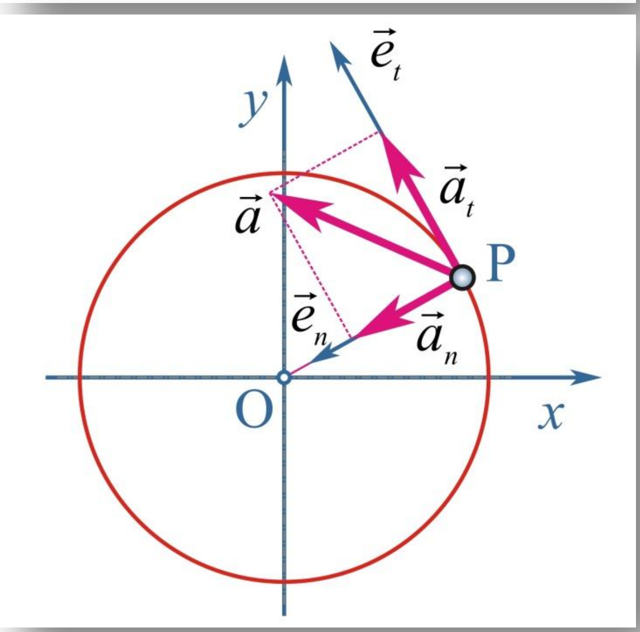
$$\vec{a}(t) = \left(R\ddot{\varphi}(t)\right)\cdot\vec{e}_t(t) + \left(R\dot{\varphi}^2(t)\right)\cdot\vec{e}_n(t)$$

$$\vec{a}(t) = a_t(t)\cdot\vec{e}_t(t) + a_n(t)\cdot\vec{e}_n(t)$$

$$a_t(t) = R\ddot{\varphi}(t)$$

$$a_n(t) = R\dot{\varphi}^2(t)$$





Primer

Kretanje tačke je opisano parametarskim jednačinama

$$\boxed{x(t)=\frac{t^2}{2}-t \qquad y(t)=\frac{t}{2}}$$

a) Odrediti trajektoriju tačke,
b) odrediti trenutak $t^*>0$ u kome će se tačka naći na osi y,
c) odrediti brzinu i ubrzanje tačke u proizvoljnom trenutku vremena t,
d) odrediti brzinu i ubrzanje tačke, i njihove intenzitete, u trenutku t^*,
e) odrediti poluprečnik krivine trajektorije u trenutku t^*.

$x(t)=\frac{t^2}{2}-t \quad y(t)=\frac{t}{2}$

ЛП

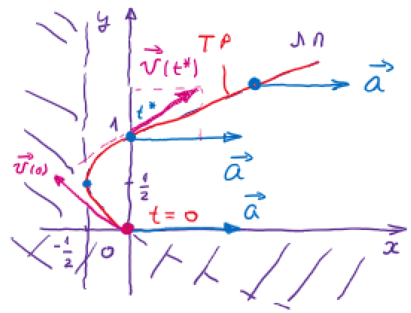
$\left.\begin{matrix} x(t)\\ y(t)\end{matrix}\right\} \xrightarrow[t]{\text{ел.}} y(x)$

$t=2y \to x=\frac{(2y)^2}{2}-2y$

$\boxed{x=2y^2-2y}$

ПАРАБОЛА

$x=2y(y-1)$



$t=0$

УСПОРЕНО

$t^*=2$

УБРЗАНО

ОК $\quad t\geq 0$

(2) $y=\frac{t}{2} \to y\geq 0$

(1) $x=t\left(\frac{t}{2}-1\right) \to -\frac{1}{2}\leq x$

$t^*=?$

$t^*>0 \to$ на y оси $\to$

$x(t^*)=0$

$x(t^*)=t^*\left(\frac{t^*}{2}-1\right)=0$

$t^*=0$

$\boxed{t^*=2}$

$t^*=2 \to x(t^*)=x(2)=\frac{2^2}{2}-2=0$

$y(t^*)=y(2)=\frac{2}{2}=1$

$x(t)=\frac{t^2}{2}-t \quad y(t)=\frac{t}{2}$

$\vec{v}=\dot{\vec{r}}=\dot{x}\,\vec{i}+\dot{y}\,\vec{j}$

$\vec{a}=\dot{\vec{v}}=\ddot{\vec{r}}=\ddot{x}\,\vec{i}+\ddot{y}\,\vec{j}$

$\dot{x}(t)=t-1 \;;\; \dot{y}(t)=\frac{1}{2}=\text{const}$

$v(t)=\sqrt{\dot{x}^2(t)+\dot{y}^2(t)}=\sqrt{(t-1)^2+\left(\frac{1}{2}\right)^2}$

$\ddot{x}(t)=1=\text{const} \;;\; \ddot{y}(t)=0=\text{const}$

$\vec{a}_{(t)}=1\cdot\vec{i}+0\,\vec{j}=1\cdot\vec{i}=\overrightarrow{\text{const}}$

$a(t)=\sqrt{\ddot{x}^2(t)+\ddot{y}^2(t)}=1=\text{const}$

$t^*=2 \qquad x(2)=0\,,\; y(2)=1$

$\dot{x}(t^*)=\dot{x}(2)=2-1=1$

$\dot{y}(t^*)=\dot{y}(2)=\frac{1}{2}$

$v(t^*)=v(2)=\sqrt{1^2+\left(\frac{1}{2}\right)^2}=\frac{\sqrt{5}}{2}$

$\ddot{x}(2)=1 \;,\; \ddot{y}(2)=0$

$a(2)=1$

$$x(t)=\frac{t^2}{2}-t \qquad y(t)=\frac{t}{2}$$

$t^* = 2$

$R_K(t^*) = R_K(2) = ?$

$v(2) = \sqrt{(2-1)^2+\frac{1}{4}} = \frac{\sqrt{5}}{2}$

$a(2) = 1$

$\dot{x}(t) = t-1 \quad , \quad \dot{y}(t) = \frac{1}{2}$

$a_T^2(2) = \dot{v}^2(2) = \left(\frac{2}{\sqrt{5}}\right)^2 = \frac{4}{5}$

$v_{(t)} = \sqrt{\dot{x}^2+\dot{y}^2} = \sqrt{(t-1)^2+\left(\frac{1}{2}\right)^2} \;\Big/\; \frac{d}{dt} \rightarrow \quad \dot{v}(t) = \frac{\cancel{2}(t-1)\cdot 1}{\cancel{2}\sqrt{(t-1)^2+\frac{1}{4}}}$

$\ddot{x}(t) = 1 \quad , \quad \ddot{y}(t) = 0$

$\dot{v}(2) = \frac{2-1}{\sqrt{(2-1)^2+\frac{1}{4}}} = \frac{1}{\frac{\sqrt{5}}{2}} = \frac{2}{\sqrt{5}}$

$a(t) = \sqrt{\ddot{x}^2+\ddot{y}^2} = 1 = const$

$a_N(2) = \sqrt{a^2_{(2)} - a^2_{T(2)}} = \sqrt{1^2 - \frac{4}{5}}$

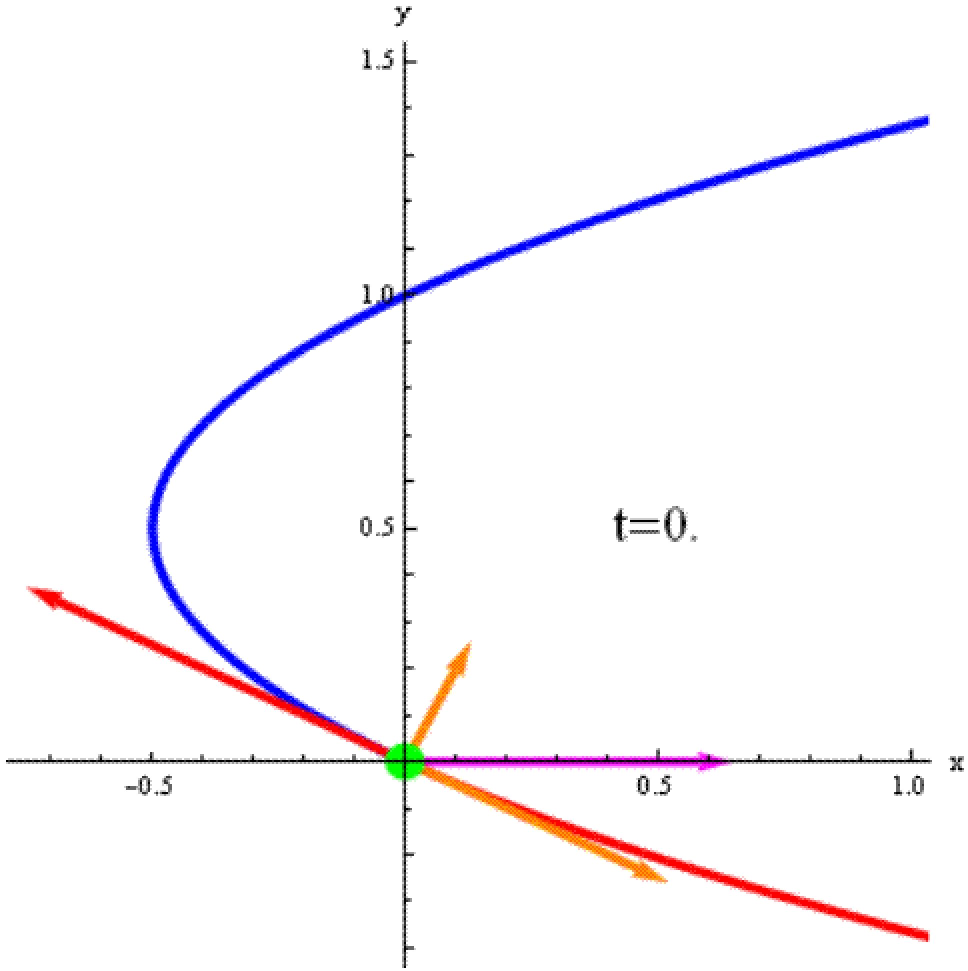
$= \sqrt{1-\frac{4}{5}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$R_K(2) = \frac{v^2(2)}{a_N(2)} = \frac{\left(\frac{\sqrt{5}}{2}\right)^2}{\frac{1}{\sqrt{5}}} = \frac{\frac{5}{4}}{\frac{1}{\sqrt{5}}} = \frac{5\sqrt{5}}{4} \quad [m]$

$$R_k(t^*) = \frac{v^2(t^*)}{a_n(t^*)} = \frac{5\sqrt{5}}{4}\, m.$$

Смисао полупречника кривине је следећи: он одређује полупречник круга чији се центар налази на правцу нормале на трајекторију, има заједничку тангенту са трајекторијом и најбоље апроксимира криву у околини посматране тачке[20].

[20]Зато је полупречник кривине круга једнак самом полупречнику круга, а полупречник кривине праве линије је бесконачно велик (односно кривина κ је једнака нули).


y
1.5
1.0
0.5
t=0.
x
-0.5
0.5
1.0

Primer

Kretanje tačke je opisano parametarskim jednačinama

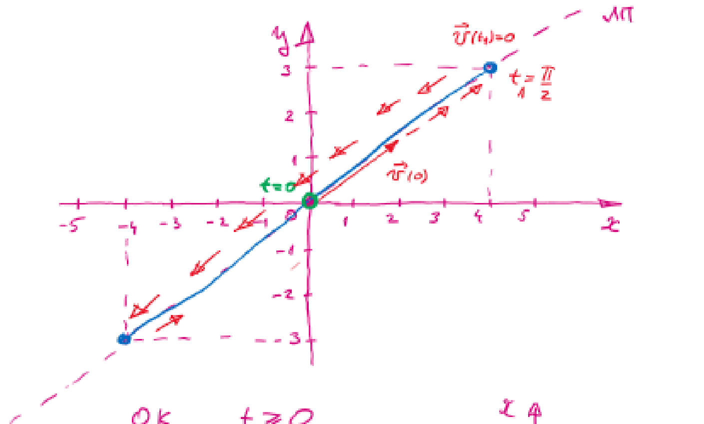
$$x(t) = 4\sin t\,,\, y(t) = 3\sin t$$

a) Odrediti i nacrtati trajektoriju kretanja tačke,
b) odrediti brzinu i ubrzanje tačke u proizvoljnom trenutku vremena t,
c) odrediti položaj, brzinu i ubrzanje tačke u trenucima $t_0 = 0, t_1 = \pi/2$ i $t_2 = \pi$,
d) odrediti pređeni put tačke do trenutka t_2.

Kretanje tačke je opisano parametarskim jednačinama

$$x(t) = 4\sin t\,,\ y(t) = 3\sin t$$

a) Odrediti i nacrtati trajektoriju kretanja tačke,
b) odrediti brzinu i ubrzanje tačke u proizvoljnom trenutku vremena t,
c) odrediti položaj, brzinu i ubrzanje tačke u trenucima $\underline{t_0 = 0}, t_1 = \pi/2$ i $t_2 = \pi$,



a) (1) $x = 4\sin t \longrightarrow \sin t = \frac{x}{4}$

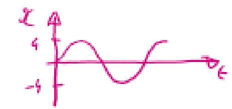
(2) $y = 3\sin t$

$$y = 3\cdot\frac{x}{4}$$

$$\boxed{y = \frac{3}{4}x} \quad ЛП$$

ПРАВА

OK $t \geq 0$



(1) $\rightarrow -4 \leq x \leq 4$

(2) $\rightarrow -3 \leq y \leq 3$

b) (1) $x(t) = 4\sin t$

(2) $y(t) = 3\sin t$

$\dot{x}(t) = 4\cos t$

$\dot{y}(t) = 3\cos t$

$v(t) = \sqrt{4^2\cos^2 t + 3^2\cos^2 t}$

$v(t) = \sqrt{25\cos^2 t}$

$v(t) = |\vec{v}(t)| = 5\sqrt{\cos^2 t}$

$\ddot{x}(t) = -4\sin t$

$\ddot{y}(t) = -3\sin t$

$a(t) = \sqrt{4^2\sin^2 t + 3^2\sin^2 t}$

$a(t) = 5\sqrt{\sin^2 t}$

Одредити $\underline{\underline{\text{смер}}}$ ($\vec{v}$) кретања у почетном тренутку.

$t = 0$

$x(0) = 4\,\cancel{\sin 0}^{\,0} = 0$

$y(0) = 4\sin 0 = 0$

$\dot{x}(0) = 4\,\cancel{\cos 0}^{\,1} = \underline{\underline{4}}$

$\dot{y}(0) = \underline{\underline{3}}$

$v(0) = 5\sqrt{\cos^2 0} = 5$

$\ddot{x}(0) = 0$

$\ddot{y}(0) = 0$

$a(0) = 0$

Kretanje tačke je opisano parametarskim jednačinama

$$x(t) = 4\sin t\,,\; y(t) = 3\sin t$$

a) Odrediti i nacrtati trajektoriju kretanja tačke,
b) odrediti brzinu i ubrzanje tačke u proizvoljnom trenutku vremena t,
c) odrediti položaj, brzinu i ubrzanje tačke u trenucima $t_0 = 0, t_1 = \pi/2$ i $t_2 = \pi$,
d) odrediti pređeni put tačke do trenutka t_2.

$P[t_0=0,\ t_2=\pi] = ?$

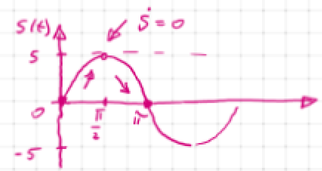
$$\dot{x}(t) = 4\cos t$$

$$\dot{y}(t) = 3\cos t$$

$$v(t) = \sqrt{4^2\cos^2 t + 3^2\cos^2 t} = 5\sqrt{\cos^2 t}$$

$$s(t_0 = 0) = 0$$

$$s(t) = \pm\int_{t_0=0}^{t} v(t)\,dt = +\int_{t_0=0}^{t} 5\cos t\,dt$$



$$s(t) = 5\sin t\,\Big|_0^t = 5(\sin t - \cancel{\sin 0}^{\,0})$$

$$s(t) = 5\sin t \;\rightarrow\; \dot{s}(t) = 5\cos t \;\rightarrow\; \dot{s}(t^*) = 5\cos t^* = 0$$

$$\cos t^* = 0$$

$$t^* = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$$P[t_0=0,\ t_2=\pi] = P[0,\tfrac{\pi}{2}] + P[\tfrac{\pi}{2},\pi]$$

$$= \left|s(\tfrac{\pi}{2}) - s(0)\right| + \left|s(\pi) - s(\tfrac{\pi}{2})\right|$$

$$P[0,\pi] = |5-0| + |0-5| = 5 + 5 = 10 \quad [m]$$

$$s(0) = 5\cancel{\sin 0}^{\,0} = 0$$

$$s(\tfrac{\pi}{2}) = 5\underbrace{\cancel{\sin}\tfrac{\pi}{2}}_{1} = 5$$

$$s(\pi) = 5\cancel{\sin\pi} = 0$$

Šta smo naučili?

5. **Kinematika tačke - polarni koordinatni sistem**
6. **Kinematika tačke - prirodni koordinatni sistem**

Kinematika
Kinematika tačke – 2. deo

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.