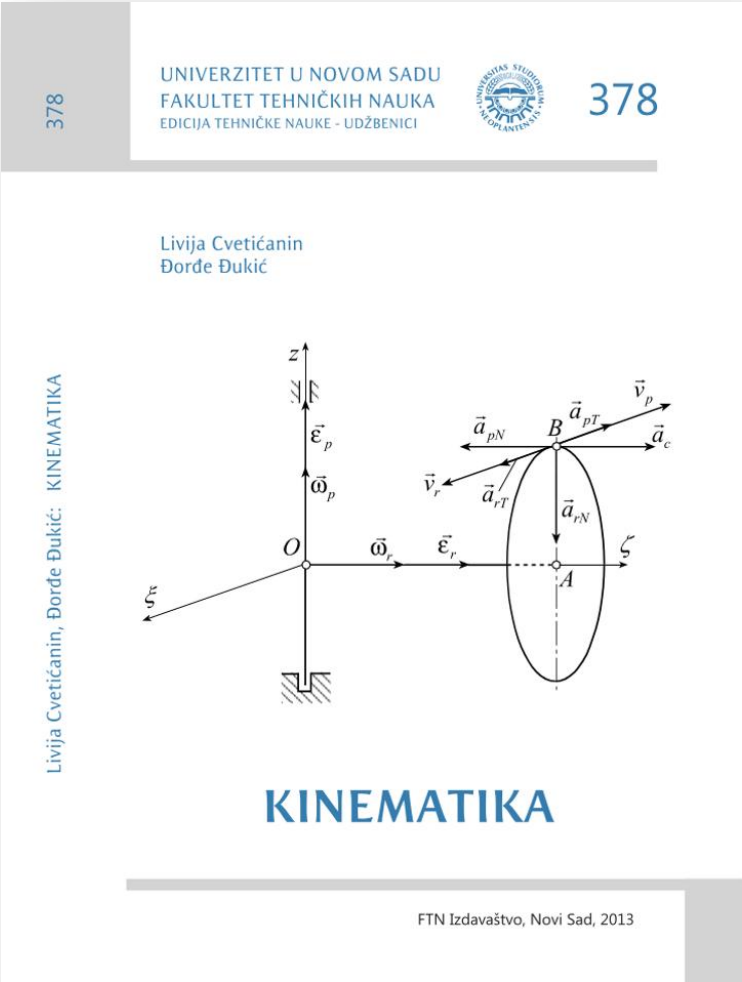
Mehanika 2 (Kinematika)
Predavanja 7

Miodrag Zuković

Novi Sad, 2023.

Literatura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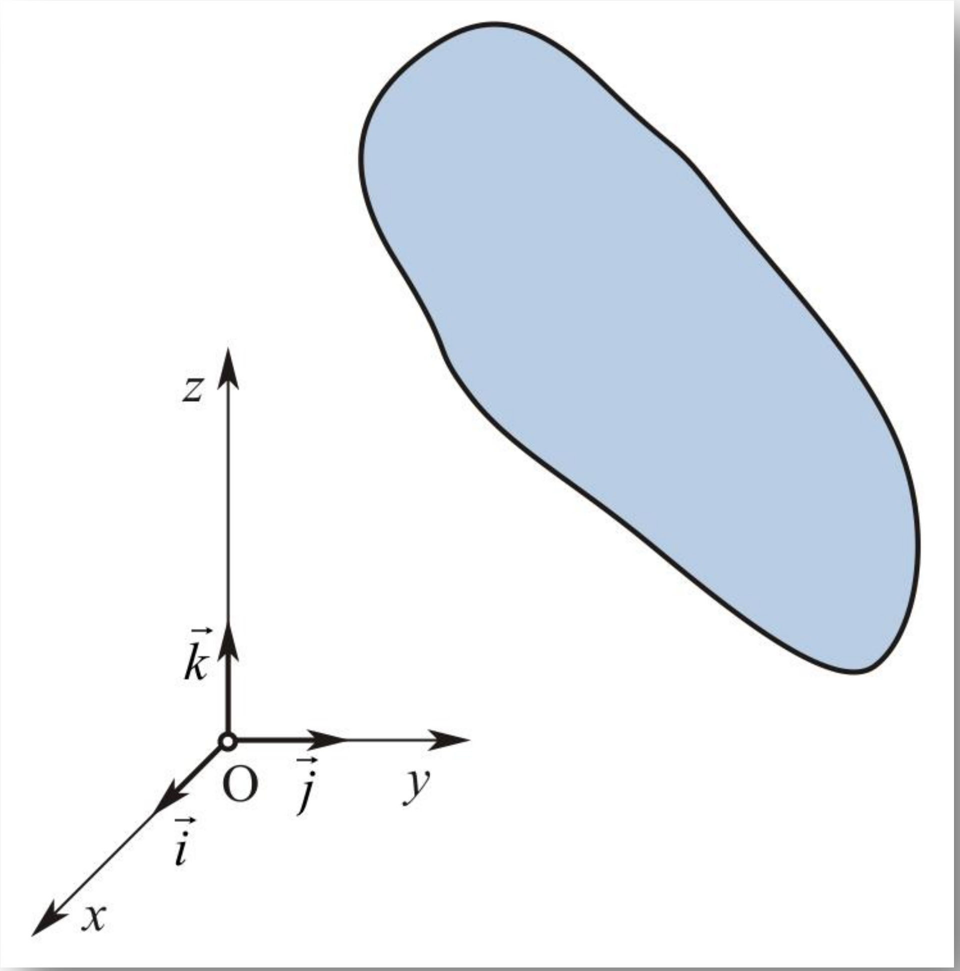
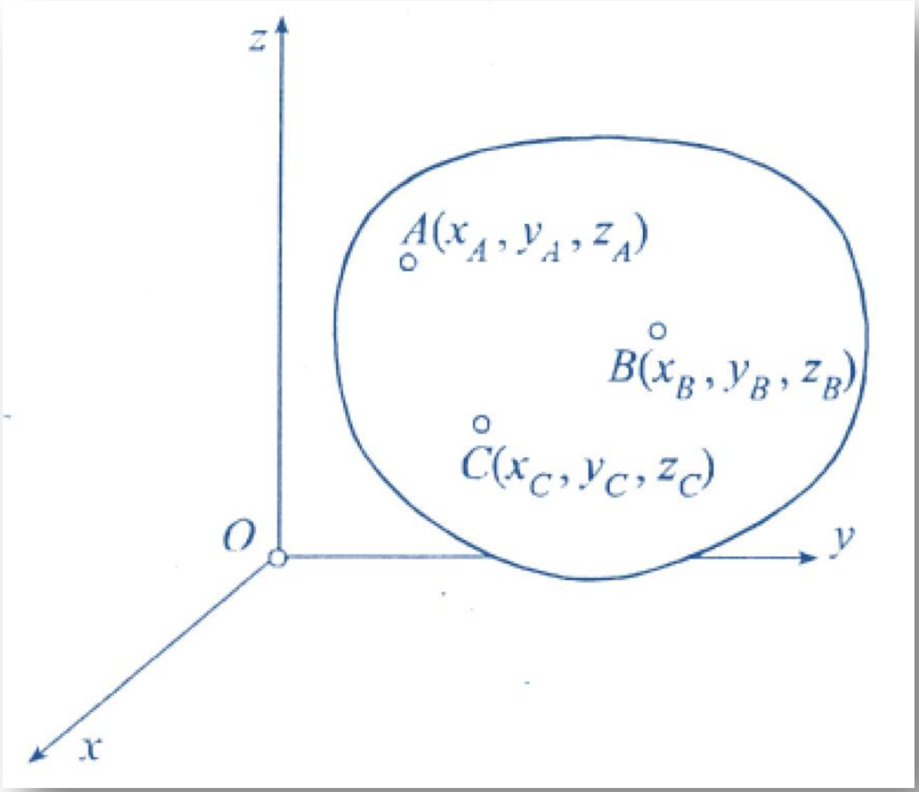
Šta ćemo naučiti?

23. **Opšte (slobodno) kretanje krutog tela**
24. **Brzine i ubrzanja tačaka krutog tela pri opštem kretanju**

23. Opšte (slobodno) kretanje krutog tela

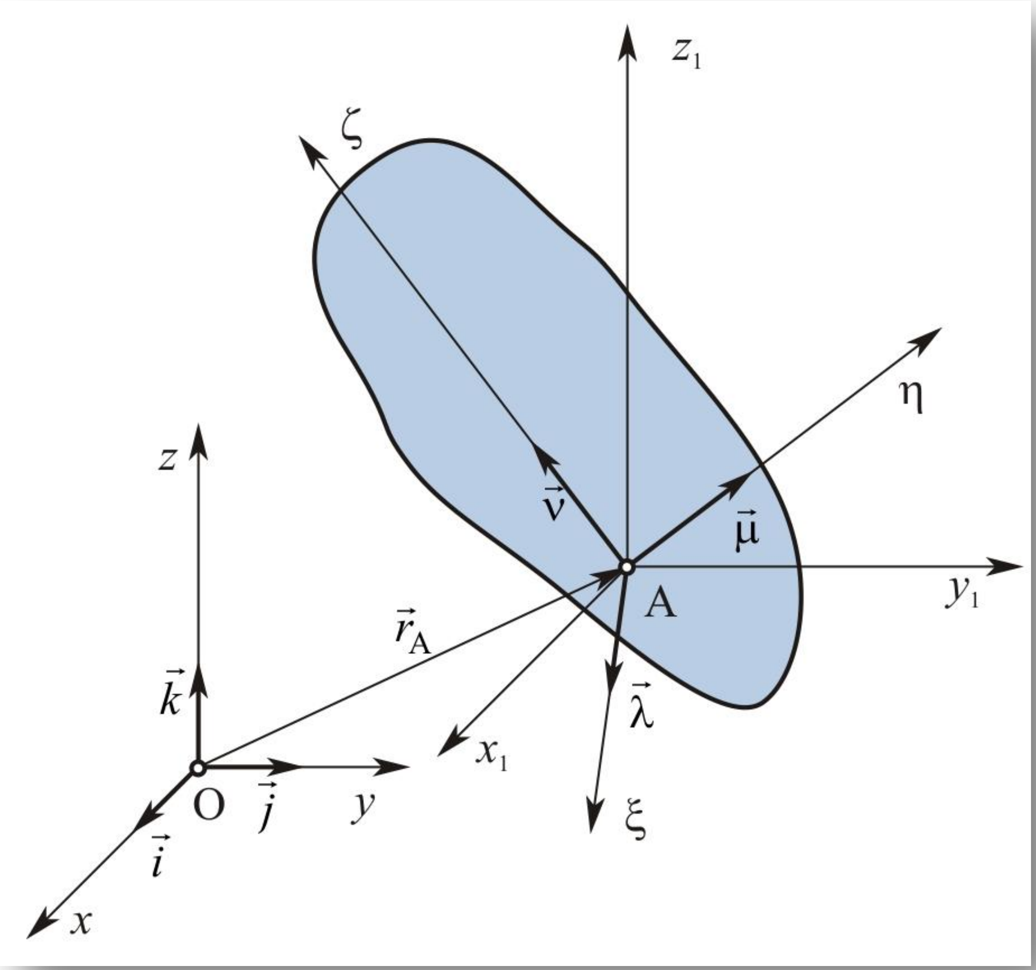


z
$\vec{k}$
O
$\vec{j}$
y
$\vec{i}$
x

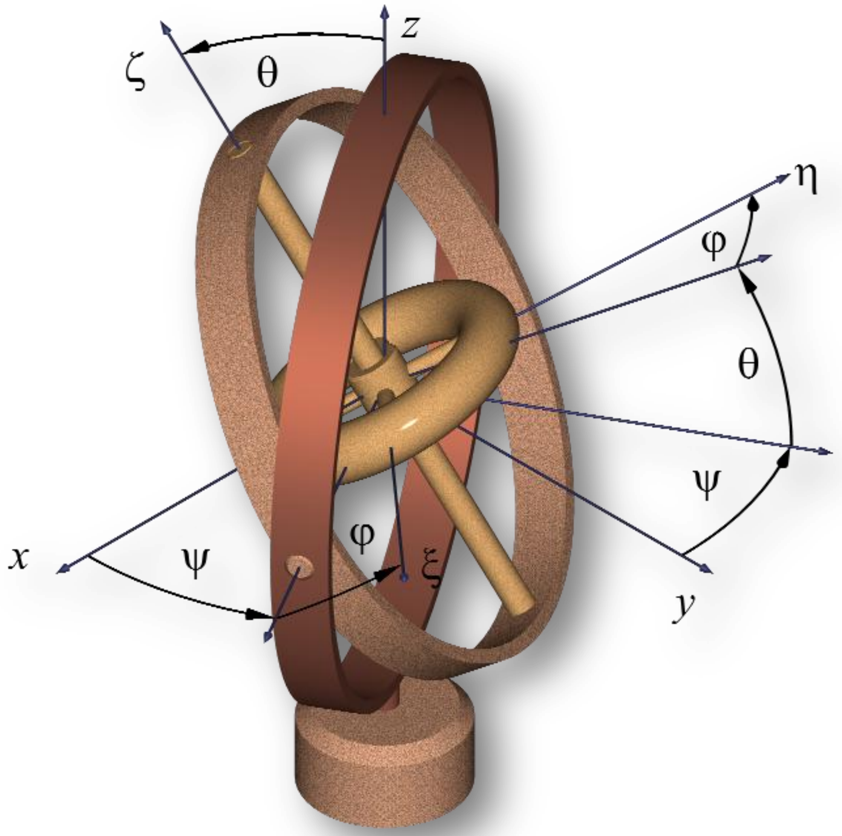


Opšte kretanje krutog tela ima (maksimalno) 6 stepeni slobode kretanja

$$
\begin{aligned}
AB &= \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2} = const., \\
AC &= \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2 + (z_A - z_C)^2} = const., \\
BC &= \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2 + (z_B - z_C)^2} = const.
\end{aligned}
$$



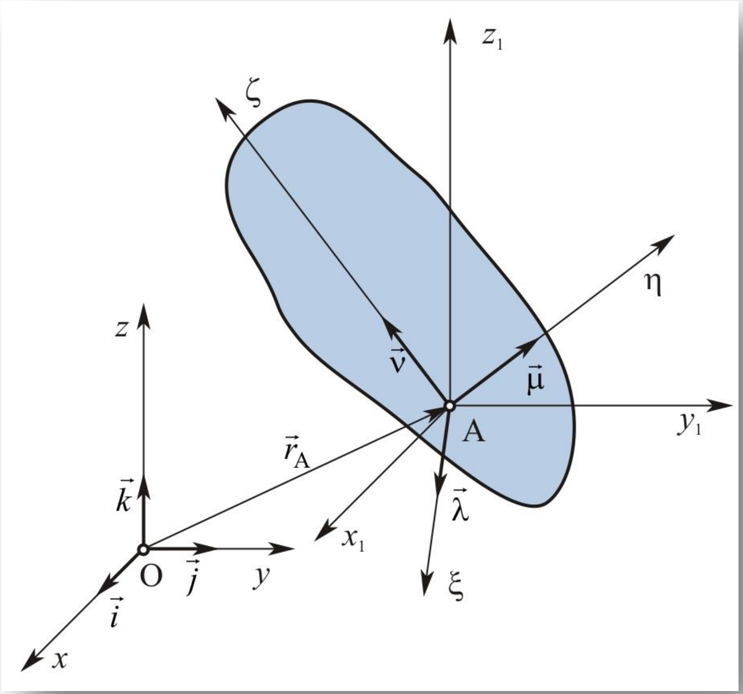
z_1
ζ
η
z
$\vec{\nu}$
$\vec{\mu}$
A
y_1
$\vec{r}_A$
$\vec{k}$
$\vec{\lambda}$
x_1
O
$\vec{j}$
y
ξ
$\vec{i}$
x



Ojlerovi uglovi:

Uglovi **precesije, nutacije i sopstvene rotacije** :

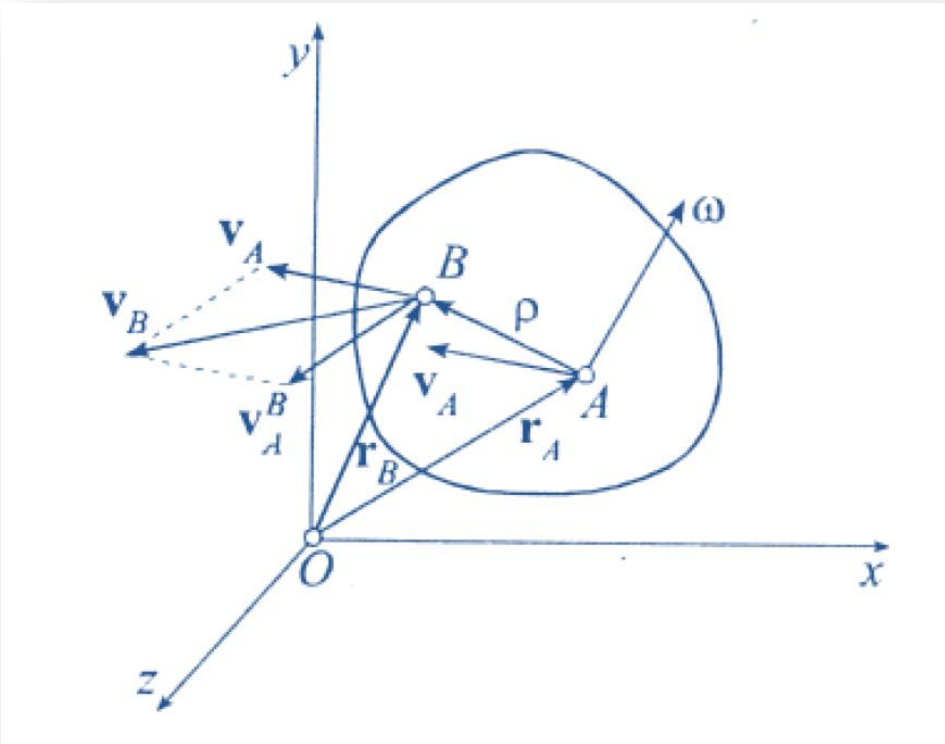
$$\psi = \psi(t); \;\; \theta = \theta(t); \; \varphi = \varphi(t)$$



Parametarske jednačine opšteg kretanja krutog tela:

$$x_A = x_A(t), y_A = y_A(t), z_A = z_A(t)$$
$$\psi = \psi(t), \theta = \theta(t), \varphi = \varphi(t)$$

24. Brzine i ubrzanja tačaka krutog tela pri opštem kretanju



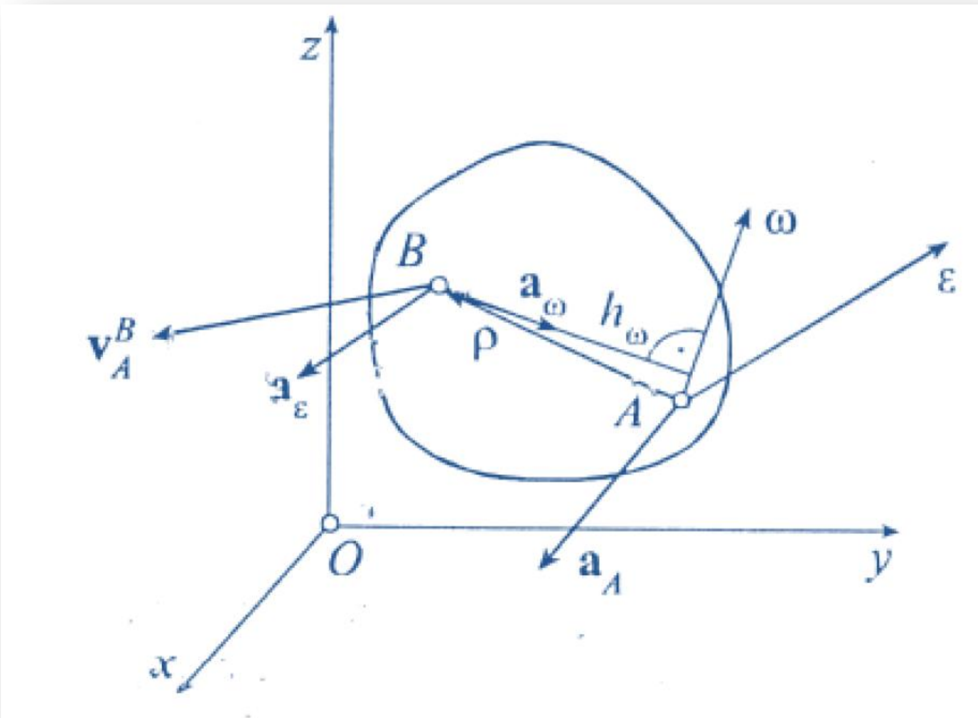
$$\vec{r}_B = \vec{r}_A + \vec{\rho}$$

$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{\rho}}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$$

$$\vec{v}_B^A = \dot{\vec{\rho}} = \vec{\omega} \times \vec{\rho}$$

$$v_B^A = \omega \rho \sin \angle(\vec{\omega}, \vec{\rho}) = \omega h_\omega$$



$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{\rho}$$

$$\dot{\vec{v}}_B = \dot{\vec{v}}_A + \dot{\vec{\omega}} \times \vec{\rho} + \vec{\omega} \times \dot{\vec{\rho}}$$

$$\vec{a}_B = \vec{a}_A + \vec{\varepsilon} \times \vec{\rho} + \vec{\omega} \times \vec{v}_B^A$$

Primer

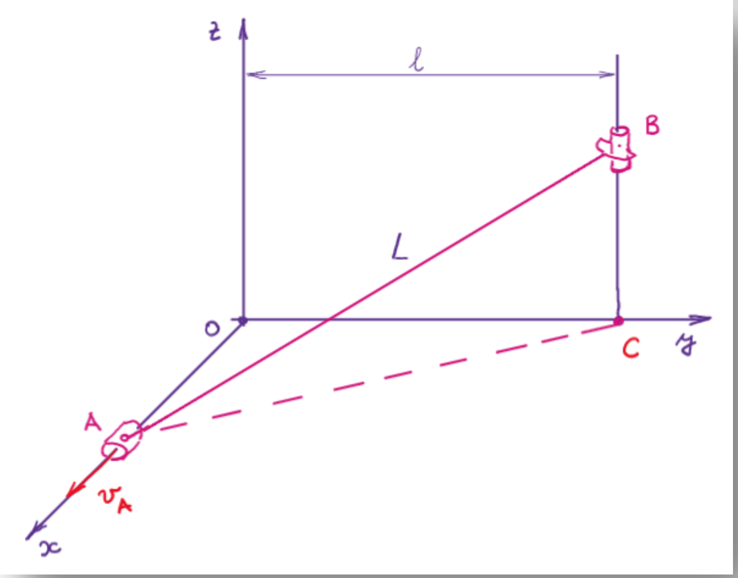
Primer 18 *Za krajeve štapa AB dužine* $L = 4$ [m] *zglobno su vezana dva klizača. Klizač A se kreće po nepokretnom pravcu OA, a klizač B po pravcu* z_1 *koji je paralelan pravcu z a nalazi se na rastojanju* $l = 2$ [m] *(Slika 8.6). U trenutku kada se kraj A štapa AB nalazi na rastojanju* $l_1 = 2$ [m] *od nepokretne tačke O brzina tačke A je*

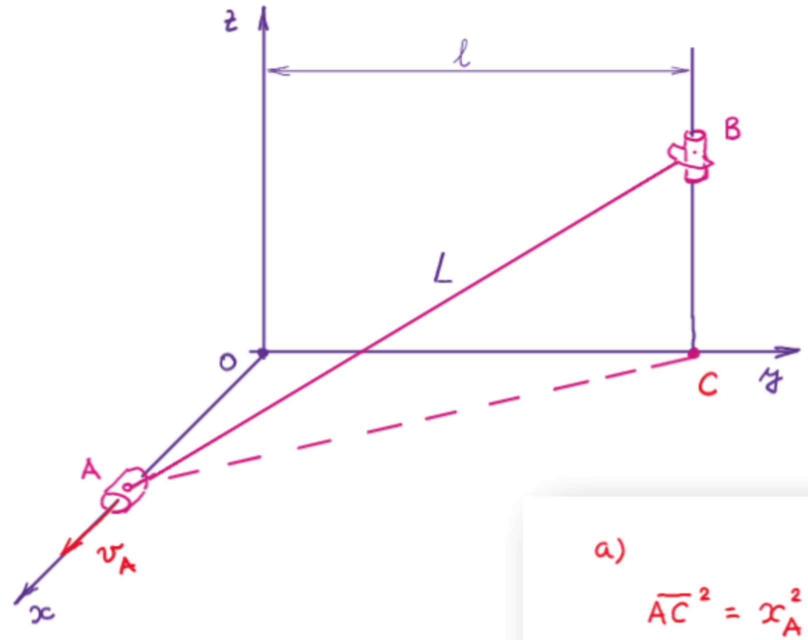
$$v_A = \sqrt{2}\ [m/s]$$

a ubrzanje

$$a_A = 0\ [m/s^2].$$

Odrediti brzinu i ubrzanje tačke B u tom trenutku vremena: a) analitički, b) vektorski.





a)

$$\overline{AC}^2 = x_A^2 + \ell^2$$

$$z_B^2 = L^2 - \overline{AC}^2$$

$$z_B^2 = L^2 - x_A^2 - \ell^2 \quad \Big/ \frac{d}{dt}$$

$$\cancel{2} z_B \dot{z}_B = -\cancel{2} x_A \dot{x}_A$$

$$v_B = \dot{z}_B = -\frac{x_A}{z_B} v_A$$

$$\dot{z}_B^2 + z_B \ddot{z}_B = -\dot{x}_A^2 - x_A \ddot{x}_A$$

$$a_B = \ddot{z}_B = -\frac{1}{z_B}\left(v_A^2 + x_A a_A + v_B^2\right)$$

$L = 4\ m,\ \ell = 2\ m$

$\ell_1 = 2\ m,$

$v_A = \sqrt{2}\ \frac{m}{s}, \quad a_A = 0$

$t = \bar{t}$

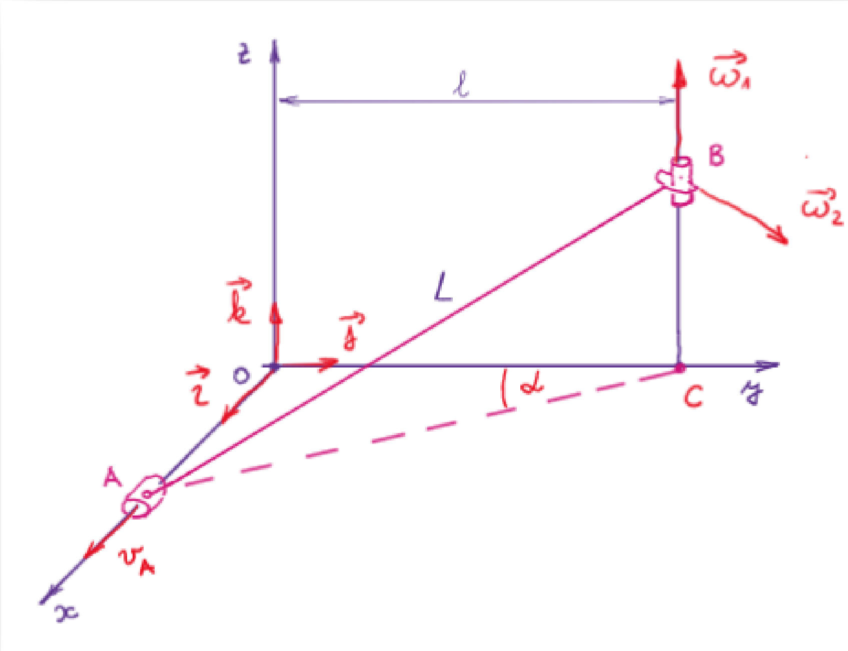
$x_A = \ell_1 = 2$

$z_B = \sqrt{4^2 - 2^2 - 2^2} = 2\sqrt{2}$

$v_B = -\frac{2}{2\sqrt{2}} \cdot \sqrt{2} = -1\ \frac{m}{s}$

$a_B = -\frac{1}{2\sqrt{2}}\left((\sqrt{2})^2 + 0 + (-1)^2\right)$

$a_B = -\frac{3\sqrt{2}}{4}\ \frac{m}{s^2}$



(1) $0 = v_A + \omega_2 \sin\alpha \cdot z_B - \omega_1 \ell$

(2) $0 = -\omega_1 x_A - \omega_2 \cos\alpha \cdot z_B$

(3) $v_B = \omega_2 \cos\alpha \cdot \ell + \omega_2 \sin\alpha \cdot x_A$

b)

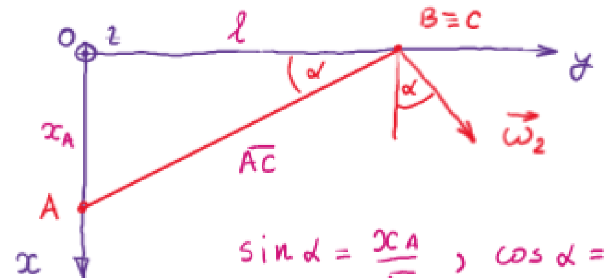
$\vec{v}_A = v_A \vec{i}$, $\vec{v}_B = v_B \vec{k}$

$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{\rho}$

$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{AB}$

$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2$

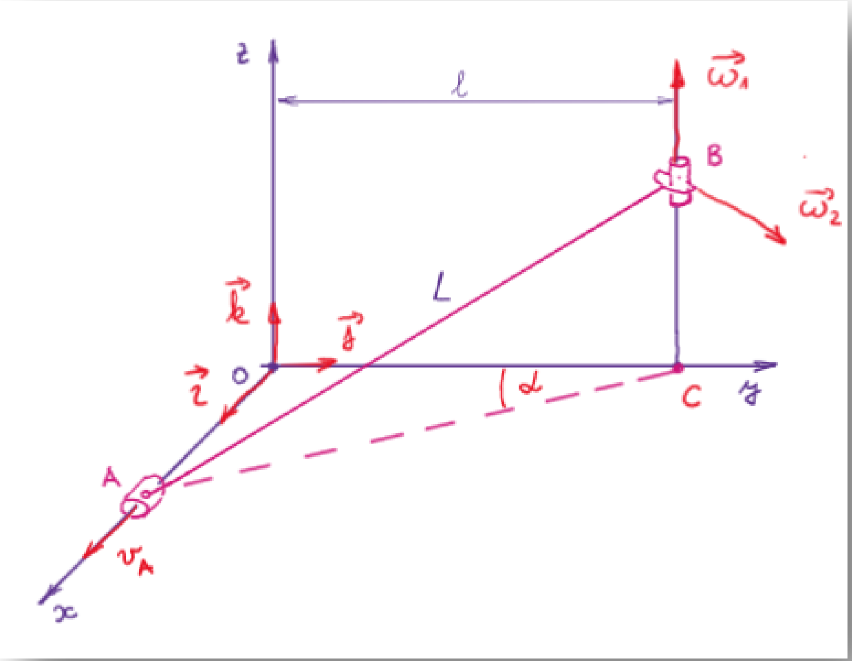
$\vec{\omega}_1 = \omega_1 \vec{k}$; $\vec{\omega}_2 = \omega_2 \cos\alpha\, \vec{i} + \omega_2 \sin\alpha\, \vec{j}$



$\sin\alpha = \dfrac{x_A}{\overline{AC}}$, $\cos\alpha = \dfrac{\ell}{\overline{AC}}$

$\vec{AB} = \vec{r}_B - \vec{r}_A = -x_A \vec{i} + \ell \vec{j} + z_B \vec{k}$

$$v_B \vec{k} = v_A \vec{i} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_2 \cos\alpha & \omega_2 \sin\alpha & \omega_1 \\ -x_A & \ell & z_B \end{vmatrix}$$



(1) $0 = v_A + \omega_2 \sin\alpha \cdot z_B - \omega_1 \ell$

(2) $0 = -\omega_1 x_A - \omega_2 \cos\alpha \cdot z_B$

(3) $v_B = \omega_2 \cos\alpha \cdot \ell + \omega_2 \sin\alpha \cdot x_A$

$t = \bar{t}$, $x_A = 2$, $z_B = 2\sqrt{2}$, $v_A = \sqrt{2}$

$\overline{AC} = 2\sqrt{2}$, $\sin\alpha = \frac{\sqrt{2}}{2}$, $\cos\alpha = \frac{\sqrt{2}}{2}$

(1) $0 = \sqrt{2} + \omega_2 \frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot \cancel{2}\cancel{\sqrt{2}} - \omega_1 \cdot 2$

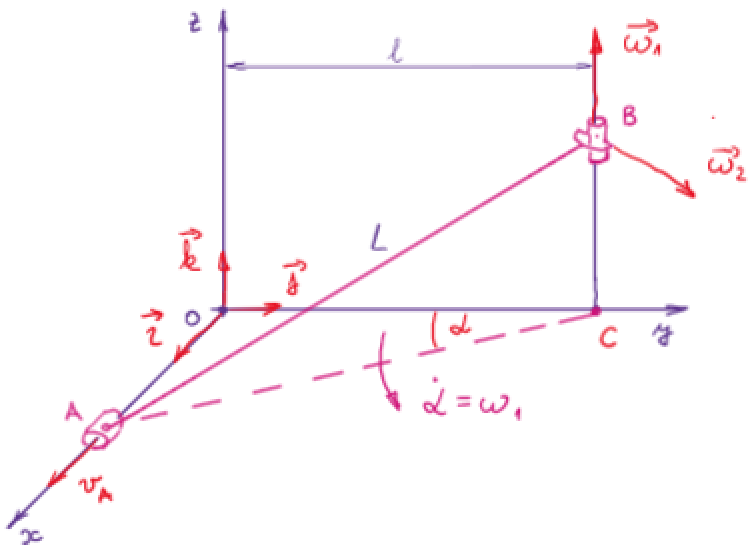
(2) $0 = -\omega_1 \cdot 2 - \omega_2 \frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot \cancel{2}\cancel{\sqrt{2}}$

(3) $v_B = \omega_2 \frac{\sqrt{2}}{\cancel{2}} \cdot \cancel{2} + \omega_2 \frac{\sqrt{2}}{2} \cdot 2 = 2\sqrt{2}\,\omega_2$

$\left.\begin{matrix}(1)\\(2)\end{matrix}\right\} \rightarrow$ $\omega_1 = \frac{\sqrt{2}}{4}\ s^{-1}$

$\omega_2 = -\frac{\sqrt{2}}{4}\ s^{-1}$

(3) $\rightarrow$ $v_B = -1\ \frac{m}{s}$



$$\vec{a}_B = a_B\,\vec{k}\,, \quad \vec{a}_A = a_A\,\vec{k}$$

$$\vec{a}_B = \vec{a}_A + \vec{\varepsilon} \times \vec{AB} + \vec{\omega} \times \vec{v}_B^{\,A}$$

$$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2 \longrightarrow \vec{\varepsilon} = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2$$

$$\vec{\omega}_1 = \omega_1\,\vec{k} \longrightarrow \vec{\varepsilon}_1 = \dot{\omega}_1\,\vec{k}$$

$$\vec{\omega}_2 = \omega_2 \cos\alpha\,\vec{i} + \omega_2 \sin\alpha\,\vec{j} \longrightarrow$$

$$\vec{\varepsilon}_2 = (\dot{\omega}_2 \cos\alpha - \omega_2 \sin\alpha \cdot \cancelto{\omega_1}{\dot{\alpha}})\,\vec{i} + (\dot{\omega}_2 \sin\alpha + \omega_2 \cos\alpha\,\cancelto{\omega_1}{\dot{\alpha}})\,\vec{j}$$

$$\vec{v}_B^{\,A} = \vec{\omega} \times \vec{AB} = \ldots$$

$t = \bar{t}$, $x_A = 2$, $z_B = 2\sqrt{2}$, $v_A = \sqrt{2}$, $a_A = 0$

$\overline{AC} = 2\sqrt{2}$, $\sin\alpha = \frac{\sqrt{2}}{2}$, $\cos\alpha = \frac{\sqrt{2}}{2}$

$\omega_1 = \frac{\sqrt{2}}{4}$, $\omega_2 = -\frac{\sqrt{2}}{4}$

$\vec{v}_B^{\,4} = -\sqrt{2}\,\vec{i} - 1\,\vec{j}$;

$\vec{AB} = -2\,\vec{i} + 2\,\vec{j} + 2\sqrt{2}\,\vec{k}$

$\vec{\omega} = -\frac{1}{4}\vec{i} - \frac{1}{4}\vec{j} + \frac{\sqrt{2}}{4}\vec{k}$

$\vec{\varepsilon} = \frac{\sqrt{2}}{2}\left(\dot{\omega}_2 + \frac{1}{8}\right)\vec{i} + \frac{\sqrt{2}}{2}\left(\dot{\omega}_2 - \frac{1}{8}\right)\vec{j} + \dot{\omega}_1\vec{k}$

$$a_B\vec{k} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\sqrt{2}}{2}\left(\dot{\omega}_2 + \frac{1}{8}\right) & \frac{\sqrt{2}}{2}\left(\dot{\omega}_2 - \frac{1}{8}\right) & \dot{\omega}_1 \\ -2 & 2 & 2\sqrt{2} \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{\sqrt{2}}{4} \\ -\sqrt{2} & -1 & 0 \end{vmatrix}$$

(1) $0 = \ldots$

(2) $0 = \ldots$

(3) $a_B = \ldots$

Šta ćemo naučiti?

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