

Kinematika

Kinematika krutog tela – 1. deo

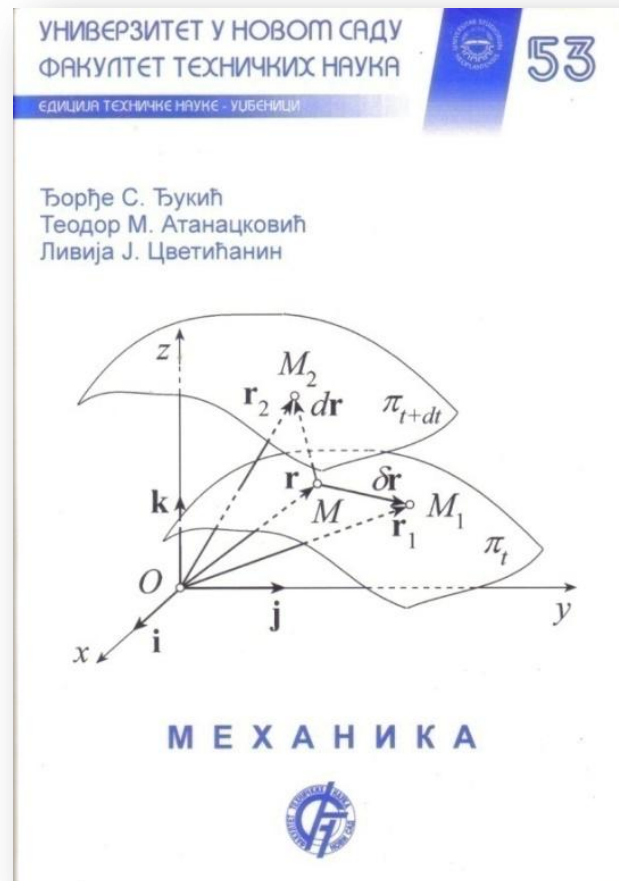
Kinematika i dinamika

Miodrag Zuković

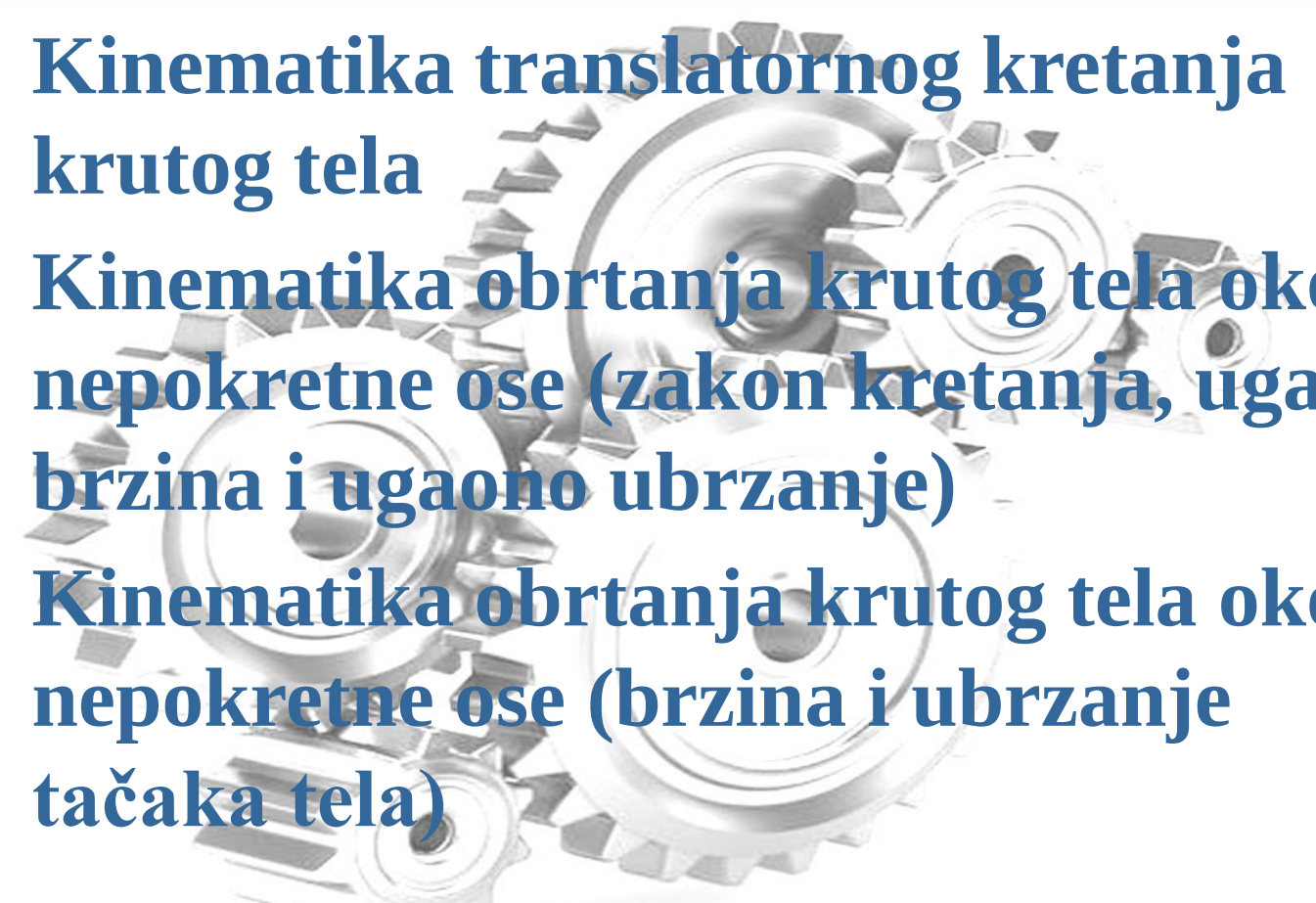
Novi Sad, 2021.

Literatura

- Đorđe S. Đukić, Teodor M. Atanacković, Livija J. Cvetićanin:
Mehanika, Fakultet tehničkih nauka u Novom Sadu, Novi Sad, 2003.



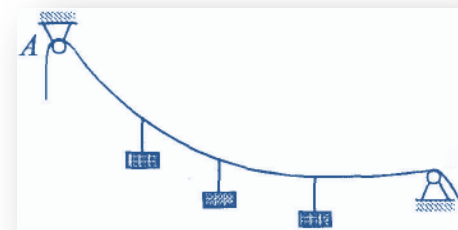
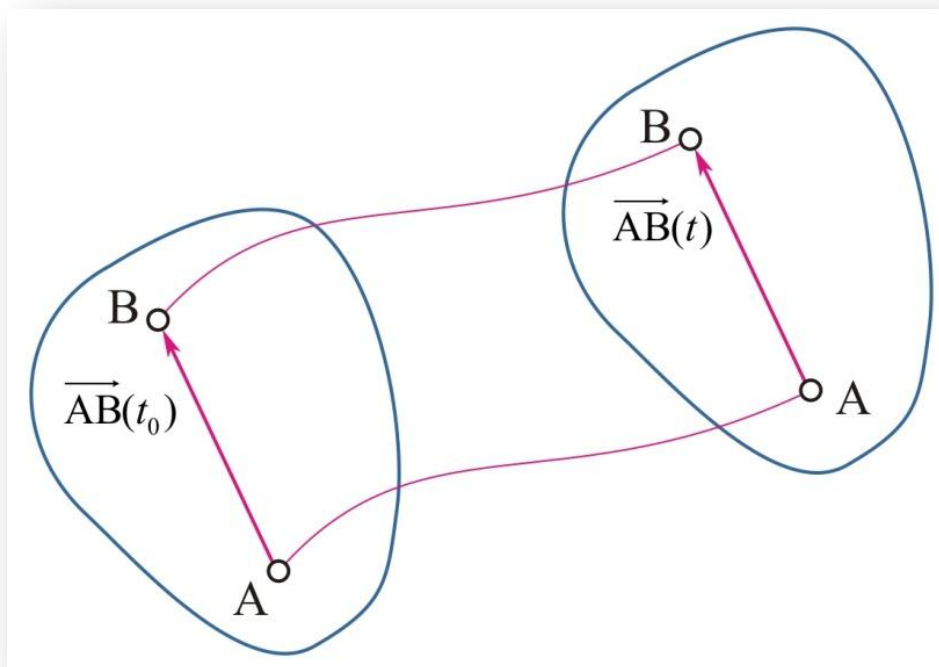
Šta ćemo naučiti?

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7. **Kinematika translatornog kretanja krutog tela**
 8. **Kinematika obrtanja krutog tela oko nepokretne ose (zakon kretanja, ugaona brzina i ugaono ubrzanje)**
 9. **Kinematika obrtanja krutog tela oko nepokretne ose (brzina i ubrzanje tačaka tela)**

7. Kinematika translatornog kretanja krutog tela

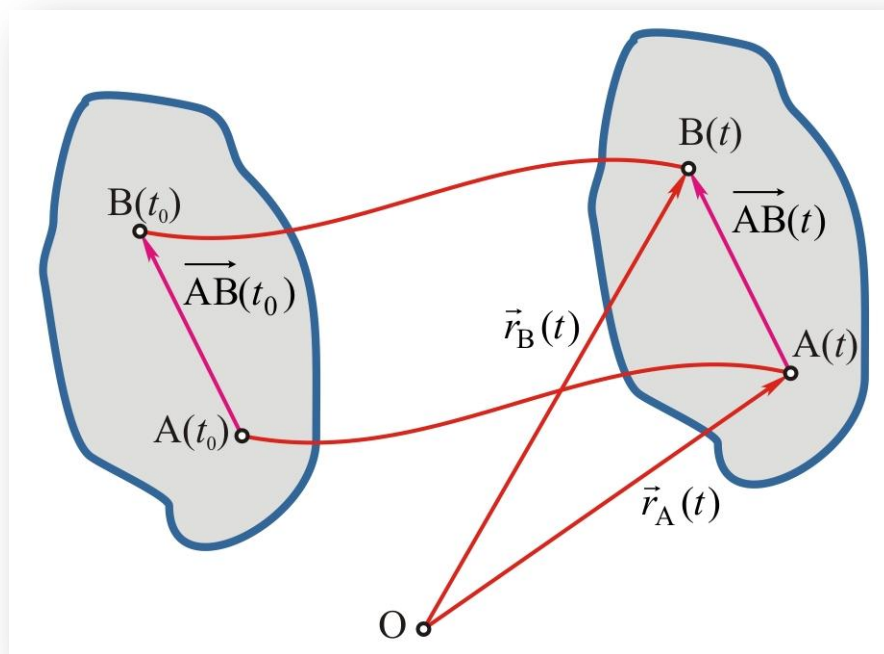
Translatorno kretanja krutog tela

- Kretanje tela je translatorno, ako svaki zamišljeni pravac u telu za vreme kretanja ostaje sam sebi paralelan. Kretanje ima tri stepena slobode.



Translatorno kretanje krutog tela

- Pri translatorsnom kretanju tela putanje, brzine i ubrzanja svih tačaka tela su u bilo kom trenutku vremena iste.



$$\overrightarrow{AB}(t) = \text{const} = \overrightarrow{AB}(t_0)$$

$$\dot{\overrightarrow{AB}}(t) = 0$$

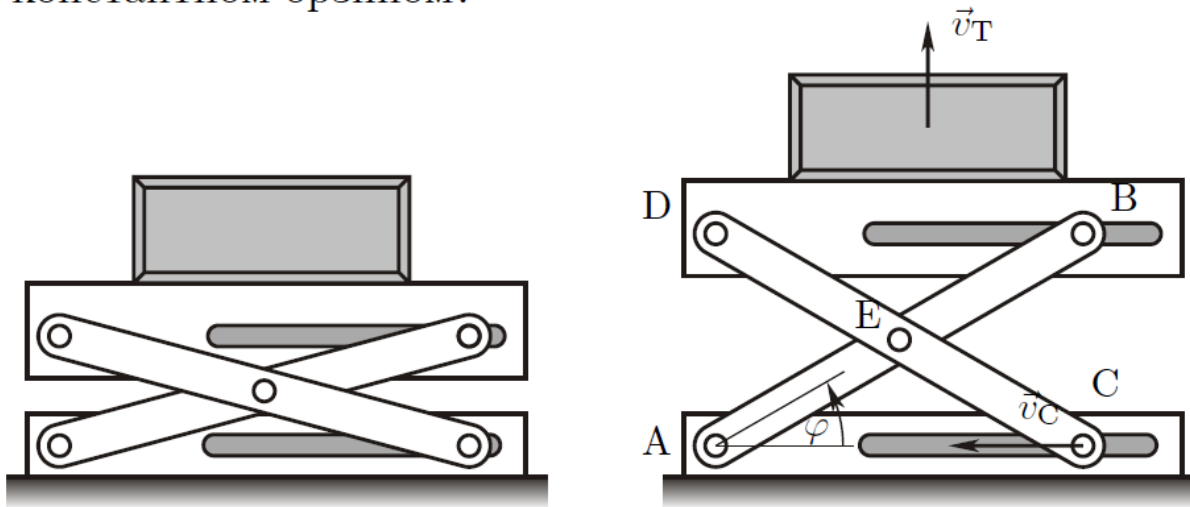
$$\vec{r}_B(t) = \vec{r}_A(t) + \overrightarrow{AB}(t)$$

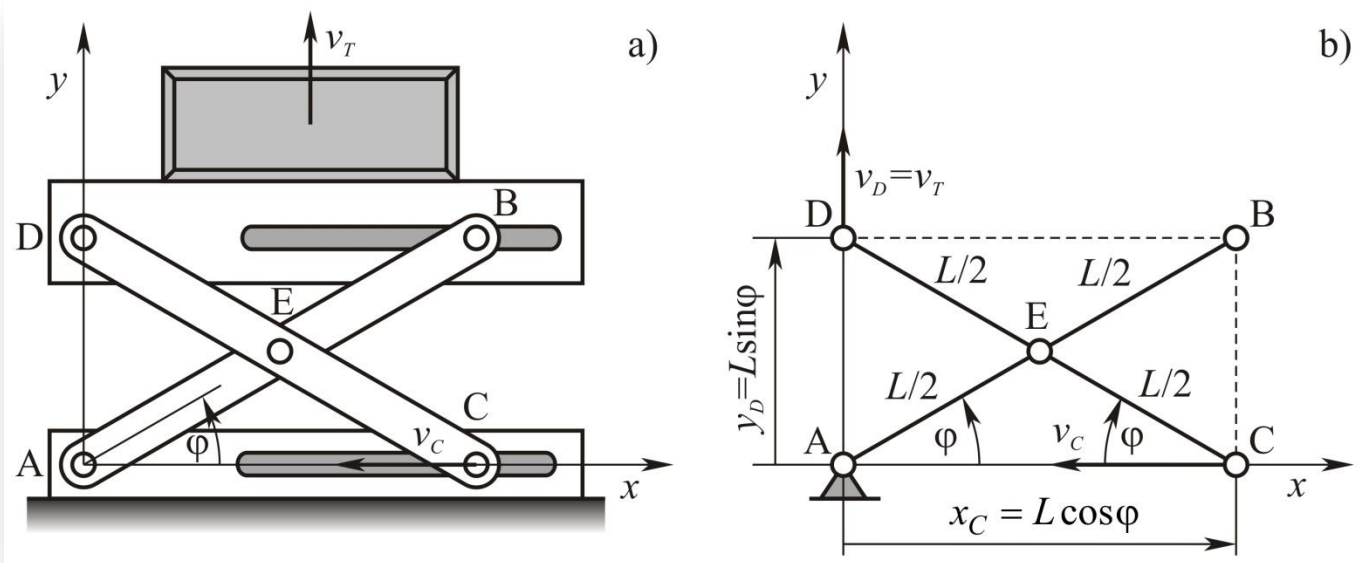
$$\dot{\vec{r}}_B(t) = \dot{\vec{r}}_A(t) \rightarrow \vec{v}_B(t) = \vec{v}_A(t)$$

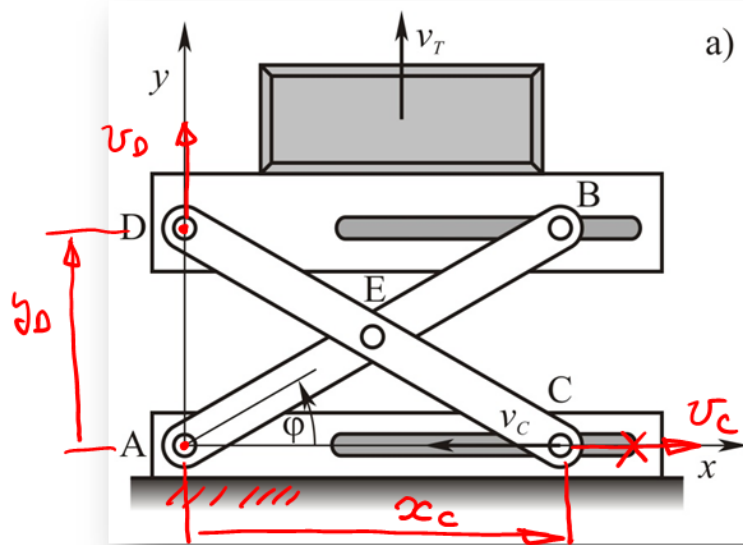
$$\ddot{\vec{r}}_B(t) = \ddot{\vec{r}}_A(t) \rightarrow \vec{a}_B(t) = \vec{a}_A(t)$$

Primer

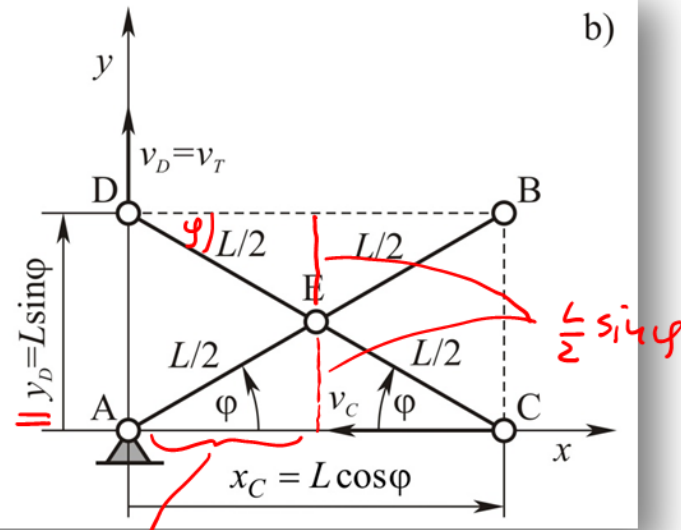
Одредити брзину и убрзање терета, ако угао који полуга АВ, механизма за подизање, гради са хоризонталом мења по закону $\varphi = \omega t$ ($\omega = \text{const.}$). Која је веза између брзине терета и брзине тачке С? Дужине полуга АВ и CD механизма су једнаке ($\overline{AB} = \overline{CD} = L$), а зглоб Е се налази на њиховој средини. Како би требало да се мења угао φ да би се терет подизао константном брзином?







a)



b)

$$\varphi = \omega t$$

$$y_D = L \sin \varphi = L \sin(\omega t)$$

$$v_D = \dot{y}_D = L \omega \cos(\omega t) = v_T$$

$$a_D = \ddot{y}_D = -L \omega^2 \sin(\omega t) = a_T$$

$$\frac{v_D}{v_C} = \frac{L \omega \cos(\omega t)}{-L \omega \sin(\omega t)} = -\cot \varphi(\omega t)$$

$$\frac{L}{2} \cos \varphi$$

$$x_c = L \cos \varphi = L \cos(\omega t)$$

$$v_C = \dot{x}_c = -L \omega \sin(\omega t)$$

$$a_C = \ddot{x}_c = -L \omega^2 \cos(\omega t)$$

$$\varphi = ? \quad \rightarrow \quad v_T = \text{const}$$

$$y_0 = L \sin \varphi$$

$$v_D = \boxed{v_T = L \dot{\varphi} \cos \varphi}$$

$$\dot{\varphi} = \frac{v_T}{L} \cdot \frac{1}{\cos \varphi}$$

$$\underline{\frac{d\varphi}{dt} = \frac{v_T}{L} \cdot \frac{1}{\cos \varphi}}$$

$$\int_{\varphi_0}^{\varphi} \cos \varphi \, d\varphi = \frac{v_T}{L} \int_0^t dt$$

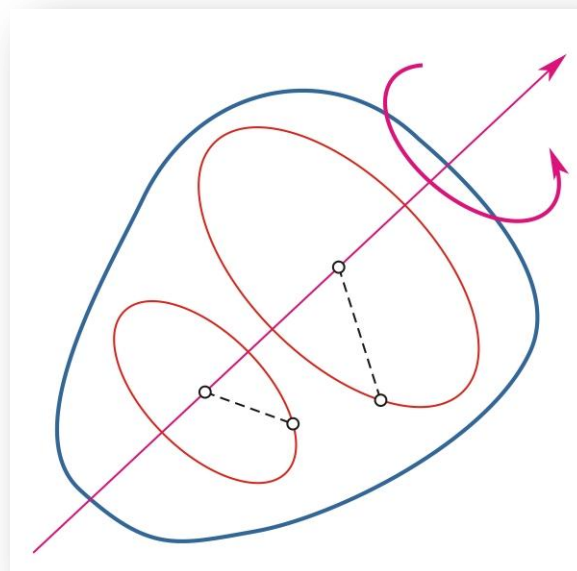
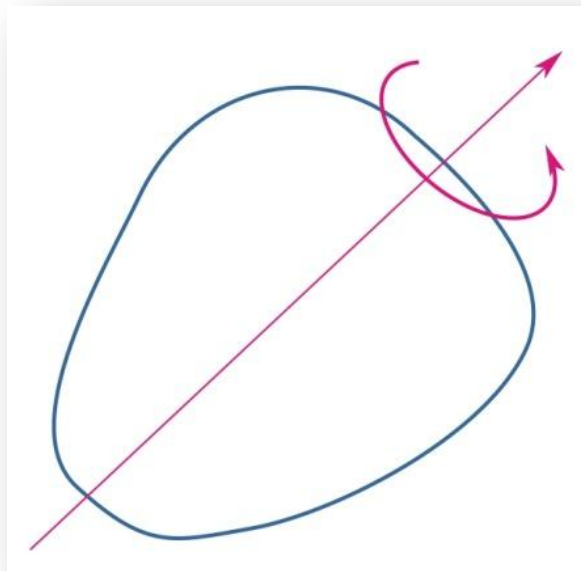
$$\sin \varphi \Big|_{\varphi_0}^{\varphi} = \frac{v_T}{L} t \Big|_0^t$$

$$\sin \varphi - \sin \varphi_0 = \frac{v_T}{L} t$$

$$\underline{\sin \varphi = \sin \varphi_0 + \frac{v_T}{L} t}$$

8. Kinematika obrtanja krutog tela oko nepokretne ose (zakon kretanja, ugaona brzina i ugaono ubrzanje)

Obrtanje krutog tela oko nepokretne ose



Obrtanje krutog tela oko nepokretne ose je takvo kretanje tela kod kog se njegove tačke kreću po kružnicama čiji je centar na nepokretnoj osi i koje su u ravnima upravnim na tu osu (osa obrtanja).

Obrtanja oko nepokretne ose – zakon kretanja

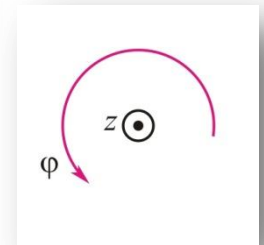
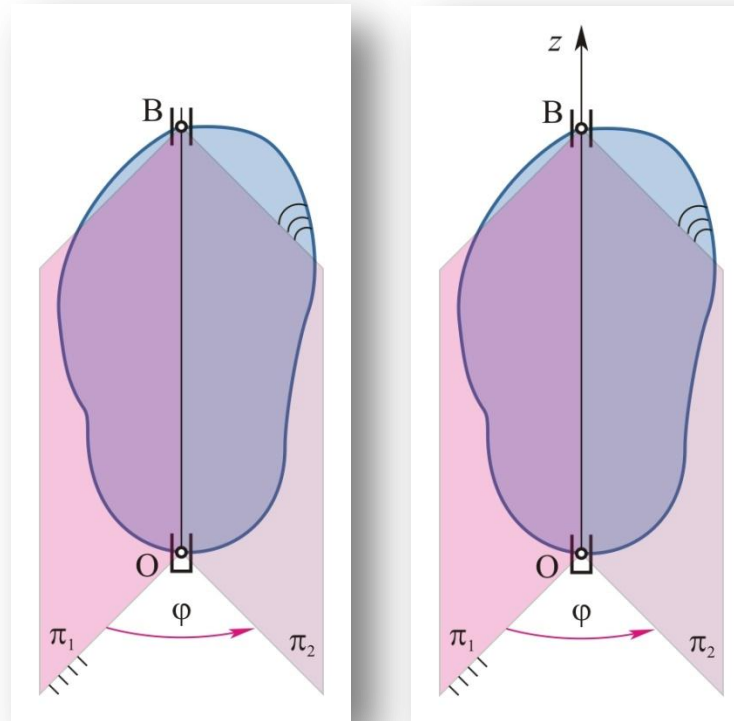
1 stepen slobode kretanja

Ugao obrtanja



Zakon kretanja –
parametarska jednačina
kretanja

$$\varphi = \varphi(t)$$



Obrtanja oko nepokretne ose – ugaona brzina

Srednja ugaona brzina

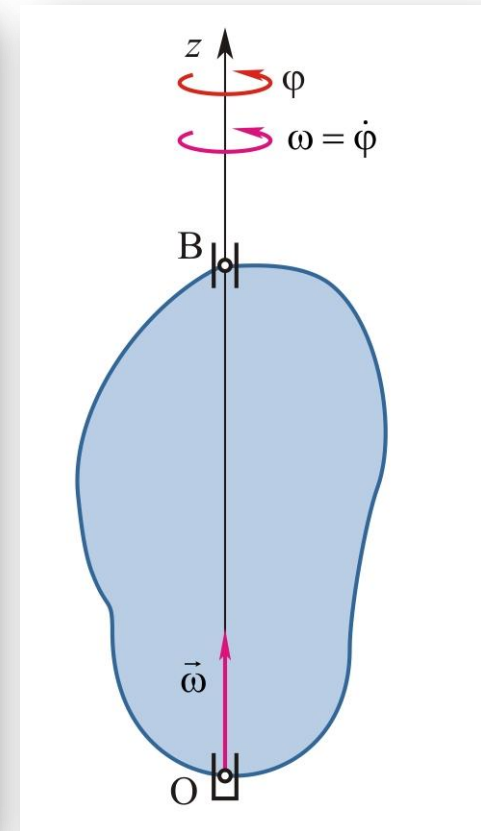
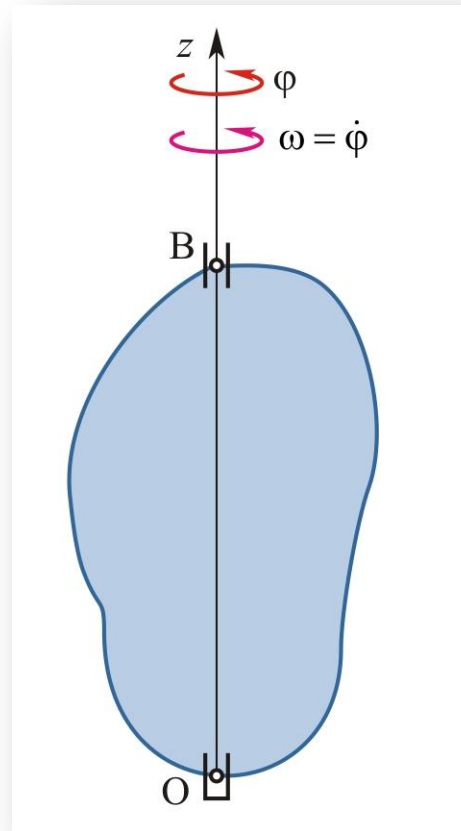
$$\omega_{sr} = \frac{\Delta\varphi}{\Delta t}$$

Ugaona brzina

$$\omega = \lim_{\Delta t \rightarrow 0} \omega_{sr} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\varphi}{\Delta t} = \frac{d\varphi}{dt} = \dot{\varphi}$$

Vektor ugaone brzine

$$\vec{\omega}(t) = \omega(t)\vec{k}$$



Obrtanja oko nepokretne ose – ugaono ubrzanje

Srednje ugaono ubrzanje

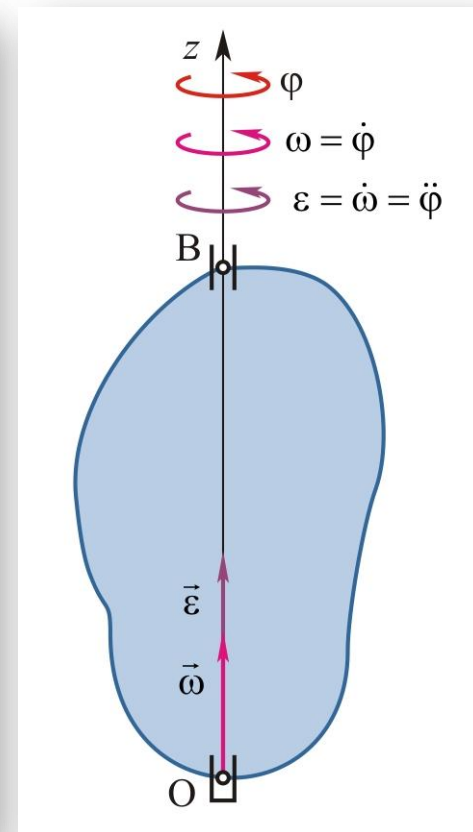
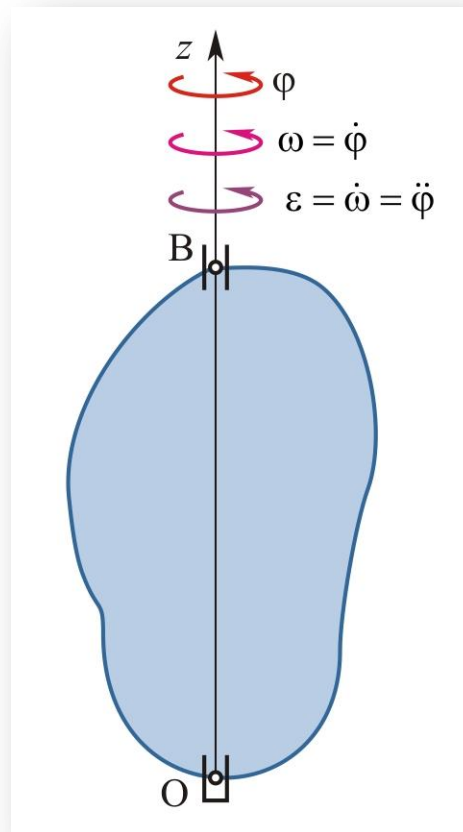
$$\varepsilon_{sr} = \frac{\Delta\omega}{\Delta t}$$

Ugaono ubrzanje

$$\varepsilon = \lim_{\Delta t \rightarrow 0} \varepsilon_{sr} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \dot{\omega} = \ddot{\phi}$$

Vektor ugaonog ubrzanja

$$\vec{\varepsilon}(t) = \varepsilon(t)\vec{k}$$



9. Kinematika obrtanja krutog tela oko nepokretne ose (brzina i ubrzanje tačaka tela)

Obrtanja oko nepokretne ose – brzine tačka tela

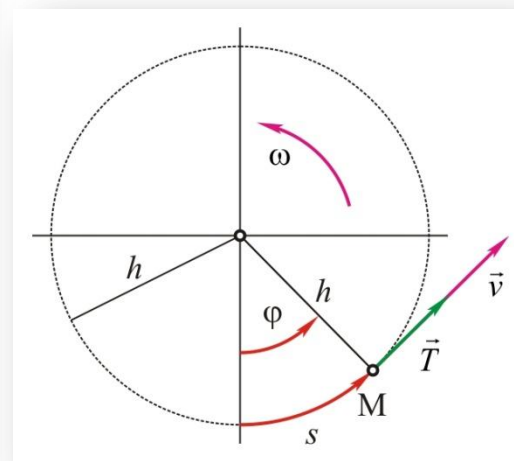
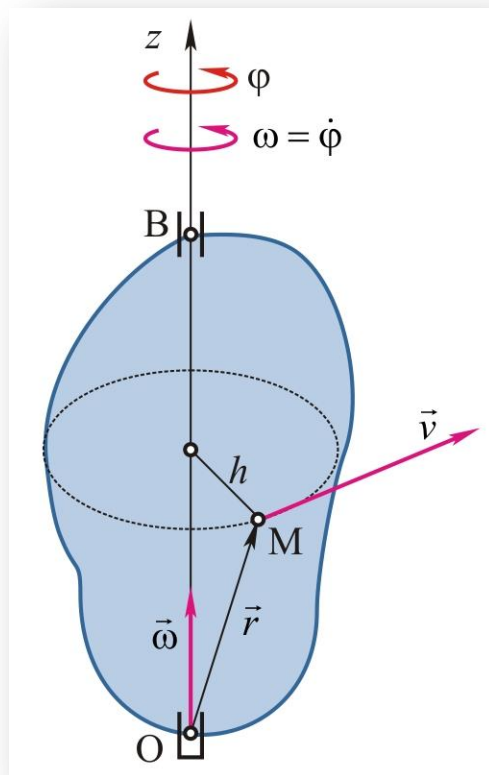
$$\vec{v} = v_T \vec{T} = \dot{s} \vec{T}$$

$$s = h\varphi$$

$$\vec{v} = h\dot{\varphi} \vec{T} = h\omega \vec{T}$$

$$v = h\omega$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$



Obrtanja oko nepokretne ose – ubrzanja tačka tela

$$\vec{a} = a_T \vec{T} + a_N \vec{N}$$

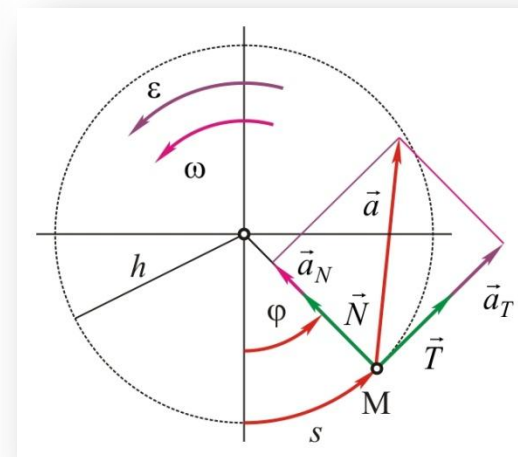
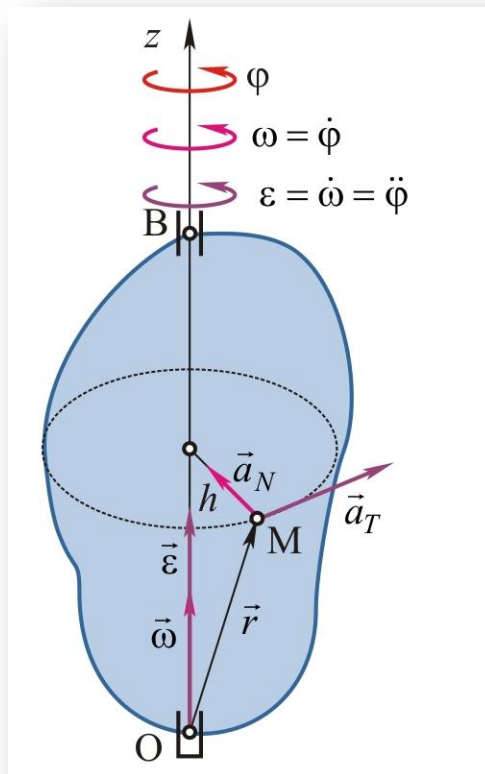
$$\vec{a} = \ddot{s} \vec{T} + \frac{\dot{s}^2}{h} \vec{N}$$

$$\vec{a} = h\ddot{\phi} \vec{T} + h\dot{\phi}^2 \vec{N}$$

$$\vec{a} = h\varepsilon \vec{T} + h\omega^2 \vec{N}$$

$$a_T = h\varepsilon$$

$$a_N = h\omega^2$$



Obrtanja oko nepokretne ose – ubrzanja tačka tela

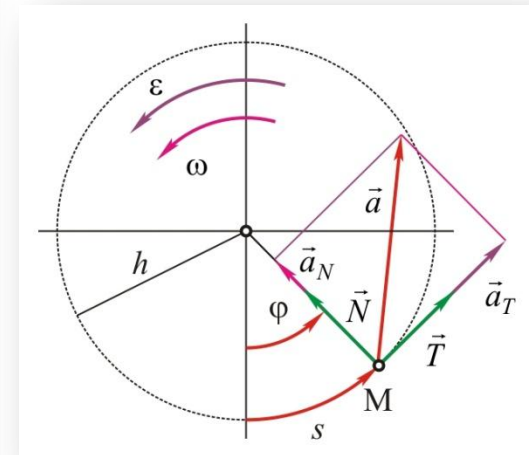
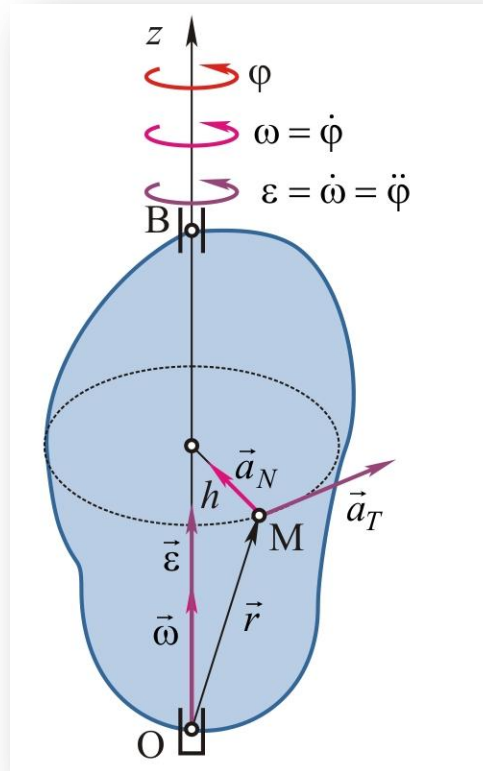
$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \dot{\vec{v}} = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}}$$

$$\vec{a} = \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times \vec{v}$$

$$\vec{a}_T = \vec{\varepsilon} \times \vec{r}$$

$$\vec{a}_N = \vec{\omega} \times \vec{v}$$



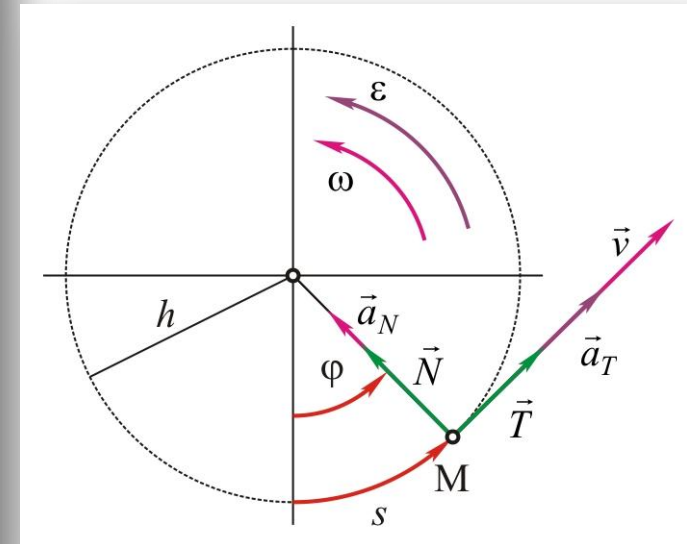
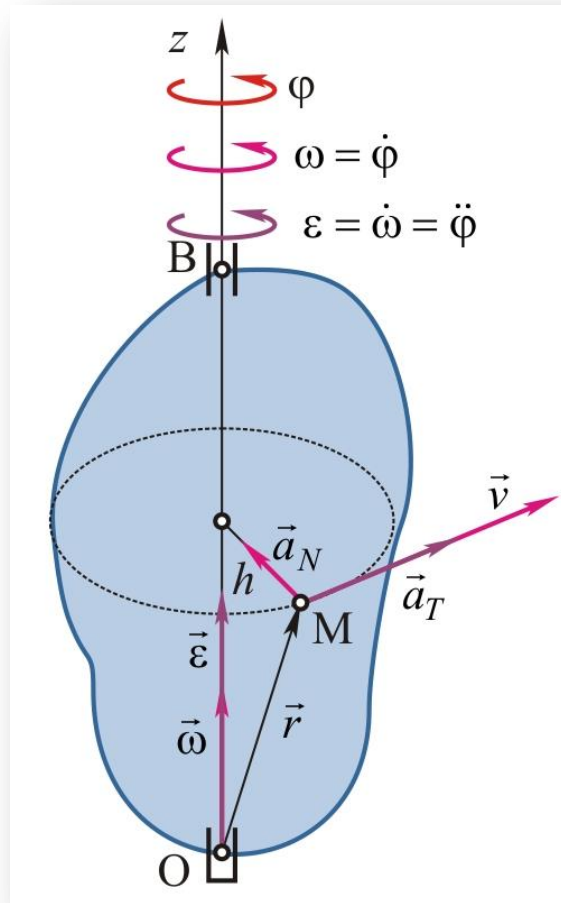
Obrtanja oko nepokretne ose – brzine i ubrzanja tačka tela

$$\vec{v} = h\omega\vec{T}$$

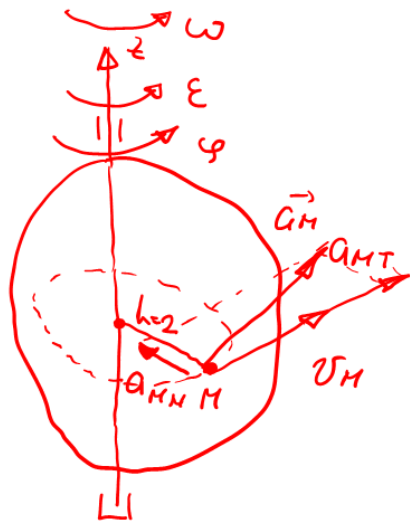
$$\vec{a} = h\varepsilon\vec{T} + h\omega^2\vec{N}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times \vec{v}$$



Primer

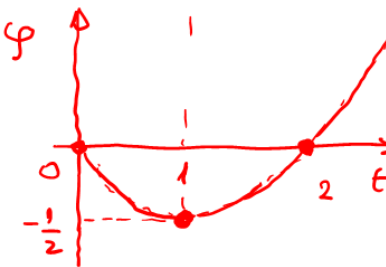
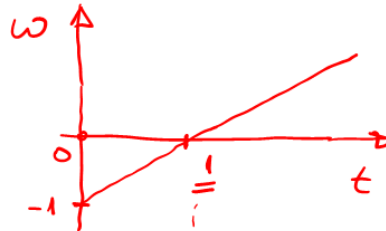
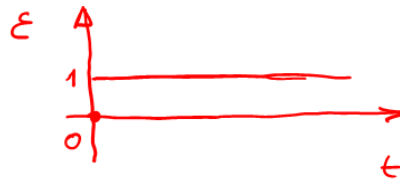


$$\varphi(t) = \frac{t^2}{2} - t = t \left(\frac{t}{2} - 1 \right)$$

$$\omega(t) = \dot{\varphi}(t) = \underline{t - 1}$$

$$\varepsilon(t) = \dot{\omega}(t) = \ddot{\varphi}(t) = 1 = \cos 0^\circ$$

$$\varphi(1) = 1 \cdot \left(\frac{1}{2} - 1 \right) = -\frac{1}{2}$$



$$\underline{\underline{t = 2}}$$

$$\varphi(2) = 0$$

$$\omega(2) = 2 - 1 = 1$$

$$\varepsilon(2) = 1$$

$$v_M(2) = h \omega(2) = 2 \cdot 1 = 2$$

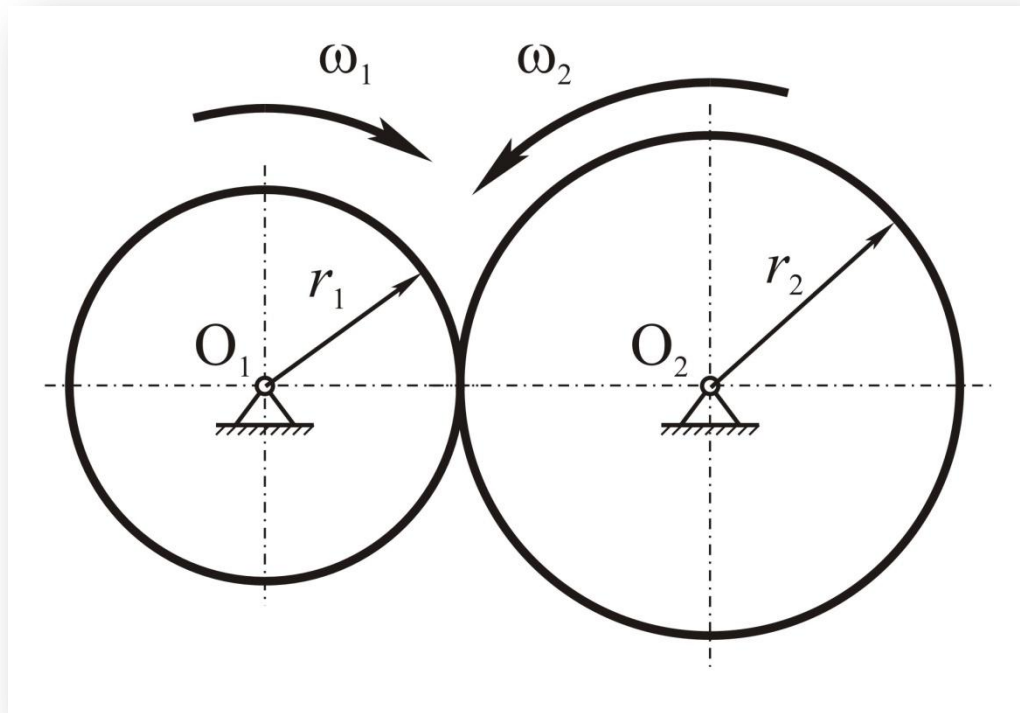
$$a_{MT}(2) = h \varepsilon(2) = 2 \cdot 1 = 2$$

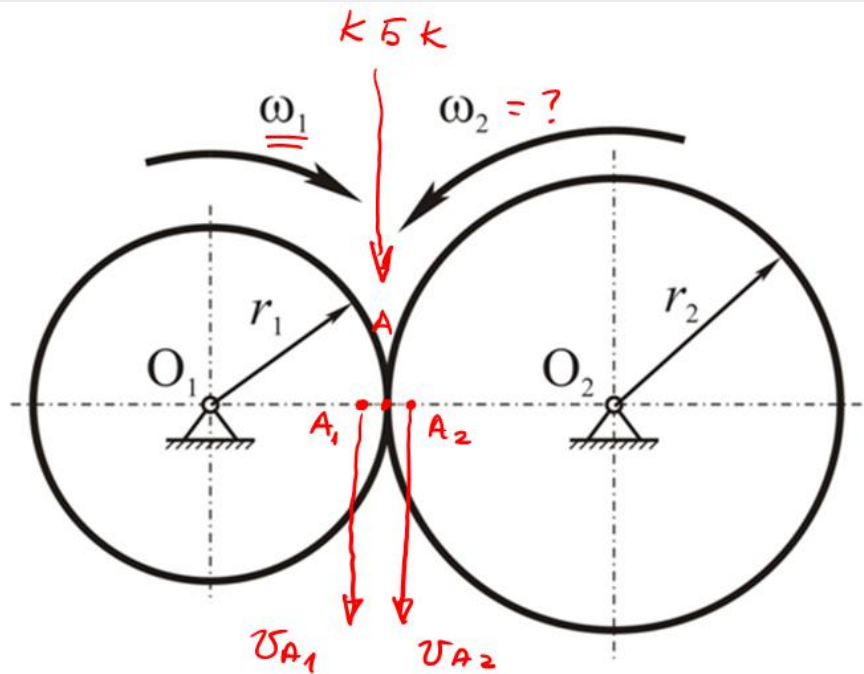
$$a_{MN}(2) = h \omega^2(2) = 2 \cdot 1^2 = 2$$

$$\begin{aligned} a_M(2) &= \sqrt{a_{MT}^2(2) + a_{MN}^2(2)} \\ &= \sqrt{2^2 + 2^2} = 2\sqrt{2} \end{aligned}$$

Primer

Ugaona brzina pogonskog zupčanika, poluprečnika r_1 , iznosi ω_1 . Odrediti ugaonu brzinu ω_2 gonjenog zupčanika, poluprečnika r_2 .



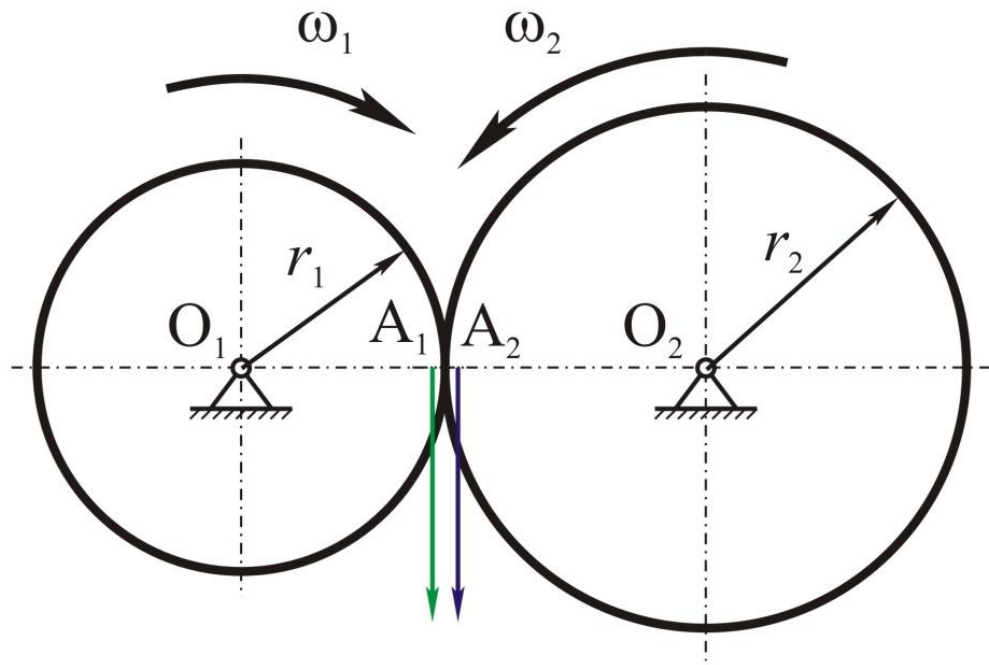


КБК → КОТРОЛАНЕ БЕЗ
КАЧАЊА

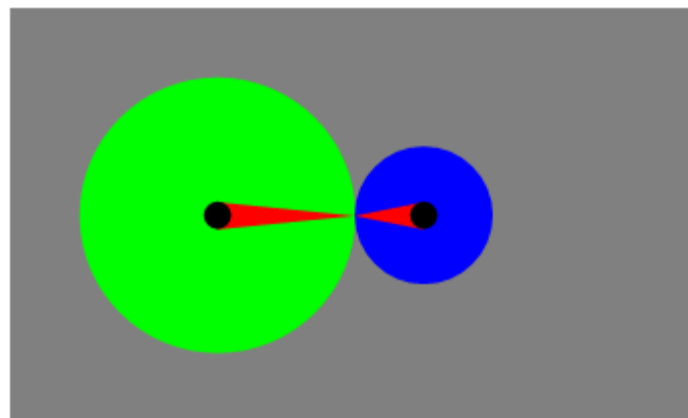
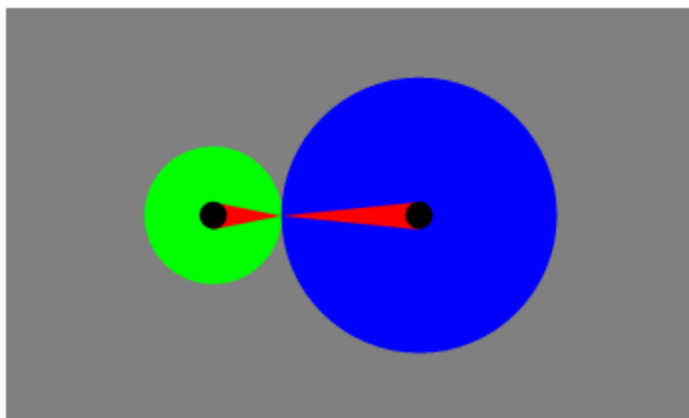
$$\left. \begin{array}{l} v_{A1} = r_1 \omega_1 \\ v_{A2} = r_2 \omega_2 \end{array} \right\} \text{КБК} \rightarrow v_{A1} = v_{A2} \rightarrow r_1 \omega_1 = r_2 \omega_2$$

$$* \left\{ \omega_2 = \frac{r_1}{r_2} \omega_1 = \frac{\omega_1}{\frac{r_2}{r_1}} = \frac{\omega_1}{i} \quad \perp \quad i = \frac{r_2}{r_1} \right.$$

$$* / \frac{d}{dt} \rightarrow \varepsilon_2 = \frac{r_1}{r_2} \varepsilon_1 = \frac{\varepsilon_1}{i}$$

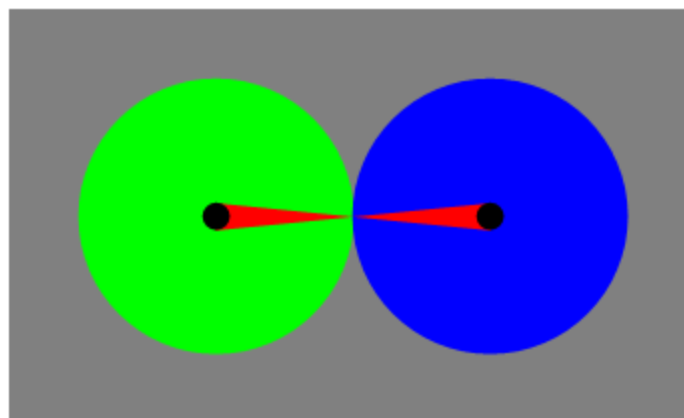


$$v_{A_1} = v_{A_2} \rightarrow r_1 \omega_1 = r_2 \omega_2 \rightarrow \omega_2 = \frac{r_1}{r_2} \omega_1$$



$$r_2 = 2r_1$$

$$\omega_2 = \frac{1}{2}\omega_1$$



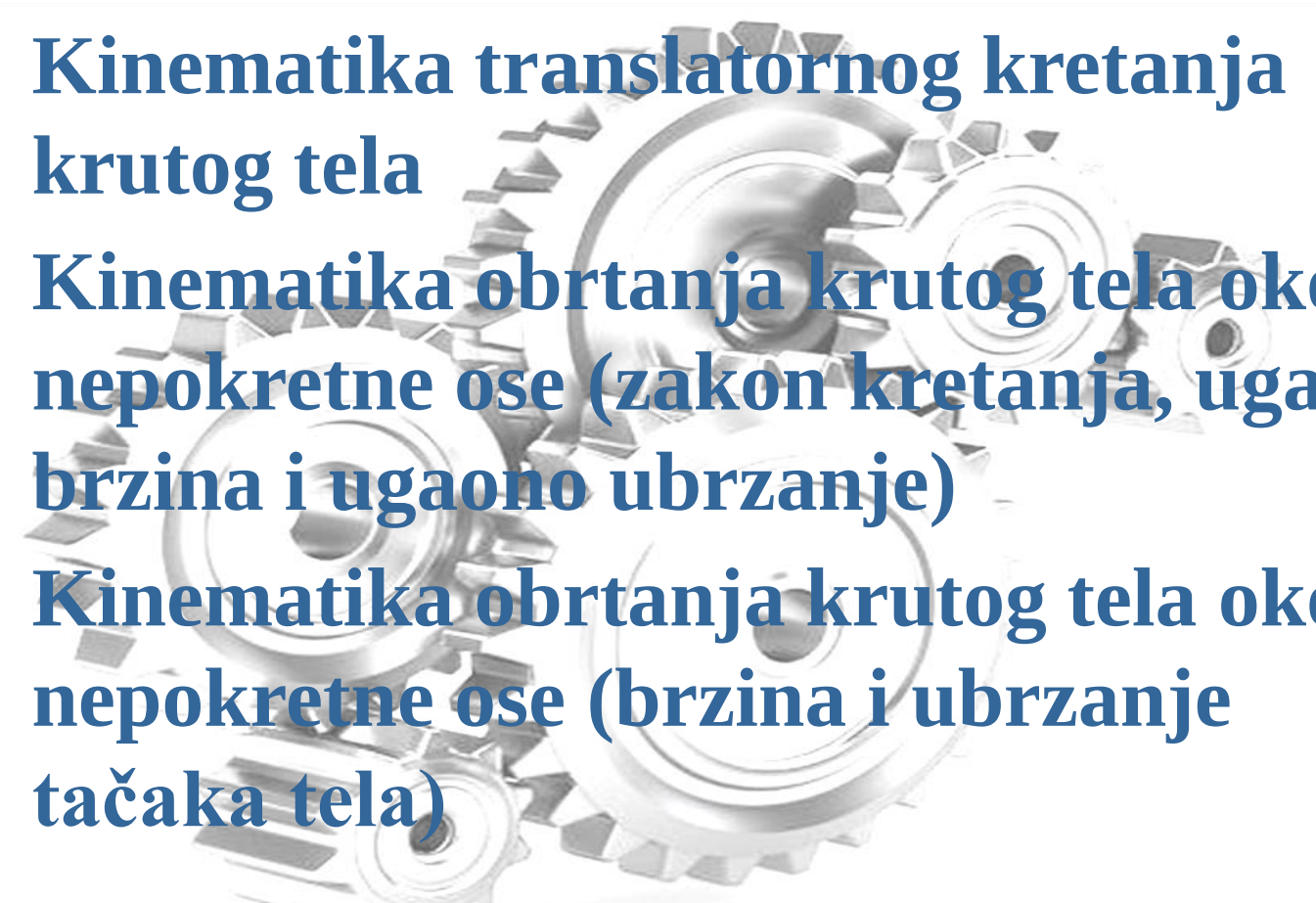
$$r_2 = \frac{r_1}{2}$$

$$\omega_2 = 2\omega_1$$

$$r_2 = r_1$$

$$\omega_2 = \omega_1$$

Šta smo naučili?

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