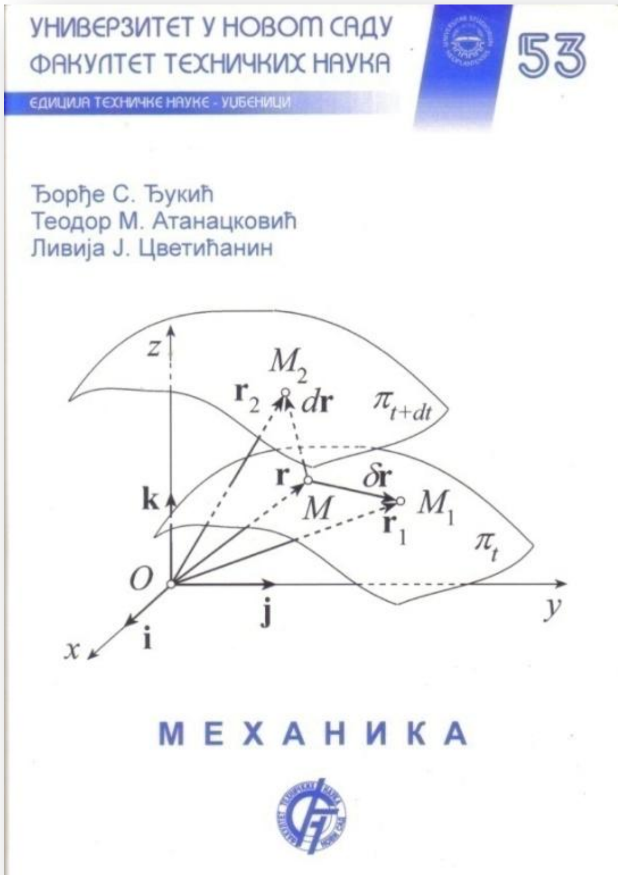
Dinamika – vežbe 4

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.

Literatura


УНИВЕРЗИТЕТ У НОВОМ САДУ
ФАКУЛТЕТ ТЕХНИЧКИХ НАУКА
ЕДИЦИЈА ТЕХНИЧКЕ НАУКЕ - УЏБЕНИЦИ
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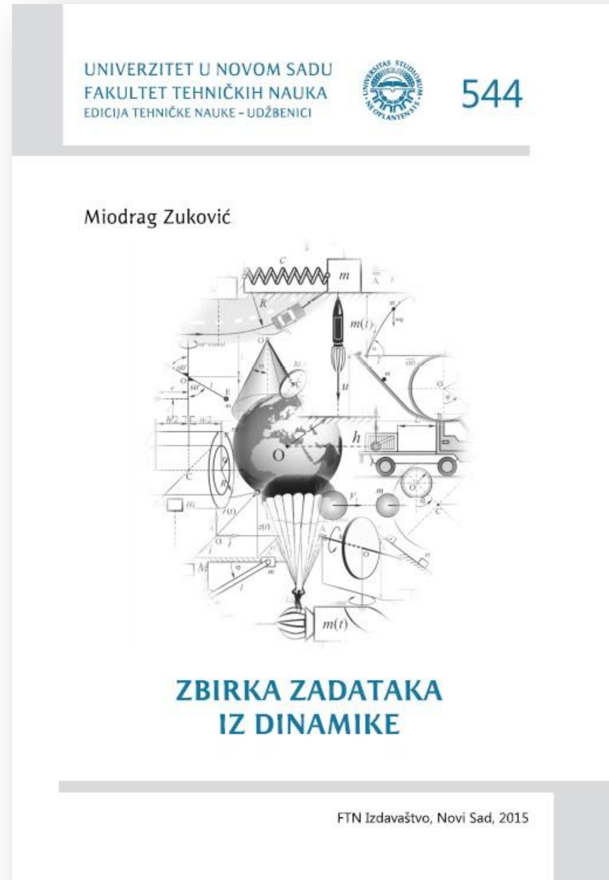
Ђорђе С. Ђукић
Теодор М. Атанацковић
Ливија Ј. Цветићанин
МЕХАНИКА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FAKULTET TEHNIČKIH NAUKA
EDICIJA TEHNIČKE NAUKE - UDŽBENICI
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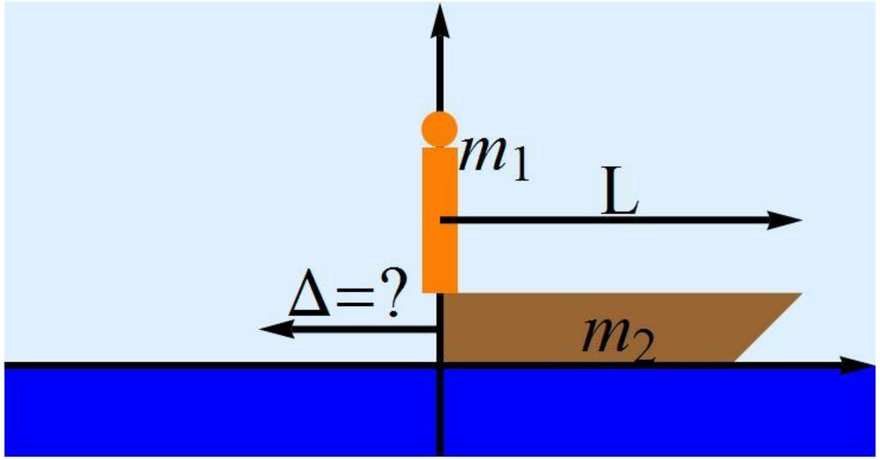
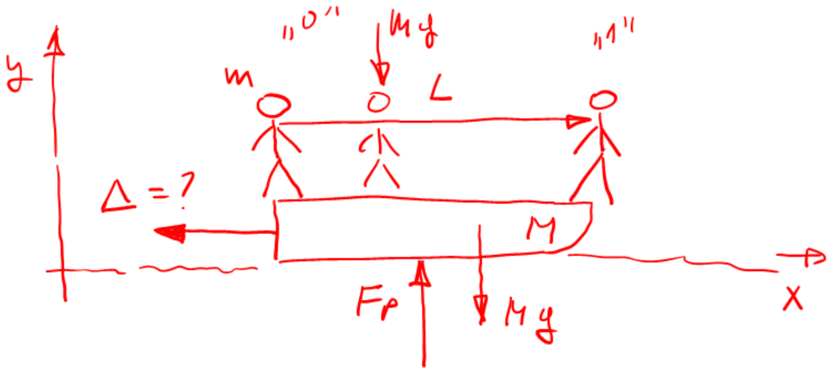
Miodrag Zuković
ZBIRKA ZADATAKA
IZ DINAMIKE
FTN Izdavaštvo, Novi Sad, 2015

Zadatak 1

Čovek, mase m_1, stoji na levom kraju čamca, mase m_2 i dužine L. Za koliko će se pomeriti čamac ako čovek pređe na drugi kraj. Otpor vode kretanju čamca zanemariti (male brzine). Sistem u početnom trenutku miruje.





$\dot{x}_c = 0 \rightarrow x_c = const$

$x_{c1} = x_{c0}$

$$\frac{m x_{m1} + M x_{M1}}{\cancel{M_s}} = \frac{m x_{m0} + M x_{M0}}{\cancel{M_s}}$$

$$m(\underbrace{x_{m1} - x_{m0}}) + M(\underbrace{x_{M1} - x_{M0}}) = 0$$

$\Delta x_m = -\Delta + L$ $\qquad$ $\Delta x_M = -\Delta$

t=0 систем мирује

$M_s \vec{a}_c = \vec{F}_g^s$; $M_s = m + M$

$/ \cdot \vec{i}$

$M_s \ddot{x}_c = 0 \rightarrow \ddot{x}_c = 0$

$\dot{x}_c = const = \dot{x}_c(0) = 0$

$\vec{r}_c = \frac{\sum m_i \cdot \vec{r}_i}{M_s}$; $x_c = \frac{\sum m_i \cdot x_i}{M_s}$

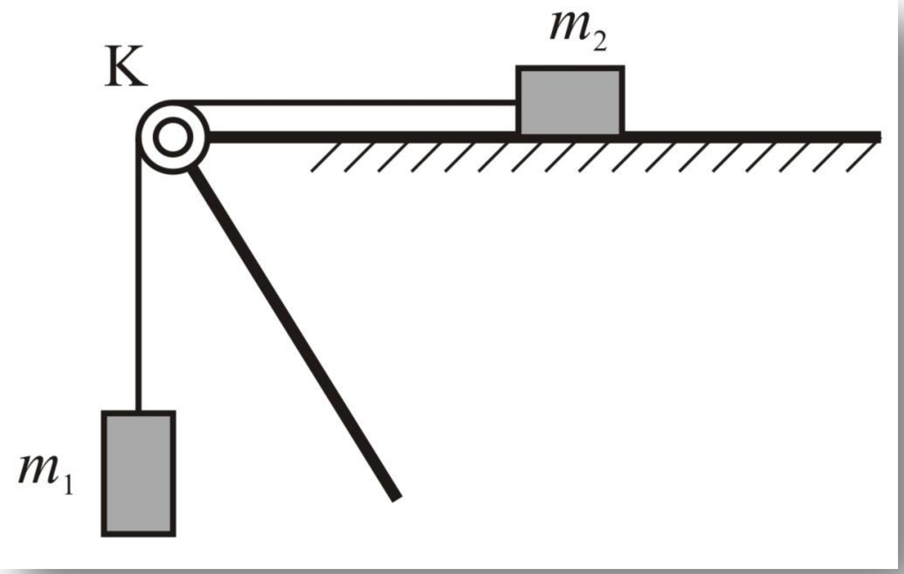
$-m\Delta + mL - M\Delta = 0$

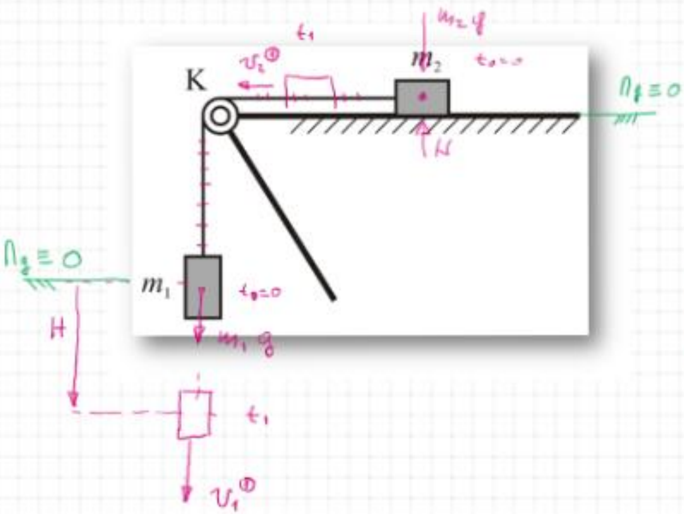
$\Delta (m + M) = mL$

$$\Delta = \frac{m}{m + M} L$$

Zadatak 2

Ako je prikazani sistem u početnom trenutku mirovao odrediti brzine tegova u položaju u kome se levi teg spustio za *H*.





$$E_{k1} + \Pi_1 = \cancelto{0}{E_{k0}} + \cancelto{0}{\Pi_0}$$

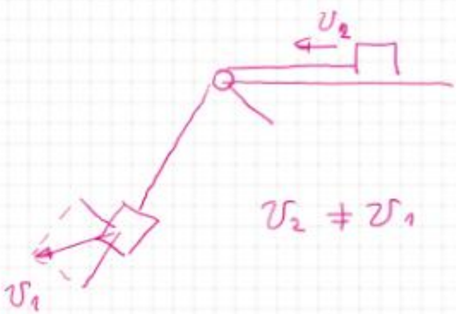
$$E_k = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\boxed{\frac{1}{2} m_1 \left(v_1^{(1)}\right)^2 + \frac{1}{2} m_2 \left(v_2^{(1)}\right)^2 - m_1 g \cdot H = 0} \quad (1)$$

ДОП. ЈЕДН. $\quad v_2 = v_1$

ИДЕАЛНО УЖЕ
+
ТРАНСЛ.

$$(2) \quad v_1^{(1)} = v_2^{(1)} = V$$



$$v_2 \neq v_1$$

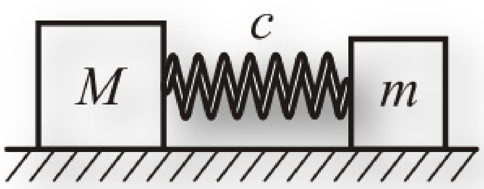
$$(2) \longrightarrow (1) \longrightarrow \frac{1}{2} m_1 V^2 + \frac{1}{2} m_2 V^2 = m_1 g H \qquad / \cdot 2$$

$$(m_1 + m_2) V^2 = 2 m_1 g H$$

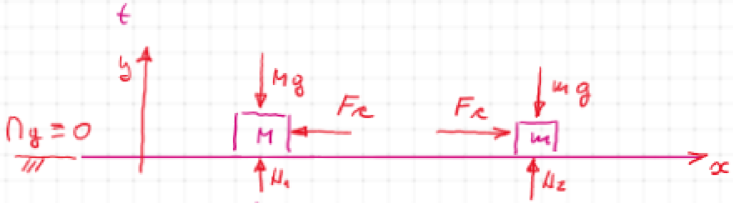
$$V = \sqrt{\frac{2 m_1 g H}{m_1 + m_2}}$$

Zadatak 3

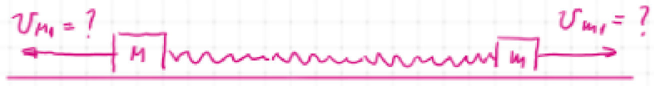
Na glatkom horizontalnom stolu nalaze se dve prizme, masa M i m. Prime se naslanjaju na krajeve opruga, krutosti c. U početnom trenutku sistem miruje, a opruga je sabijena za dužinu δ. Odrediti brzine prizmi u trenutku u kom opruga dostigne dužinu koju ima u nenapregnutom stanju.



$t_0=0$, $v_{M0}=v_{m0}=0$, $(\Delta l)_0=-\delta$



t_1 , $(\Delta l)_1=0$



I $\quad \vec{K}_1 - \cancel{\vec{K}_0}^{\,0} = \vec{I}^{\,s}_{01}$

$$\vec{K} = M\vec{v}_M + m\vec{v}_m$$

$$M\vec{v}_{M1} + m\vec{v}_{m1} = \int_{t_0}^{t_1} (M\vec{g} + m\vec{g} + \vec{N}_1 + \vec{N}_2)\,dt \quad /\cdot\vec{i}$$

$$\boxed{M(-v_{M1}) + m v_{m1} = 0} \quad (1)$$

$$v_{M1} = \frac{m}{M} v_{m1}$$

II $\quad E_{k1} + \Pi_1 = \cancel{E_{k0}}^{\,0} + \Pi_0$

$$E_k = \frac{1}{2} M v_M^2 + \frac{1}{2} m v_m^2$$

$$\Pi_0 = \cancel{\Pi_{g0}}^{\,0} + \Pi_{c0} = \frac{1}{2} c(-\delta)^2 = \frac{1}{2} c\delta^2$$

$$\Pi_1 = \cancel{\Pi_{g1}}_{\,0} + \Pi_{c1} = \frac{1}{2} c(0)^2 = 0$$

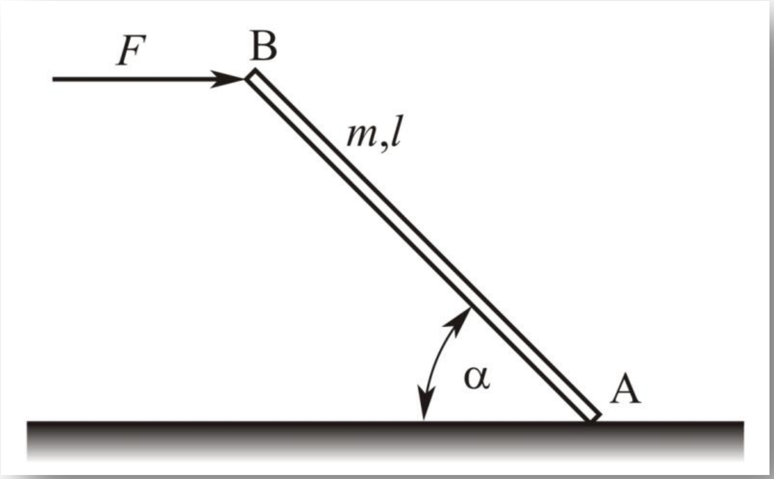
$$\Pi_c = \frac{1}{2} c(\Delta l)^2$$

$$(2) \quad \boxed{\frac{1}{2} M v_{M1}^2 + \frac{1}{2} m v_{m1}^2 = \frac{1}{2} c\delta^2}$$

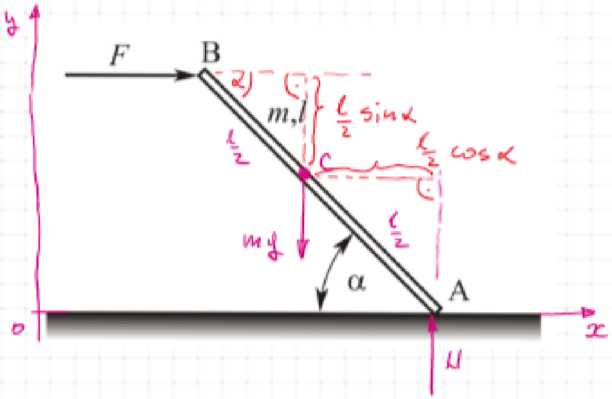
$$\left.\begin{matrix}(1)\\(2)\end{matrix}\right\} \rightarrow \quad \begin{matrix} v_{M1} = \\ v_{m1} = \end{matrix}$$

Zadatak 4

Štap AB, mase *m* i dužine *l*, krajem A klizi po glatkoj, horizontalnoj, nepokretnoj podlozi. Na kraj B deluje horizontalna sila konstantnog intenziteta *F*. Koliki treba da je intenzitet ove sile da bi se štap kretao translatorno, pri čemu gradi ugao α sa horizontalom. Odrediti kretanje štapa u tom slučaju.



$F = ?$



ТРАНСЛ. $\alpha = const \rightarrow y_c = const$

$$(4)\quad \boxed{\dot{y}_c = \ddot{y}_c = 0}$$

$$(4) \rightarrow (2) \quad 0 = -mg + N$$

$$\boxed{N = mg}$$

$$(3) \rightarrow \quad \boxed{F = N \cdot \frac{\cos\alpha}{\sin\alpha} = mg\, ctg\,\alpha}$$

$$(1) \quad \underline{\cancel{m}\, \ddot{x}_c = \cancel{m}\, g\, ctg\alpha} = const$$

T1 $\quad m\vec{a}_c = \Sigma \vec{F}_i^s$

$$m\vec{a}_c = \vec{F} + m\vec{g} + \vec{N} \quad /\cdot\vec{i}/\cdot\vec{j}$$

$$(1) \quad m\ddot{x}_c = F$$

$$(2) \quad m\ddot{y}_c = -mg + N$$

УСЛОВ ТРАНСЛ. КР.

$$\left.\begin{array}{l} \dfrac{d\vec{L}_c}{dt} = \Sigma \vec{M}_{Ci}^s \\ \text{ТРАНСЛ.} \rightarrow \vec{L}_c = 0 \end{array}\right\} \rightarrow \Sigma \vec{M}_{Ci}^s = 0$$

$$(3) \quad \Sigma M_c = 0$$

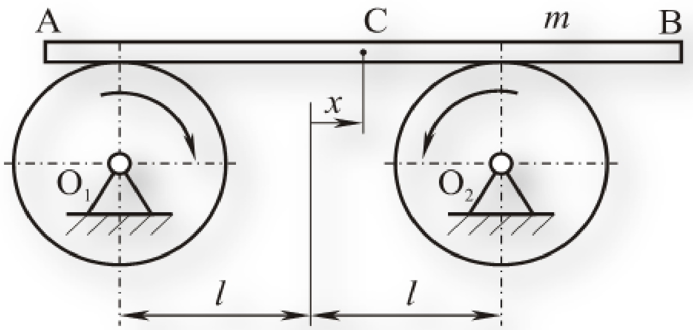
$$\boxed{(3) \quad -F\cdot\frac{\cancel{l}}{\cancel{2}}\sin\alpha + N\cdot\frac{\cancel{l}}{\cancel{2}}\cos\alpha = 0}$$

$$\ddot{x}_c = g\, ctg\alpha \rightarrow \dot{x}_c = g\, ctg\alpha \cdot t + c_1$$

$$\boxed{x_c = g\, ctg\alpha \cdot \frac{t^2}{2} + c_1 t + c_2}$$

Zadatak 5

Homogeni štap AB, mase m, položen je na dva kružna valjka, jednakih poluprečnika, koji se obrću oko paralelnih nepokretnih osa u suprotnim smerovima. Ose valjaka su horizontalne i nalaze se na istoj visini, na rastojanju $2l$. Usled delovanja sila suvog trenja, u dodirnim tačkama štapa i valjaka, štap se kreće, pri čemu stalno proklizava u odnosu na valjke. Koeficijent trenja između štapa i oba valjka je isti i iznosi μ. Odrediti kretanje štapa, ako je u početnom trenutku štap mirovao, a centar štapa bio pomeren za x_0 u odnosu na svoj simetrični položaj prema valjcima.

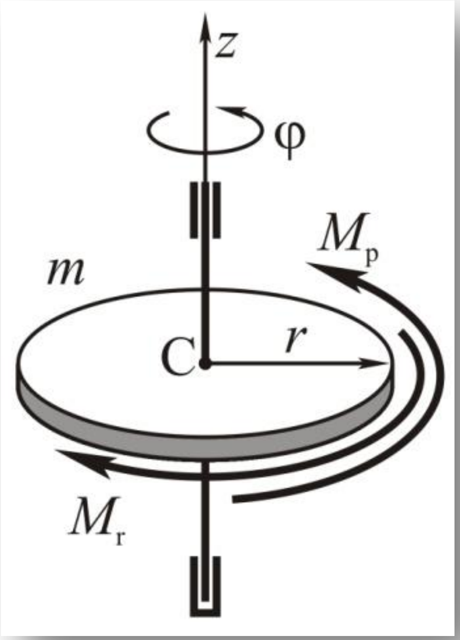


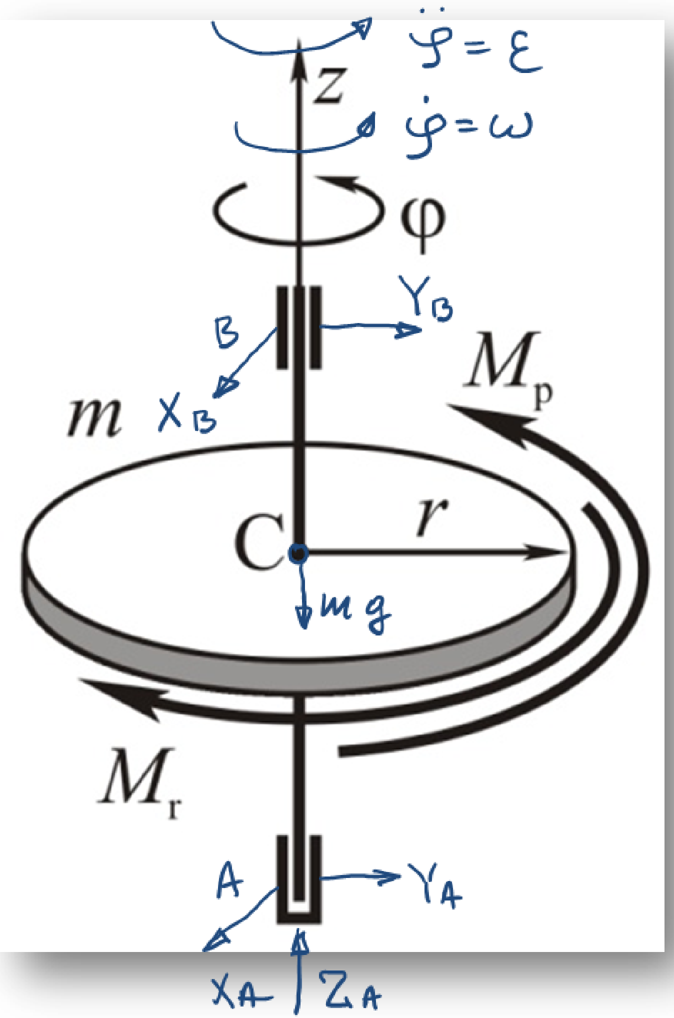
Zadatak 6

Disk, mase m i poluprečnika r, obrće se oko nepokretne ose. Na njega deluje pogonski moment koji je funkcija ugaone brzine

$$M_p = M_0 \left(1 - \frac{\dot{\varphi}}{\Omega}\right)$$

i konstantan radni moment M_r. Odrediti zakon promene ugaone brzine diska, koji kreće iz stanja mirovanja.





ДЈК

$$\boxed{J_z \cdot \varepsilon = \Sigma M_z}$$

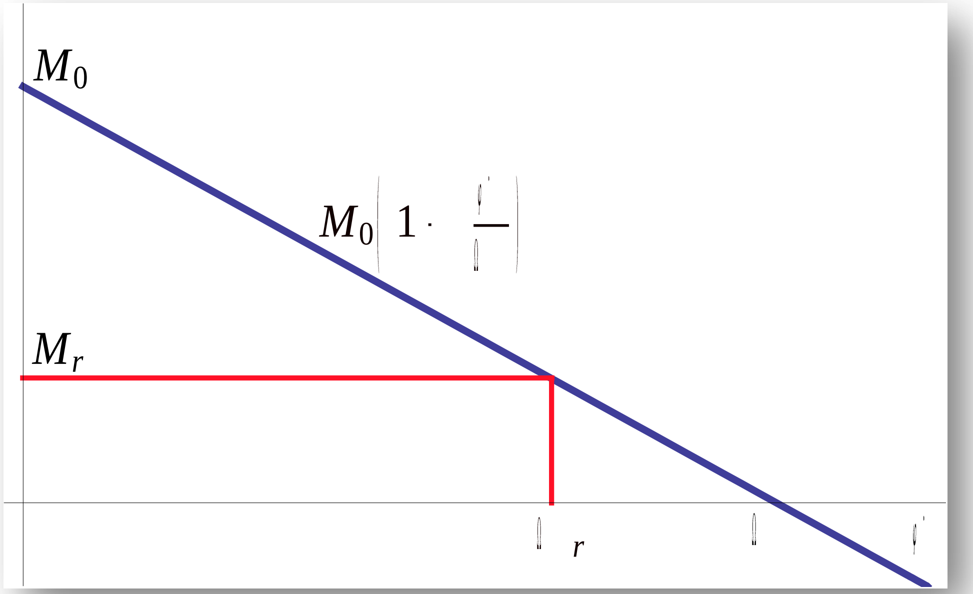
$$J_z = \frac{m r^2}{2}$$

$$J_z \ddot{\varphi} = M_p - M_r$$

$$J_z \frac{d\dot{\varphi}}{dt} = \left(M_0\left(1 - \frac{\dot{\varphi}}{\Omega}\right) - M_r\right)$$

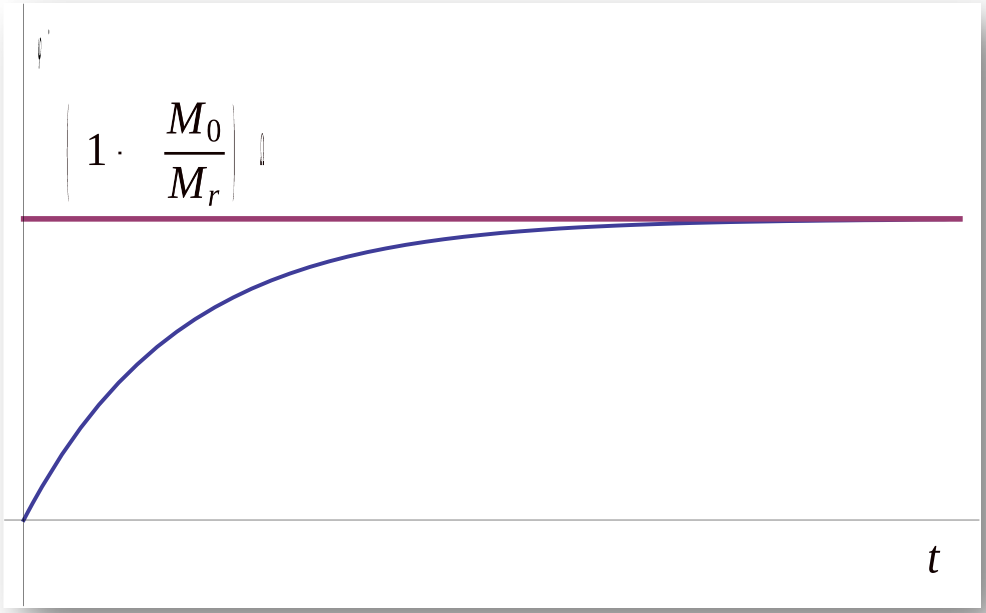
$$\int_{\dot{\varphi}(0)=0}^{\dot{\varphi}} \frac{J_z \, d\dot{\varphi}}{M_0\left(1 - \frac{\dot{\varphi}}{\Omega}\right) - M_r} = \int_0^t dt$$

$$-\frac{J_z\Omega}{M_0}\ln\left(\frac{M_0\left(1-\frac{\dot{\varphi}}{\Omega}\right)-M_r}{M_0-M_r}\right)=t$$



$$\frac{M_0\left(1-\frac{\dot{\varphi}}{\Omega}\right)-M_r}{M_0-M_r}=e^{-\frac{M_0}{J_z\Omega}t}$$

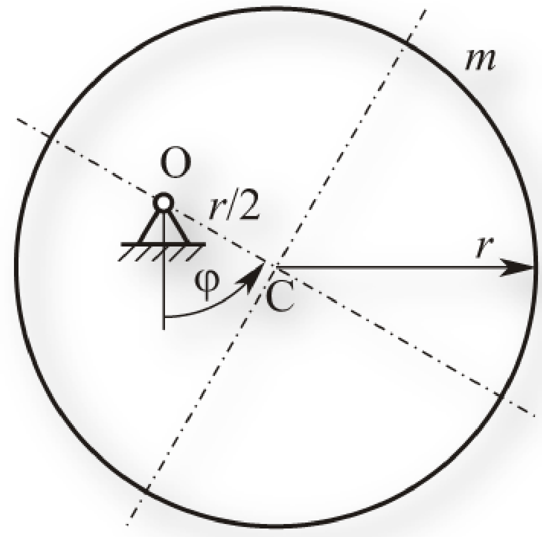
$$\dot{\varphi}=\Omega\left(1-\frac{M_r}{M_0}\right)-\Omega\left(1-\frac{M_r}{M_0}\right)e^{-\frac{M_0}{J_z\Omega}t}$$



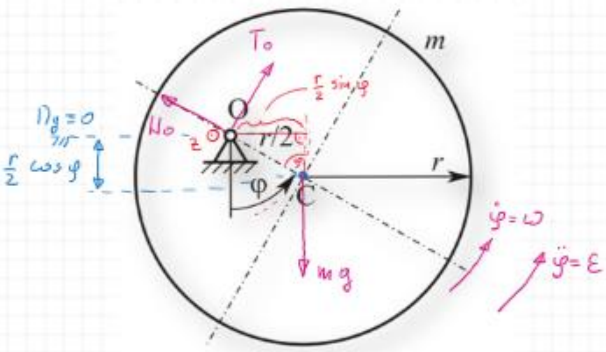
$$\lim_{t\to\infty}\dot{\varphi}=\Omega\left(1-\frac{M_r}{M_0}\right)=\Omega_r$$

Zadatak 7

Disk, mase m i poluprečnika r, vezan je u tački O, koja se nalazi na rastojanju $\overline{OC} = r/2$ od centra diska, nepokretnim cilindričnim zglobom čija osa je horizontalna. Disk kretanje započinje iz položaja $\varphi(0) = \varphi_0$ iz stanja mirovanja. Odrediti ugaonu brzinu diska u funkciji položaja, ugla φ.



$\omega(\varphi) = ?$



ОБРТ. ОКО НЕП. ОСЕ

ДЈК

$$\boxed{J_z \cdot \varepsilon = \Sigma M_z}$$

$$\frac{3}{4} m r^2 \cdot \ddot{\varphi} = - m g \cdot \frac{r}{2} \sin\varphi$$

$$\boxed{\ddot{\varphi} = -\frac{2}{3}\frac{g}{r}\sin\varphi}$$

$$\omega(\varphi) = \dot{\varphi}(\varphi) = ?$$

$$\ddot{\varphi} = \frac{d\dot{\varphi}}{dt} = \frac{d\varphi}{dt}\cdot\frac{d\dot{\varphi}}{d\varphi} = \dot{\varphi}\frac{d\dot{\varphi}}{d\varphi}$$

$$\dot{\varphi}\frac{d\dot{\varphi}}{d\varphi} = -\frac{2}{3}\frac{g}{r}\sin\varphi$$

$$\int_{\dot{\varphi}(0)=0}^{\dot{\varphi}} \dot{\varphi}\, d\dot{\varphi} = -\frac{2}{3}\frac{g}{r}\int_{\varphi(0)=\varphi_0}^{\varphi} \sin\varphi\, d\varphi$$

$$\left.\frac{\dot{\varphi}^2}{2}\right|_0^{\dot{\varphi}} = +\frac{2}{3}\frac{g}{r}\left.\cos\varphi\right|_{\varphi_0}^{\varphi}$$

$$\frac{\dot{\varphi}^2}{2} - \frac{0^2}{2} = \frac{2}{3}\frac{g}{r}(\cos\varphi - \cos\varphi_0)$$

$$\dot{\varphi}^2 = \frac{4}{3}\frac{g}{r}(\cos\varphi - \cos\varphi_0)$$

$$\dot{\varphi} = \pm\sqrt{}$$

ДИСК

$$J_{z'} = J_c = \frac{mr^2}{2}$$

ШТАЈНЕРОВА Т.

$$J_z = J_{z'} + m\,\overline{OC}^2$$

$$J_O = J_c + m\left(\frac{r}{2}\right)^2$$

$$J_O = \frac{mr^2}{2} + m\frac{r^2}{4} = \frac{3}{4} m r^2$$

II $\quad E_k + \Pi = \cancel{E_{k0}}^{\,0} + \Pi_0$

$$E_k = \frac{1}{2} J_z \omega^2 = \frac{1}{2}\frac{3}{4} m r^2 \omega^2$$

$$\Pi = - m g \cdot \frac{r}{2}\cos\varphi$$

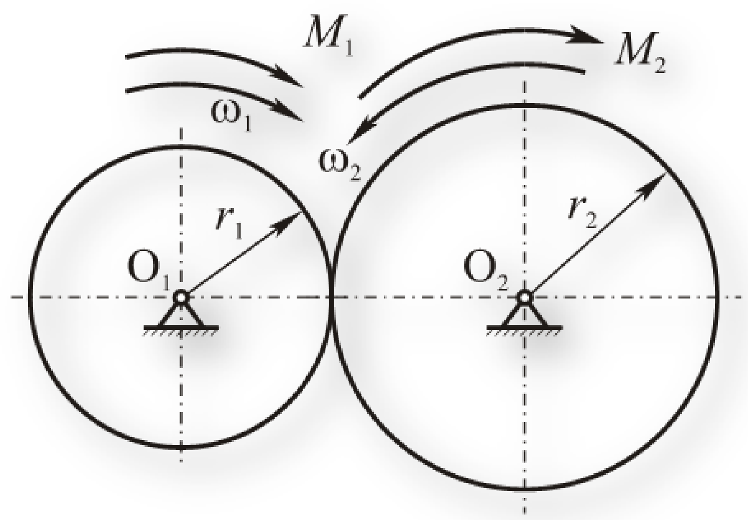
$$\Pi_0 = - m g \frac{r}{2}\cos\varphi_0$$

$$\cancel{\frac{1}{2}}\frac{3}{4}\cancel{m}r^{\cancel{2}}\omega^2 - \cancel{m} g \cancel{\frac{r}{2}}\cos\varphi = - \cancel{m} g \cancel{\frac{r}{2}}\cos\varphi_0$$

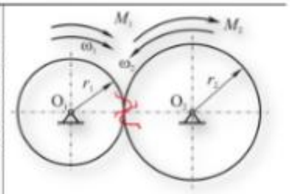
$$\boxed{\omega^2 = \frac{4}{3}\frac{g}{r}(\cos\varphi - \cos\varphi_0)}$$

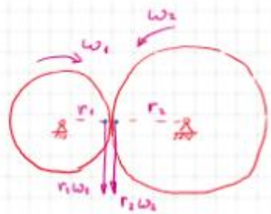
Zadatak 8

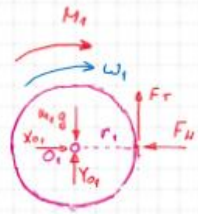
Odrediti obrtne momente, M_1 i M_2 , koji deluju na zupčanike zupčastog para pri njegovom stacionarnom kretanju – kretanju konstantnim ugaonim brzinama ω_1 i ω_2 . Poluprečnici zupčanika zupčastog prenosnika su r_1 i r_2.

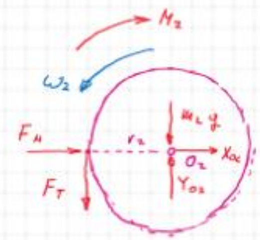


Odrediti vezu između obrtnih momenata M_1 i M_2, koji deluju na zupčanike zupčastog para pri njegovom stacionarnom kretanju – kretanju konstantnim ugaonim brzinama ω_1 i ω_2. Poluprečnici zupčanika zupčastog prenosnika su r_1 i r_2.









KBK → $r_1\omega_1 = r_2\omega_2$

$$\omega_2 = \frac{r_1}{r_2}\omega_1 = \frac{\omega_1}{\frac{r_2}{r_1}} = \frac{\omega_1}{i}$$

$$\varepsilon_2 = \frac{\varepsilon_1}{i}$$

(1) $J_{O_1}\varepsilon_1 = \Sigma M_{O_1}$

$$J_{O_1}\varepsilon_1 = M_1 - F_T\cdot r_1$$

(2) $J_{O_2}\varepsilon_2 = \Sigma M_{O_2}$

$$J_{O_2}\cdot\varepsilon_2 = -M_2 + F_T\cdot r_2$$

$$\left.\begin{aligned} (1)\rightarrow\ & F_T = \frac{M_1}{r_1} - J_{O_1}\frac{\varepsilon_1}{r_1} \\ (2)\rightarrow\ & F_T = \frac{M_2}{r_2} + J_{O_2}\frac{\varepsilon_2}{r_2} \end{aligned}\right\} \rightarrow \frac{M_2}{r_2} + J_{O_2}\frac{\varepsilon_2}{r_2} = \frac{M_1}{r_1} - J_{O_1}\frac{\varepsilon_1}{r_1} \quad /\cdot r_2$$

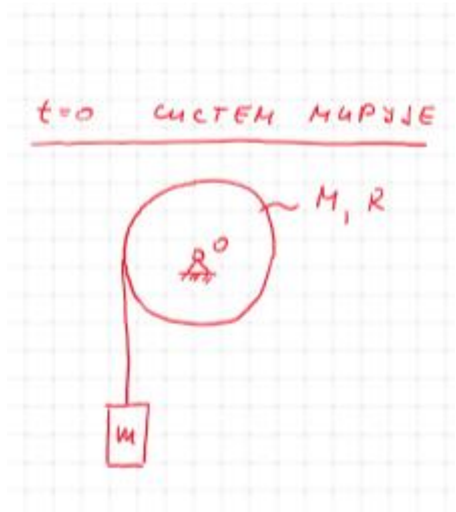
$$M_2 = \frac{r_2}{r_1}M_1 - \left(J_{O_1}\frac{r_2}{r_1}\varepsilon_1 + J_{O_2}\varepsilon_2\right)$$

СТАЦ. КР. $\omega_1 = const$, $\omega_2 = const$ → $\varepsilon_1 = \varepsilon_2 = 0$

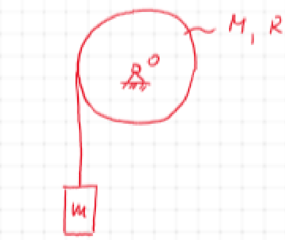
$$M_2 = \frac{r_2}{r_1}M_1$$

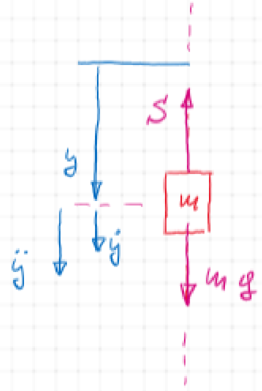
$$\boxed{M_2 = i\,M_1}$$

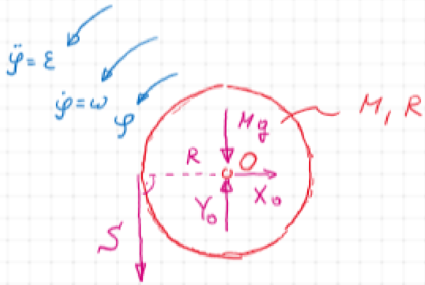
Zadatak 9

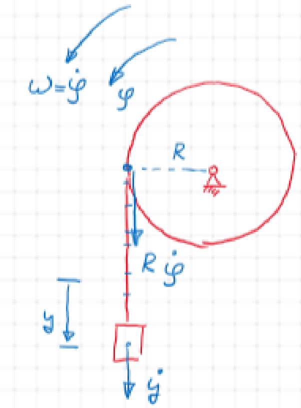


$t=0$ СИСТЕМ МИРУЈЕ









(1) $J_O \, \varepsilon = \Sigma M_O$

$$\frac{MR^2}{2}\cdot\ddot{\varphi} = +S\cdot R$$

(2) $m\ddot{y} = mg - S$

$$y,\ \varphi,\ S = ?$$

ДОП. ЈЕД. (3) $\dot{y} = R\dot{\varphi} \rightarrow \ddot{y} = R\ddot{\varphi}$

(3) → (2) → $mR\ddot{\varphi} = mg - S \rightarrow$ * $S = mg - mR\ddot{\varphi} = \dots$

(1) $\frac{MR^{\cancel{2}}}{2}\ddot{\varphi} = (mg - mR\ddot{\varphi})\cdot\cancel{R}$

$$\left(\frac{MR}{2} + mR\right)\ddot{\varphi} = mg \rightarrow \ddot{\varphi} = \frac{mg}{\left(\frac{M}{2}+m\right)R} = const \ \int \rightarrow \dot{\varphi} \ \int \rightarrow \varphi$$

$$* \quad S = mg - m\cancel{R}\cdot\frac{mg}{\left(\frac{M}{2}+m\right)\cancel{R}} = \dots$$

Dinamika – vežbe 4

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.