

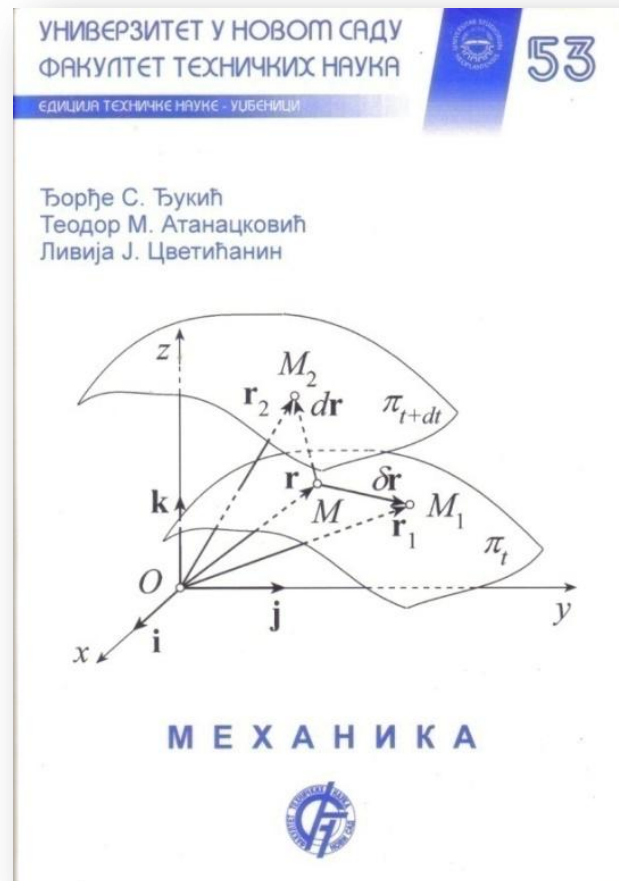
Mehanika 2 (Kinematika)

Predavanja 2

Miodrag Zuković
Novi Sad, 2023.

Literatura

- Đorđe S. Đukić, Teodor M. Atanacković, Livija J. Cvetićanin:
Mehanika, Fakultet tehničkih nauka u Novom Sadu, Novi Sad, 2003.



Ispit

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	
	Br. Indeksa	Prezime	Ime	Pr.	Vež.	D. 1	D. 2	Zad. 1	Zad. 2	Zad. 3	Zad.	Teor. 1	Teor. 2	Teor. 3	Teor.	Bod.	O
1	MM xx/xxxx	Marković	Marko	2	2	3	3	10	10	10	30	10	10	10	30	100	
2	MM xx/xxxx	Petrović	Petar	2		3		10			10		10		10	35	

Šta ćemo naučiti?

- 5. Pravolinijsko kretanje tačke. Ravnomerno i ravnomerno promenljivo kretanje
- 6. Kinematika tačke - polarni koordinatni sistem

5. Pravolinijsko kretanje tačke. Ravnomerno i ravnomerno promenljivo kretanje

Pravolinijsko kretanje tačke

- Vektor položaja

$$\vec{r}(t) = x(t)\vec{i}$$

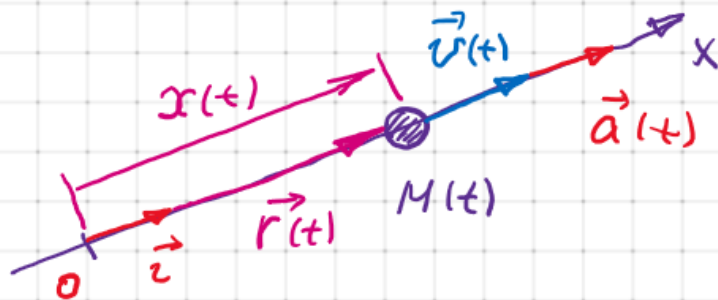
- Brzina

$$\vec{v}(t) = v_x(t)\vec{i} = \dot{x}(t)\vec{i} = \dot{\vec{r}}(t)$$

- Ubrzanje

$$\vec{a}(t) = a_x(t)\vec{i} = \dot{v}_x(t)\vec{i} = \ddot{x}(t)\vec{i} = \dot{\vec{v}}(t) = \ddot{\vec{r}}(t)$$

Pravolinijsko kretanje tačke



$$\vec{r}(t) = x(t) \vec{e}$$

$$\vec{v}(t) = \dot{\vec{r}}(t) = \dot{x}(t) \vec{e}$$

$$\vec{a}(t) = \dot{\vec{v}}(t) = \ddot{x}(t) \vec{e}$$

УБРЗАНО КР.

$$|\vec{v}| \uparrow$$

$$\vec{v} \cdot \vec{a} > 0$$

$$\dot{x} \cdot \ddot{x} > 0$$



УСЛОРЕНО КР.

$$\vec{v} \cdot \vec{a} < 0$$

$$\dot{x} \cdot \ddot{x} < 0$$



Primer 3

- Pravolinijsko kretanje tačke opisano je parametarskom jednačinom

$$x(t) = 6t^2 - t^3$$

- Odrediti brzinu i ubrzanje tačke i nacrtati dijagrame njihove promene,
- odrediti trenutak t^* u kom tačka menja smer kretanja,
- odrediti put koji će tačka preći tokom prvih 6s kretanja,
- odrediti intervale vremena, tokom prvih 6s kretanja, u kojima će se tačka kretati ubrzano, odnosno, usporeno.

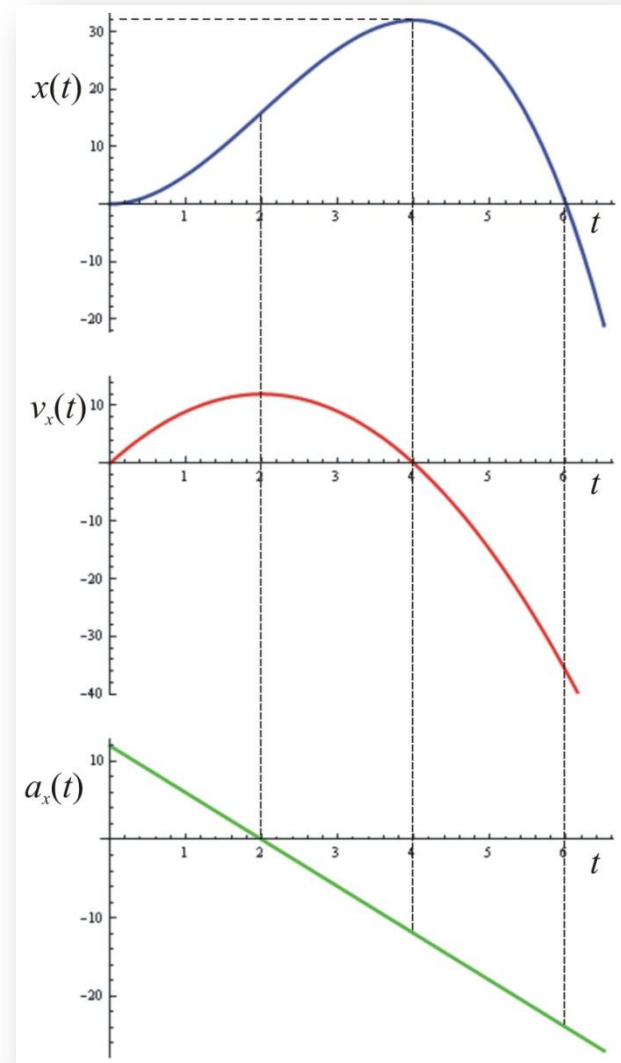
$$x(t) = 6t^2 - t^3 = t^2(6 - t)$$

↓

$$\dot{x}(t) = 12t - 3t^2 = 3t(4 - t)$$

↓

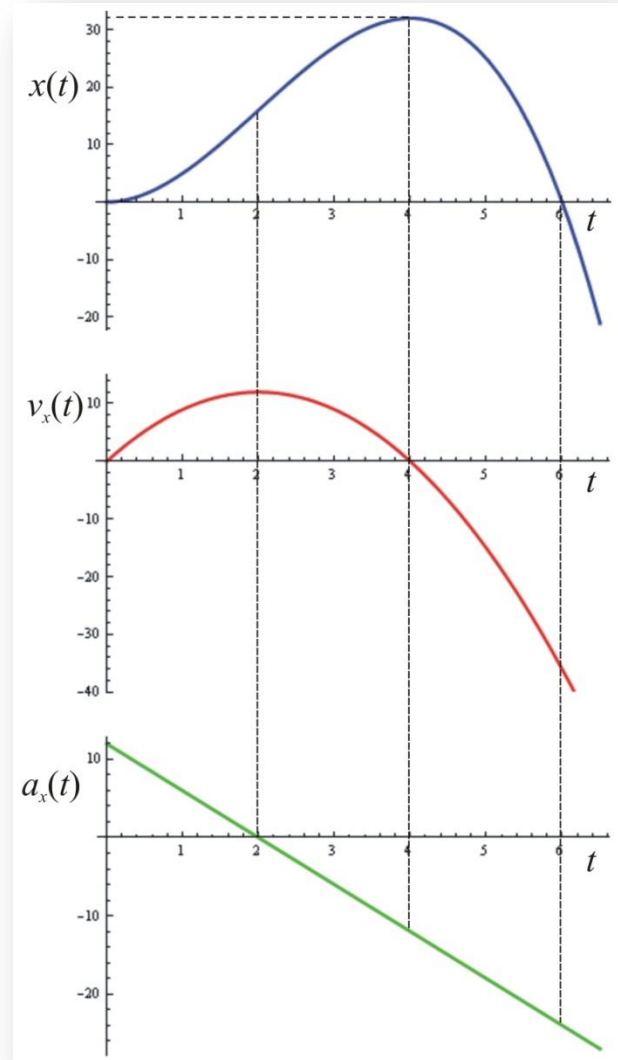
$$\ddot{x}(t) = 12 - 6t = 6(2 - t)$$



$$\dot{x}(t^*) = 3t^*(4 - t^*) = 0$$

↓

$$t^* = 4$$



$$P[0,6] = P[0,4] + P[4,6]$$

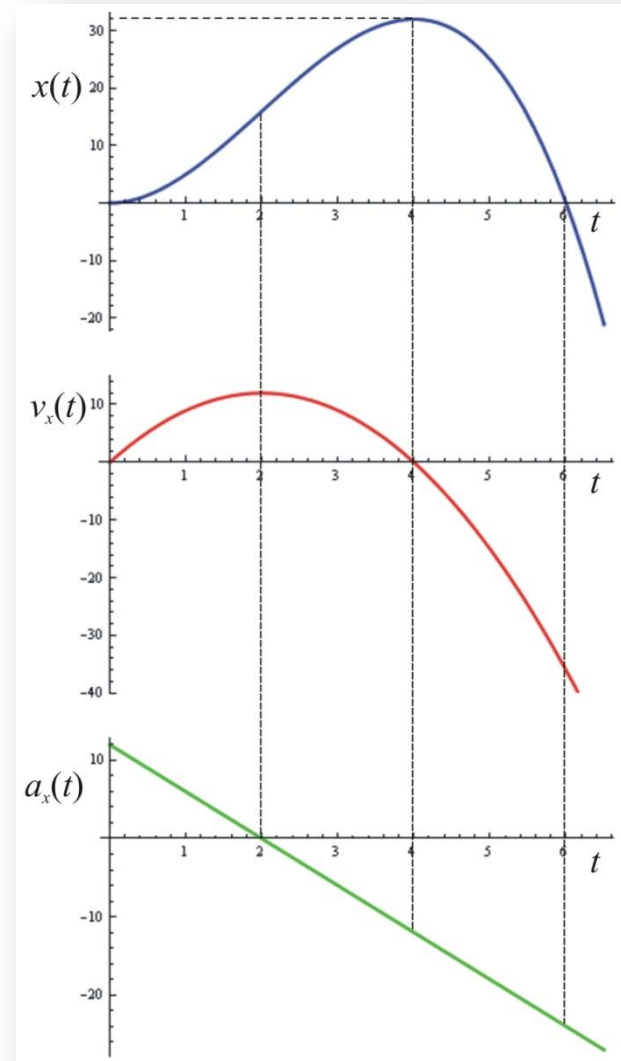
↓

$$P[0,6] = |x(4) - x(0)| + |x(6) - x(4)|$$

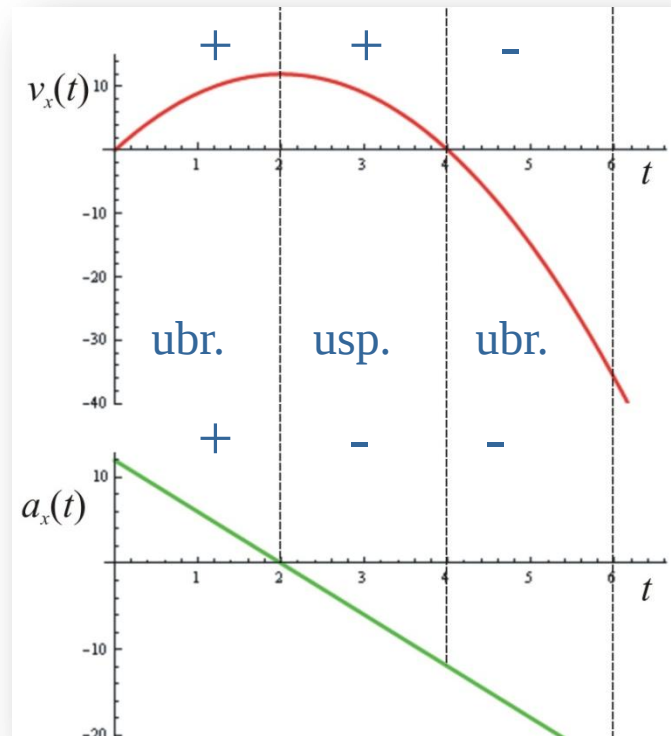
↓

$$P[0,6] = |32 - 0| + |0 - 32| = 64$$

$$x(t) = 6t^2 - t^3 = t^2(6 - t)$$



$t \in (0,2)$, $\dot{x}(t) > 0, \ddot{x}(t) > 0 \rightarrow$ ubrzano kretanje
 $t \in (0,2)$, $\dot{x}(t) > 0, \ddot{x}(t) < 0 \rightarrow$ usporeno kretanje
 $t \in (4,6)$, $\dot{x}(t) < 0, \ddot{x}(t) < 0 \rightarrow$ ubrzano kretanje

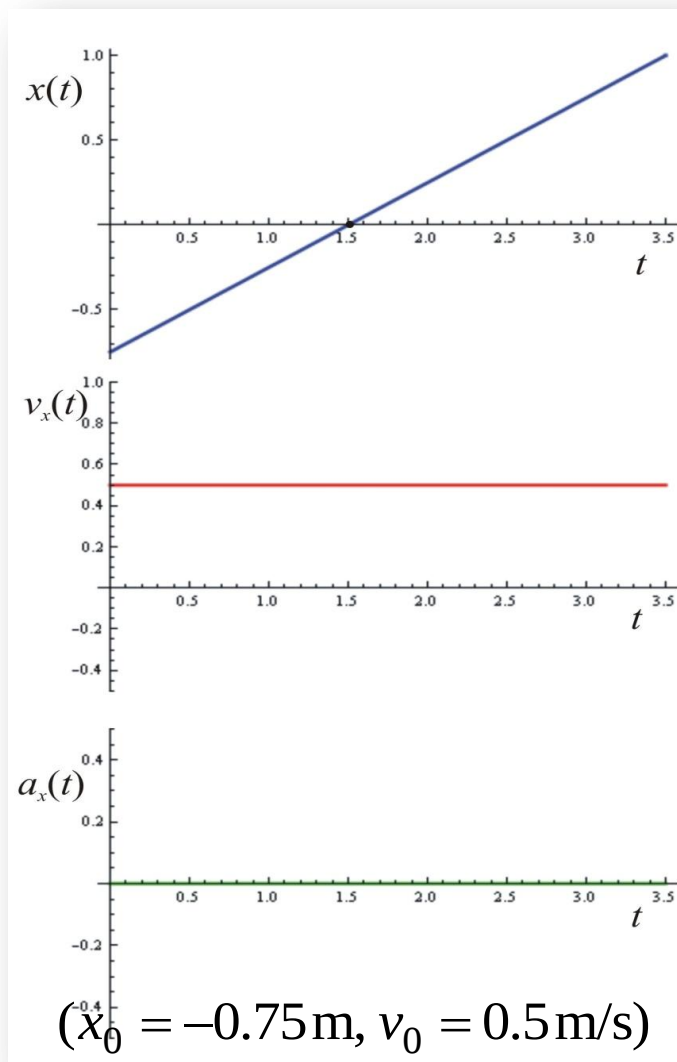


Ravnomerno kretanje tačke

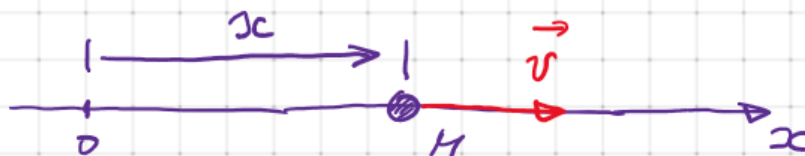
$$v_x(t) = v_0 = \text{const.}$$

$$a_x(t) = 0 = \text{const.}$$

$$x(t) = v_0 t + x_0$$



Ravnomerno kretanje tačke



$$v = \text{const} = v_0 \quad ; \quad x(0) = x_0$$

$$\dot{x} = v_0 = \text{const} \quad \rightarrow \quad \ddot{x} = 0$$

↓

$$\frac{dx}{dt} = v_0 \quad \rightarrow \quad \int dx = v_0 \int dt$$

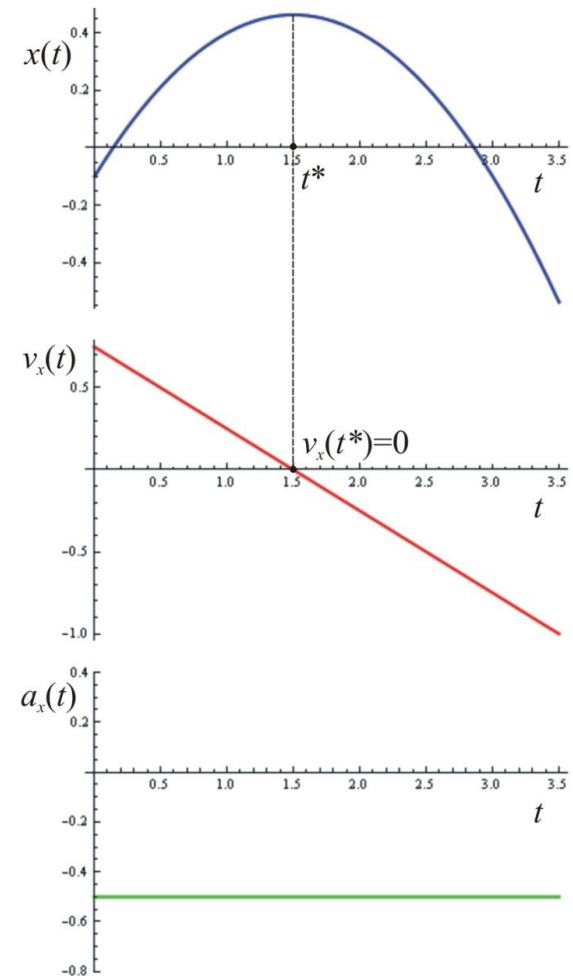
$$x(t) = v_0 \cdot t + x_0$$

Ravnomerno promenljivo kretanje tačke

$$a_x(t) = a_0 = \text{const}$$

$$v_x(t) = a_0 t + v_0$$

$$x(t) = \frac{1}{2} a_0 t^2 + v_0 t + x_0$$



$$(x_0 = -0.1\text{m}, v_0 = 0.75\text{m/s}, a_0 = -0.5\text{m/s}^2)$$

Ravnomerno promenljivo kretanje tačke



$$\ddot{x} = a_0 = \text{const} \quad \underline{\text{ny}} \quad \begin{cases} t = 0 \\ x(0) = x_0 \\ \dot{x}(0) = v_0 \end{cases}$$

$$\frac{d\dot{x}}{dt} = a_0 \rightarrow \int d\dot{x} = a_0 \int dt \rightarrow$$

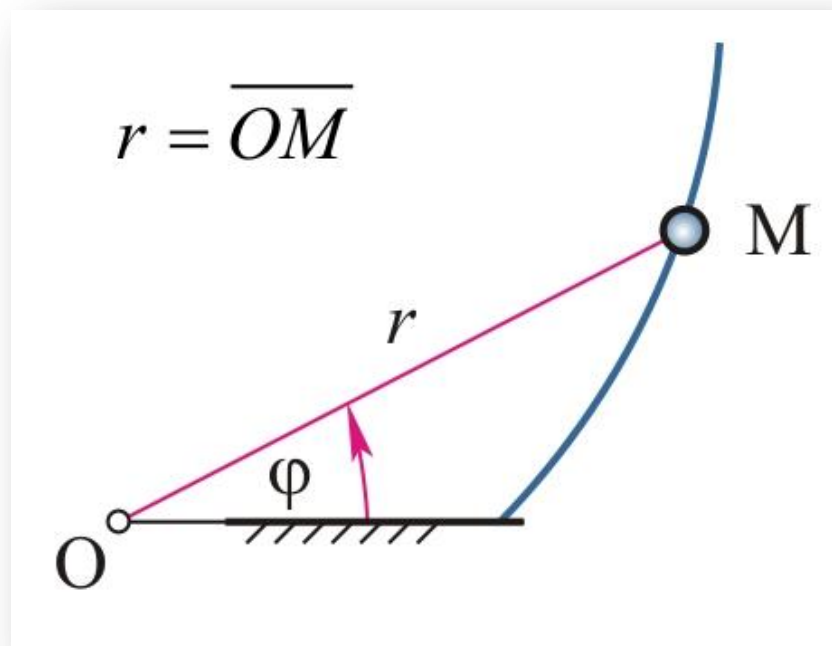
$$(a) \quad \left\{ \dot{x} = a_0 \cdot t + c_1 \right\} \rightarrow \frac{dx}{dt} = a_0 t + c_1 \rightarrow$$

$$\int dx = a_0 \int t dt + c_1 \int dt \rightarrow (b) \quad \left\{ x = a_0 \frac{t^2}{2} + c_1 t + c_2 \right\}$$

$$\underline{\text{ny}} \quad \left. \begin{array}{l} (a) \quad \dot{x}(0) = c_1 = v_0 \\ (b) \quad x(0) = c_2 = x_0 \end{array} \right\} \rightarrow \begin{array}{l} (A) \quad \dot{x}(t) = a_0 t + v_0 \\ (B) \quad x(t) = a_0 \frac{t^2}{2} + v_0 t + x_0 \end{array}$$

6. Kinematika tačke - polarni koordinatni sistem

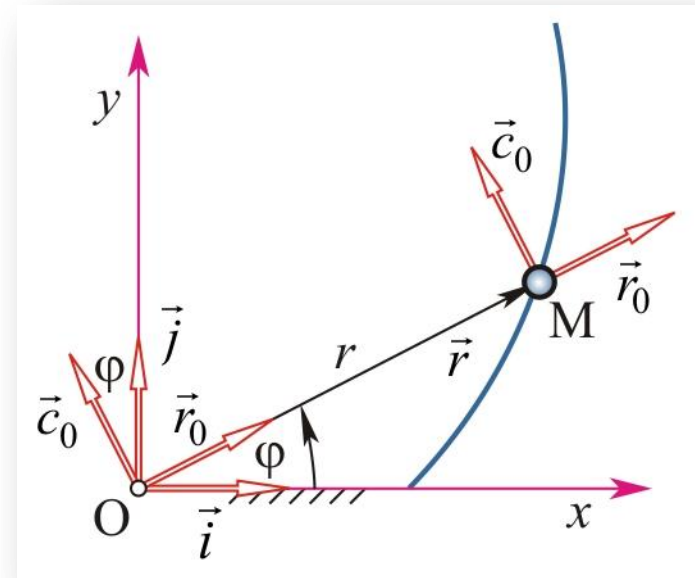
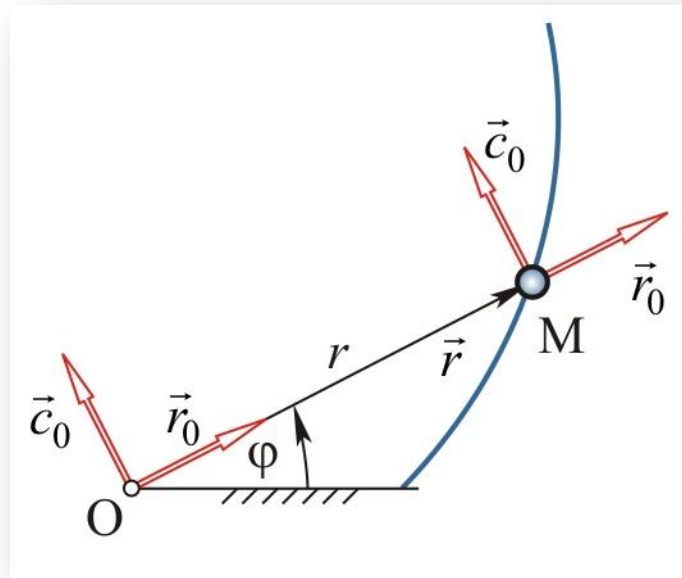
Polarne koordinate – parametarske jednačine kretanja



$r(t)$

$\varphi(t)$

Polarne koordinate – jedinični vektori



$\vec{r}_0$ - jedinični vektor radijalnog pravca

$\vec{c}_0$ - jedinični vektor cirkularnog pravca

Vektor položaja

$$\vec{r} = r \vec{r}_0$$

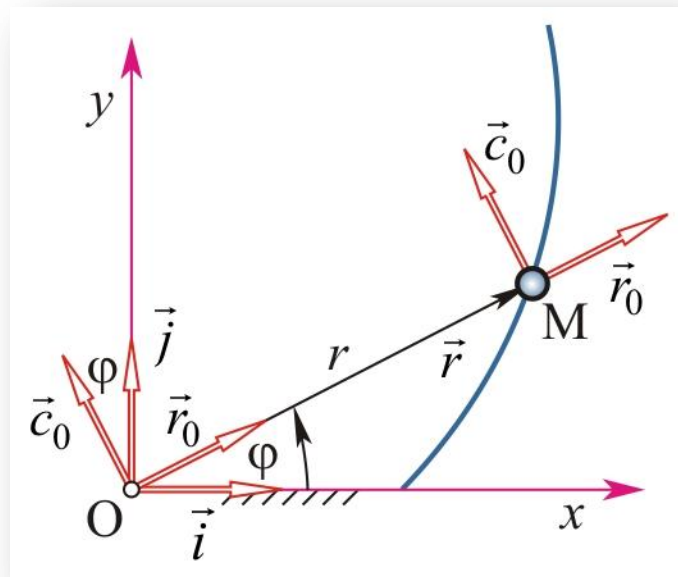
$$\vec{r}_0 = \cos \varphi \vec{i} + \sin \varphi \vec{j}$$

$$\vec{c}_0 = -\sin \varphi \vec{i} + \cos \varphi \vec{j}$$

$$\dot{\vec{r}}_0 = -\sin \varphi \dot{\varphi} \vec{i} + \cos \varphi \dot{\varphi} \vec{j} = \dot{\varphi} \vec{c}_0$$

$$\dot{\vec{c}}_0 = -\cos \varphi \dot{\varphi} \vec{i} - \sin \varphi \dot{\varphi} \vec{j} = -\dot{\varphi} \vec{r}_0$$

Polarne koordinate – veza sa Dekartovim koordinatama



$$\begin{aligned}x &= r \cos \varphi \\y &= r \sin \varphi\end{aligned}$$

$$\begin{aligned}r &= \sqrt{x^2 + y^2} \\ \tan \varphi &= \frac{y}{x}\end{aligned}$$

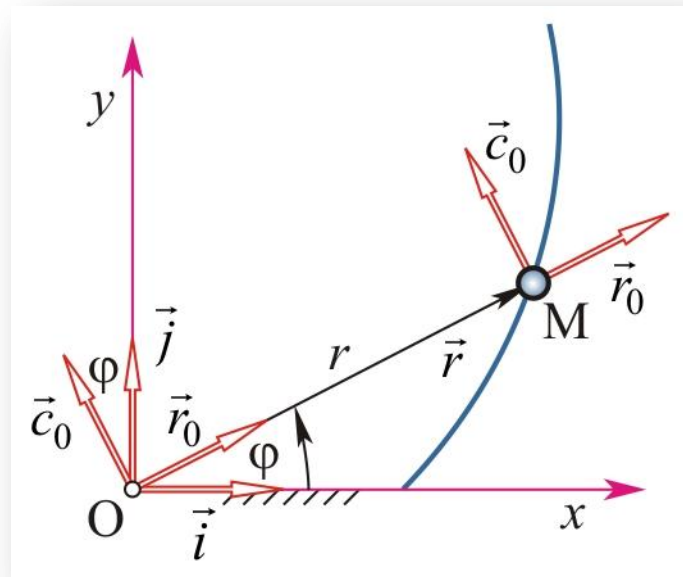
Polarne koordinate – brzina

Vektor položaja

$$\vec{r} = r \vec{r}_0$$

$$\dot{\vec{r}}_0 = \dot{\phi} \vec{c}_0$$

$$\dot{\vec{c}}_0 = -\dot{\phi} \vec{r}_0$$



Brzina

$$\vec{v} = \dot{\vec{r}} = \frac{d}{dt}(r \vec{r}_0) = \dot{r} \vec{r}_0 + r \dot{\vec{r}}_0$$

$$\vec{v} = \dot{r} \vec{r}_0 + r \dot{\phi} \vec{c}_0$$

$$\vec{v} = \dot{r} \vec{r}_0 + r \dot{\phi} \vec{c}_0$$

$$\vec{v} = v_r \vec{r}_0 + v_c \vec{c}_0$$

$$v_r = \dot{r}$$

$$v_c = r \dot{\phi}$$

$$v = \sqrt{v_r^2 + v_c^2} = \sqrt{\dot{r}^2 + r^2 \dot{\phi}^2}$$

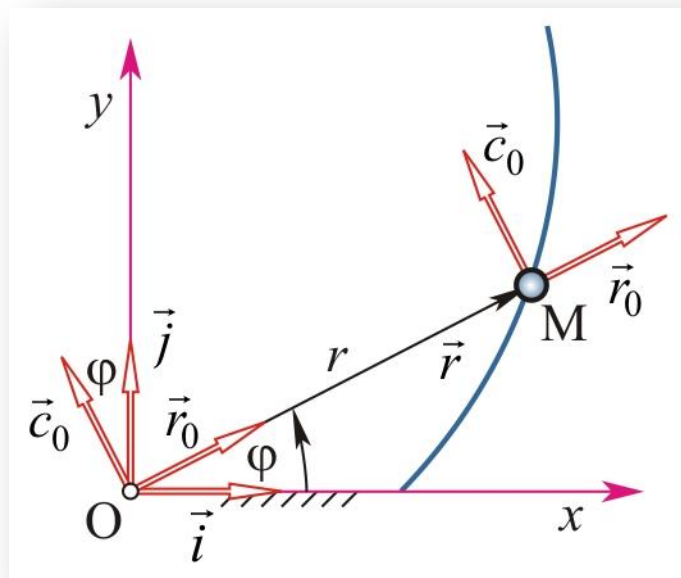
Polarne koordinate – ubrzanje

Brzina

$$\vec{v} = \dot{r} \vec{r}_0 + r \dot{\phi} \vec{c}_0$$

$$\dot{\vec{r}}_0 = \dot{\phi} \vec{c}_0$$

$$\dot{\vec{c}}_0 = -\dot{\phi} \vec{r}_0$$



Ubrzanje

$$\vec{a} = \dot{\vec{v}} = \ddot{r} \vec{r}_0 + \dot{r} \dot{\vec{r}}_0 + \dot{r} \dot{\phi} \vec{c}_0 + r \ddot{\phi} \vec{c}_0 + r \dot{\phi} \dot{\vec{c}}_0$$

$$\vec{a} = (\ddot{r} - r \dot{\phi}^2) \vec{r}_0 + (r \ddot{\phi} + 2\dot{r} \dot{\phi}) \vec{c}_0$$

$$\vec{a} = a_r \vec{r}_0 + a_c \vec{c}_0$$

$$a_r = \ddot{r} - r \dot{\phi}^2$$

$$a_c = r \ddot{\phi} + 2\dot{r} \dot{\phi}$$

$$a = \sqrt{a_r^2 + a_c^2}$$

Primer

Kretanje tačke je opisano parametarskim jednačinama

$$r(t) = t, \varphi(t) = \frac{\pi}{6}t$$

- a) Odrediti trajektoriju tačke,
- b) odrediti brzinu i ubrzanje tačke u proizvoljnom trenutku vremena t ,
- c) odrediti položaj, brzinu i ubrzanje tačke u trenutku $t^*=3s$.

Kretanje tačke je opisano parametarskim jednačinama

$$r(t) = t, \varphi(t) = \frac{\pi}{6}t$$

- Odrediti trajektoriju tačke.
- odrediti brzinu i ubrzanje tačke u proizvoljnom trenutku vremena t .
- odrediti položaj, brzinu i ubrzanje tačke u trenutku $t^* = 3s$.



ТРАЈ. = ?

$$\left. \begin{matrix} r(t) \\ \varphi(t) \end{matrix} \right\} \xrightarrow[t]{\text{ЕЛИМ.}} \underline{r(\varphi)} \quad \text{и}$$

$$\begin{aligned} r &= t \\ \varphi &= \frac{\pi}{6}t \end{aligned} \rightarrow \varphi = \frac{\pi}{6}r \rightarrow \boxed{r = \frac{6}{\pi}\varphi} \quad \text{и}$$

АРХИМЕДОВА
СПИРАЛА

$$\varphi = \frac{\pi}{6} \rightarrow r = 1$$

$$\varphi = \frac{\pi}{3} \rightarrow r = 2$$

$$\varphi = \frac{\pi}{2} \rightarrow r = 3$$

$$\underline{\text{OK}} \quad t \geq 0$$

$$r(t) = t \rightarrow r \geq 0$$

$$\varphi(t) = \frac{\pi}{6}t \rightarrow \varphi \geq 0$$

$$\left. \begin{aligned} r(t) &= t \\ \varphi(t) &= \frac{\pi}{6} t \end{aligned} \right\} \rightarrow \begin{aligned} \dot{r} &= 1 \\ \dot{\varphi} &= \frac{\pi}{6} \end{aligned} \rightarrow \begin{aligned} \ddot{r} &= 0 \\ \ddot{\varphi} &= 0 \end{aligned}$$

$$\left. \begin{aligned} v_r &= \dot{r} = 1 = \text{const} \\ v_c &= r \dot{\varphi} = \frac{\pi}{6} t \end{aligned} \right\} \rightarrow v = \sqrt{v_r^2 + v_c^2} = \sqrt{1^2 + \left(\frac{\pi}{6} t\right)^2}$$

$$\left. \begin{aligned} a_r &= \cancel{\ddot{r}} - r \dot{\varphi}^2 = -\frac{\pi^2}{36} t \\ a_c &= r \cancel{\ddot{\varphi}} + 2 \dot{r} \dot{\varphi} = \frac{\pi}{3} \end{aligned} \right\} \rightarrow a = \sqrt{a_r^2 + a_c^2} = \sqrt{\frac{\pi^4}{36^2} t^2 + \frac{\pi^2}{9}}$$

$$\underline{t^* = 3} \rightarrow \begin{aligned} v_r(3) &= 1 \\ v_c(3) &= \frac{\pi}{6} \cdot 3 = \frac{\pi}{2} \end{aligned} \quad v(3) = \sqrt{1 + \frac{\pi^2}{4}}$$

$$\left. \begin{aligned} a_r(3) &= -\frac{\pi^2}{36} \cdot 3 = -\frac{\pi^2}{12} \\ a_c(3) &= \frac{\pi}{3} \end{aligned} \right\} a(3) = \sqrt{a_r^2(3) + a_c^2(3)} =$$

Šta smo naučili?

- 5. Pravolinijsko kretanje tačke. Ravnomerno i ravnomerno promenljivo kretanje
- 6. Kinematika tačke - polarni koordinatni sistem

Mehanika 2 (Kinematika)

Predavanja 2

Miodrag Zuković
Novi Sad, 2023.