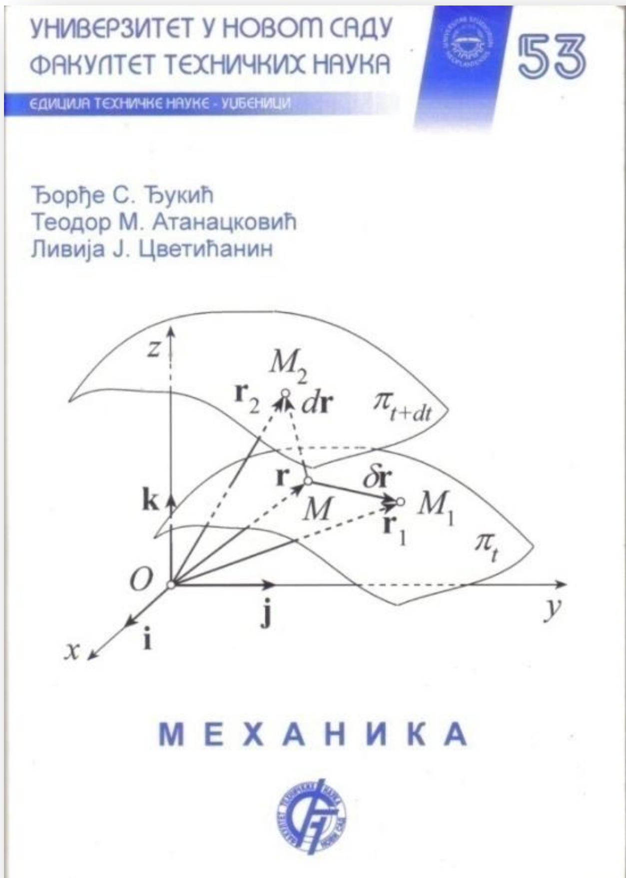
Dinamika – vežbe 3

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.

Literatura



УНИВЕРЗИТЕТ У НОВОМ САДУ
ФАКУЛТЕТ ТЕХНИЧКИХ НАУКА
ЕДИЦИЈА ТЕХНИЧКЕ НАУКЕ - УЏБЕНИЦИ
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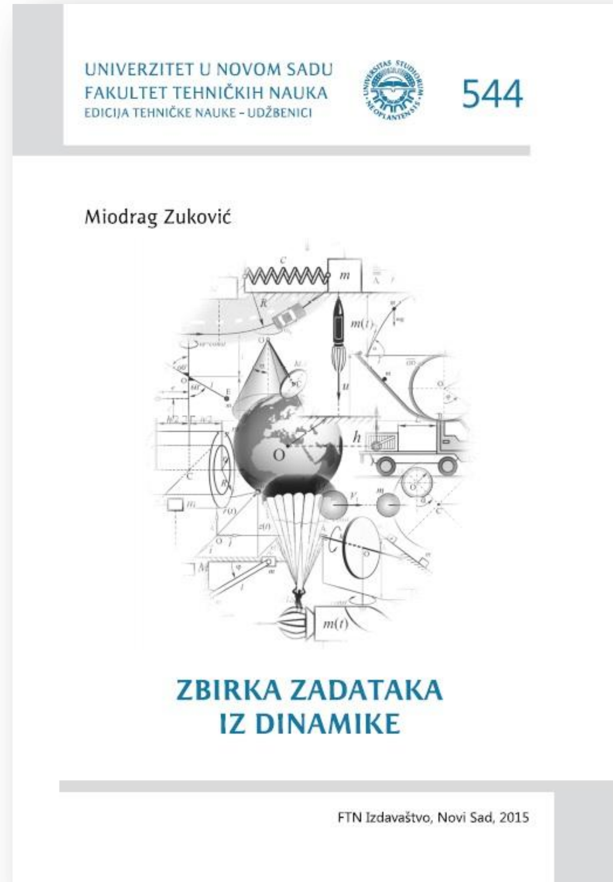

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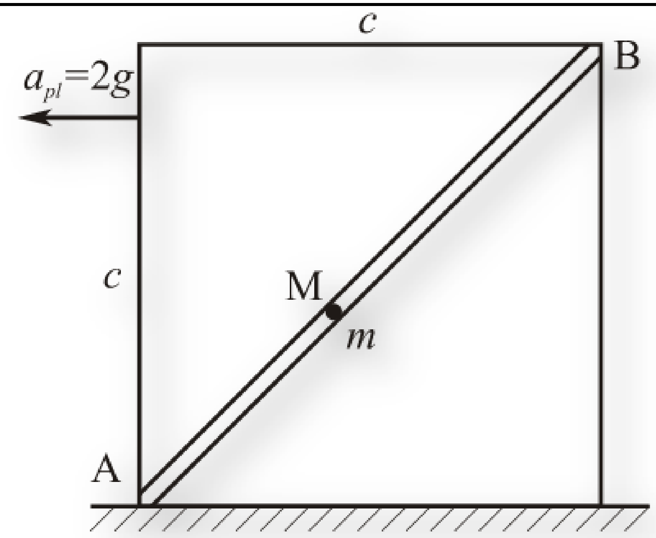
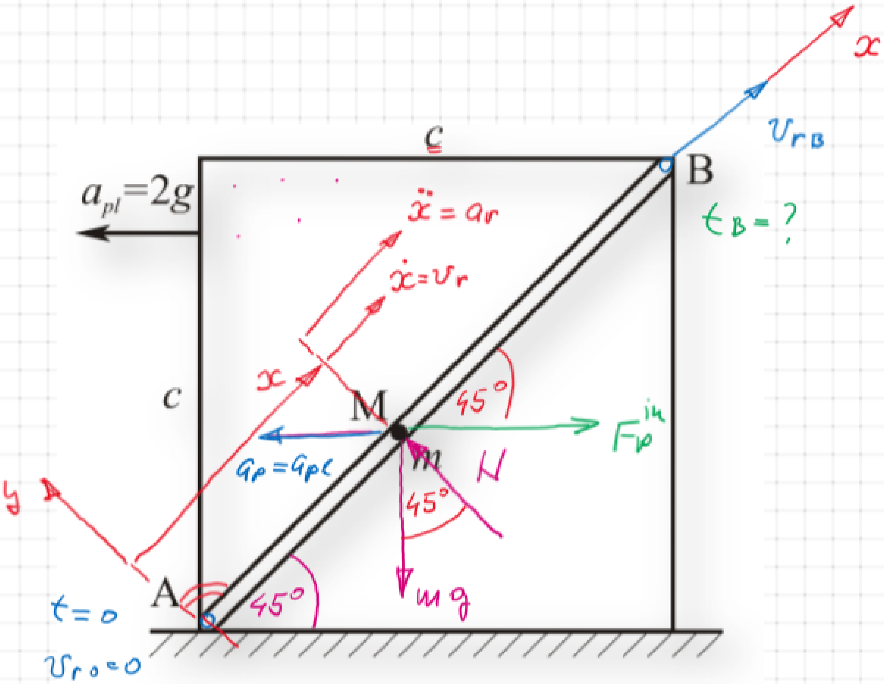
ZBIRKA ZADATAKA
IZ DINAMIKE
FTN Izdavaštvo, Novi Sad, 2015

Zadatak 1

Duž žleba u pravcu dijagonale kvadratne ploče, koja se kreće translatorno pravolinijski konstantnim ubrzanjem $a_{pl} = 2g = const$, kreće se materijalna tačka M, mase m. Odrediti relativnu brzinu tačke, u odnosu na ploču, u položaju B. Tačka započinje kretanje bez relativne brzine iz položaja A. Sve otpore kretanju tačke zanemariti.





$$\boxed{m\vec{a}_r = \vec{F} + \vec{F}_p^{\,in} + \cancel{\vec{F}_c^{\,in}}^{\,0}}$$

ПРЕН. КР. — ПЛОЧА — ТРАНСЛ. → $\vec{\omega}_p = 0$ →

→ $\vec{a}_c = 2\vec{\omega}_p \times \vec{v}_r = 0$

→ $\vec{F}_c^{\,in} = -m\vec{a}_c = 0$

РЕЛ. КР. → ПРАВОЛ.

$$\boxed{m\vec{a}_r = m\vec{g} + \vec{N} + \vec{F}_p^{\,in}} \quad /\cdot\vec{i} \quad /\cdot\vec{j}$$

$$\vec{F}_p^{\,in} = -m\vec{a}_p = -m\vec{a}_{pl}$$

$$F_p^{\,in} = m a_p = m a_{pl} = m\cdot 2g$$

$$(1)\quad m\ddot{x} = -mg\sin 45^\circ + F_p^{in}\cdot\cos 45^\circ$$

$$(2)\quad 0 = -mg\cos 45^\circ + N - F_p^{in}\sin 45^\circ$$

$$(2) \rightarrow \boxed{N} = mg\cdot\frac{\sqrt{2}}{2} + F_p^{in}\frac{\sqrt{2}}{2} = mg\frac{\sqrt{2}}{2} + 2mg\cdot\frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}mg$$

$$(1)\quad \cancel{m}\ddot{x} = -\cancel{m}g\frac{\sqrt{2}}{2} + 2\cancel{m}g\cdot\frac{\sqrt{2}}{2} \rightarrow \boxed{\ddot{x} = g\frac{\sqrt{2}}{2}} = const$$

$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{dx}{dt}\frac{d\dot{x}}{dx} = \dot{x}\frac{d\dot{x}}{dx}$$

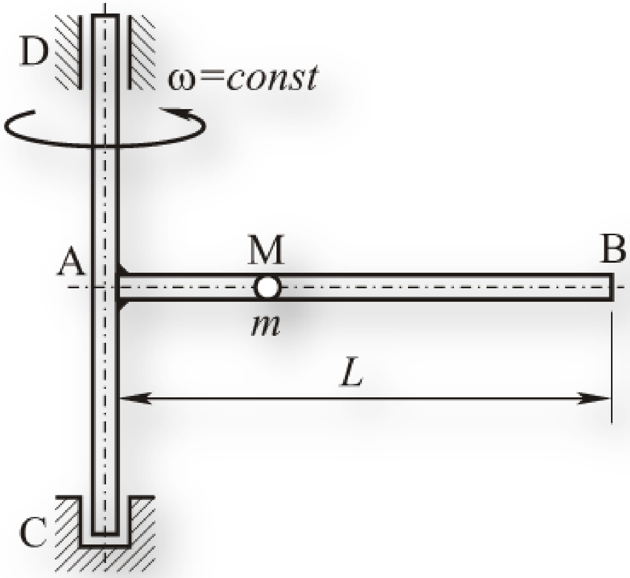
$$\dot{x}\frac{d\dot{x}}{dx} = g\frac{\sqrt{2}}{2} \rightarrow$$

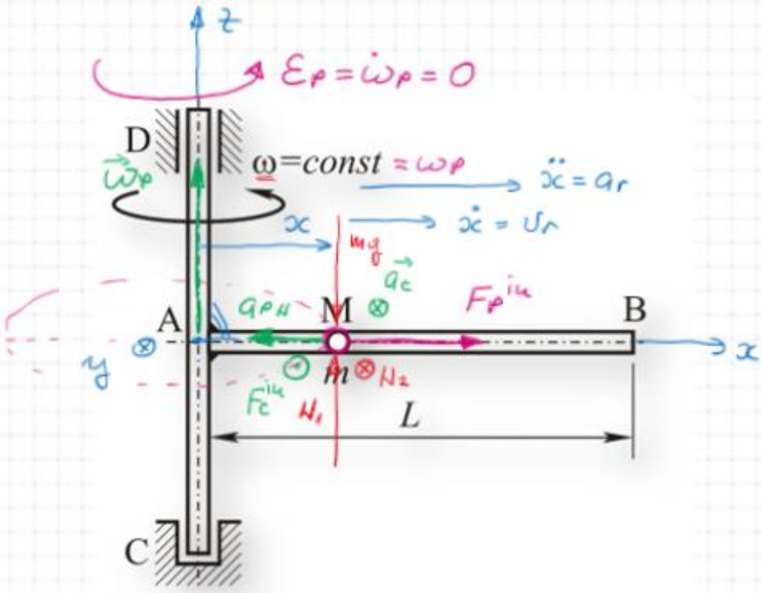
$$\int_{\dot{x}_A = v_{r0} = 0}^{\dot{x}_B = v_{rB}} \dot{x}\,d\dot{x} = g\frac{\sqrt{2}}{2}\int_{x_A=0}^{x_B = r\sqrt{2}} dx \rightarrow \left.\frac{\dot{x}^2}{\cancel{2}}\right|_0^{v_{rB}} = g\frac{\sqrt{2}}{\cancel{2}}\left.x\right|_0^{r\sqrt{2}} \rightarrow v_{rB}^2 = g\sqrt{2}\cdot r\sqrt{2} = 2gr$$

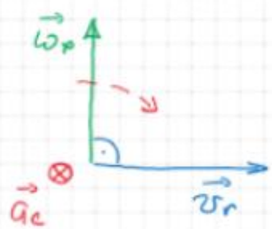
$$\boxed{v_{rB} = \sqrt{2gr}}$$

Zadatak 2

Kuglica, mase m, može da se kreće u horizontalnoj cevi, dužine L, koja se obrće oko vertikalne ose konstantnom ugaonom brzinom $\omega = const$. Kuglica kretanje započinje sa sredine cevi iz stanja mirovanja u odnosu na cev. Kolika je relativna brzina kuglice na izlasku iz cevi.







$$\boxed{m\vec{a}_r = m\vec{g} + \vec{N}_1 + \vec{N}_2 + \vec{F}_p^{in} + \vec{F}_c^{in}}$$

ПРЕН. КР. → ЦЕВ → ОБРТАЊЕ ОКО НЕП. ОСЕ (z) →

→ $TP_{np} = \mathcal{K}\{A, \overline{AM} = x\}$

РЕЛ. КР. → ПРАВОЛ.

$$\vec{F}_p^{in} = -m\vec{a}_p = -m(\overset{0}{\cancel{\vec{a}_{pT}}} + \vec{a}_{pN}) = -m\vec{a}_{pN}$$

$$F_p^{in} = m a_p = m a_{pN} = m \cdot \overline{AM}\,\omega_p^2 = m x \omega^2$$

$$a_{pT} = \overline{AM}\,\varepsilon_p = 0$$

$$a_{pN} = \overline{AM}\,\omega_p^2$$

$$\vec{F}_c^{in} = -m\vec{a}_c =$$

$$F_c^{in} = m a_c = m \cdot 2\omega\dot{x}$$

$$\vec{a}_c = 2\vec{\omega}_p \times \vec{v}_r$$

$$a_c = 2\omega_p v_r \sin\angle(\vec{\omega}_p, \vec{v}_r)$$

$$= 2\omega\dot{x}\underbrace{\cancel{\sin 90^\circ}}_{1}$$

$$\boxed{m\vec{a}_r = m\vec{g} + \vec{N}_1 + \vec{N}_2 + \vec{F}_p^{in} + \vec{F}_c^{in}} \quad / \cdot \vec{i} \; / \cdot \vec{j} \; / \cdot \vec{k}$$

$$\boxed{(1) \quad m\ddot{x} = F_p^{in}}$$

$$(2) \quad 0 = N_2 - F_c^{in} \longrightarrow N_2 = F_c^{in} = m\,2\omega\dot{x}$$

$$(3) \quad 0 = -mg + N_1 \longrightarrow N_1 = mg$$

$$\vec{N} = \vec{N}_1 + \vec{N}_2$$

$$(1) \quad \cancel{m}\ddot{x} = \cancel{m}x\omega^2 \longrightarrow \boxed{\ddot{x} = \omega^2 x}$$

$$\ddot{x} = \dot{x}\frac{d\dot{x}}{dx}$$

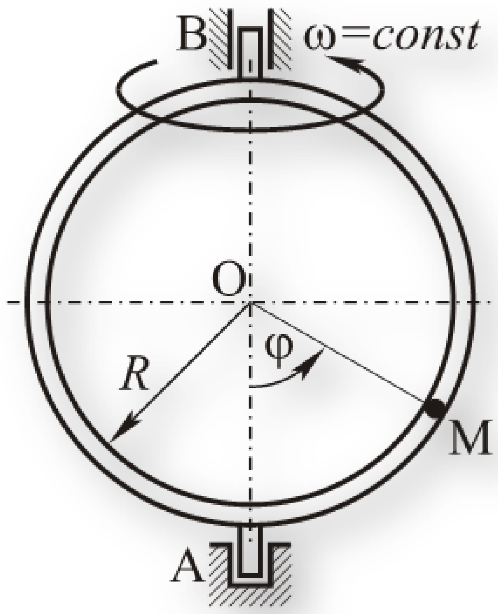
$$\boxed{\dot{x}\frac{d\dot{x}}{dx} = \omega^2 x}$$

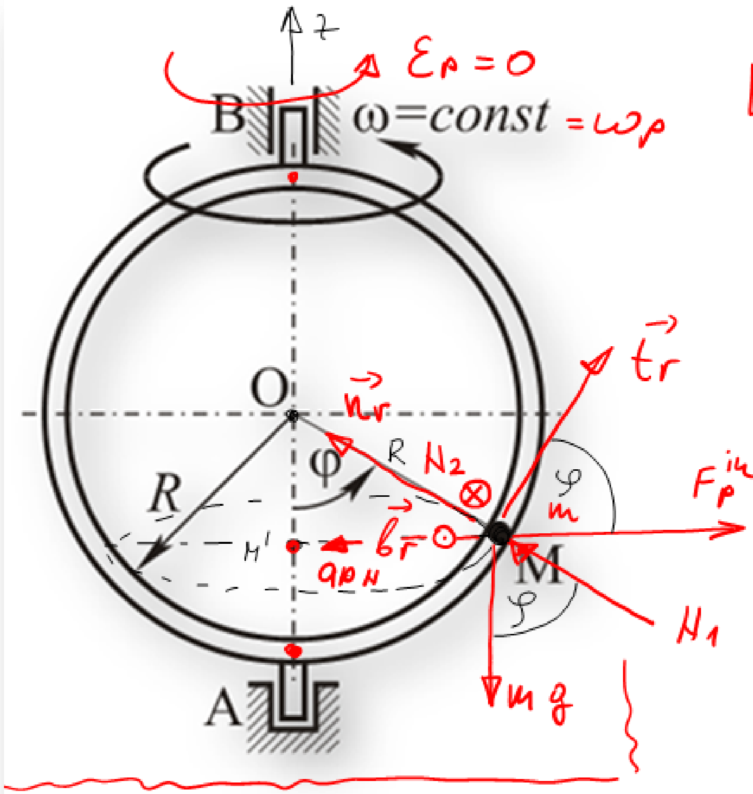
$$\int_{\dot{x}(0)=v_{r0}=0}^{\dot{x}_B = v_{rB}} \dot{x}\,d\dot{x} = \omega^2 \int_{x(0)=\frac{L}{2}}^{x_B = L} x\,dx \longrightarrow \frac{\dot{x}^2}{\cancel{2}}\Bigg|_0^{v_{rB}} = \omega^2 \frac{x^2}{\cancel{2}}\Bigg|_{\frac{L}{2}}^{L}$$

$$\boxed{v_{rB}^2 = \omega^2\left(L^2 - \left(\frac{L}{2}\right)^2\right)} \quad \ldots \quad v_{rB} = \sqrt{}$$

Zadatak 3

Kuglica M, mase m, može da se kreće u kružnoj cevi, poluprečnika R, koja se obrće oko vertikalnog prečnika AB konstantnom ugaonom brzinom $\omega = const$. Odrediti položaje relativne ravnoteže kuglice. Sve otpore kretanju kuglice zanemariti.





$$\boxed{m\vec{a}_r = \vec{F} + \vec{F}_p^{in} + \vec{F}_c^{in}}$$

ПОЛ. РЕЛ. РАВН. :

$\vec{v}_r = \vec{a}_r = 0$, $(\vec{a}_c = 2\vec{\omega}_p \times \vec{v}_r = 0,\ \vec{F}_c^{in} = 0)$

$\boxed{\vec{F} + \vec{F}_p^{in} = 0}$ УСЛОВ РЕЛ. РАВН.

$$\boxed{m\vec{g} + \vec{N}_1 + \vec{N}_2 + \vec{F}_p^{in} = 0} \quad *$$

ПР. КР. → ЦЕВ → ОБРТ. ОКО НЕП. ОСЕ (z)

→ $K\{M',\ M'M = R\sin\varphi\}$ $\quad \omega_p = \omega = const$

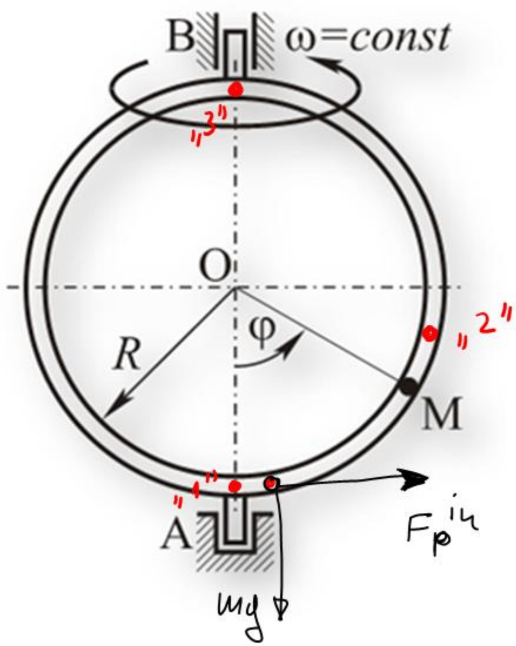
РЕЛ. КР. → $K\{O, R\}_\varphi$ $\quad \varepsilon_p = \dot{\omega}_p = 0$

$$\vec{F}_p^{in} = -m\vec{a}_p = -m(\underbrace{\cancel{\vec{a}_{pT}}}_{0\ (\varepsilon_p = 0)} + \vec{a}_{pN}) = -m\vec{a}_{pN}$$

$$\boxed{\begin{aligned} F_p^{in} &= m \cdot a_{pN} = m(\overline{M'M} \cdot \omega_p^2) \\ &= m R \sin\varphi\, \omega^2 \end{aligned}}$$

$* / \cdot \vec{t}_r \ \rightarrow \ -mg\sin\varphi + F_p^{in} \cdot \cos\varphi = 0$

$$\boxed{-mg\sin\varphi + mR\sin\varphi\,\omega^2 \cdot \cos\varphi = 0}$$



$$\boxed{-\cancel{m}g\sin\varphi + \cancel{m}R\sin\varphi\,\omega^2\cdot\cos\varphi = 0}$$

$$\sin\varphi\cdot(-g + R\omega^2\cos\varphi) = 0$$

$$\sin\varphi = 0$$

$$\varphi_1 = 0$$

$$\varphi_3 = \pi$$

$$-g + R\omega^2\cos\varphi = 0$$

$$\boxed{\cos\varphi_2 = \frac{g}{R\omega^2}} \le 1$$

$$\varphi_2 = \arccos\frac{g}{R\omega^2}$$

$$\frac{g}{R\omega^2} \le 1 \longrightarrow \omega \ge \sqrt{\frac{g}{R}}$$

	$\omega > \sqrt{\frac{g}{R}}$	$\omega < \sqrt{\frac{g}{R}}$
$\varphi_1 = 0$	НЕСТ.	СТАБ.
φ_2	СТАБ.	X
$\varphi_3 = \pi$	НЕСТ.	НЕСТ.

Zadatak 4

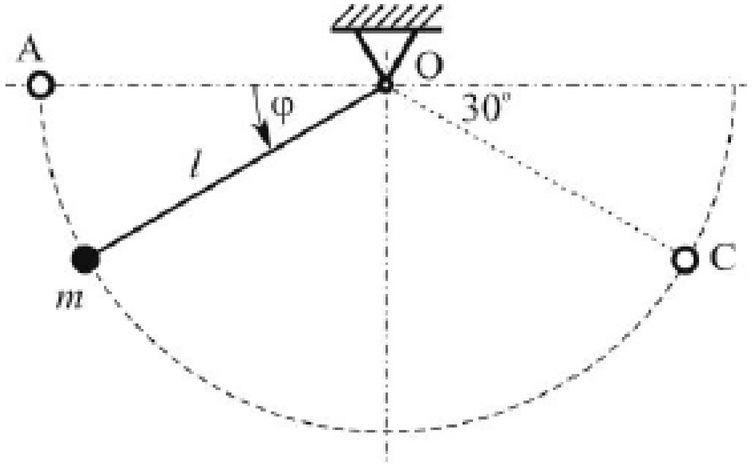
Matematičko klatno, materijalna tačka mase m obešena o nepokretnu tačku neistegljivim užetom dužine l, započinje kretanje iz položaja A bez početne brzine.

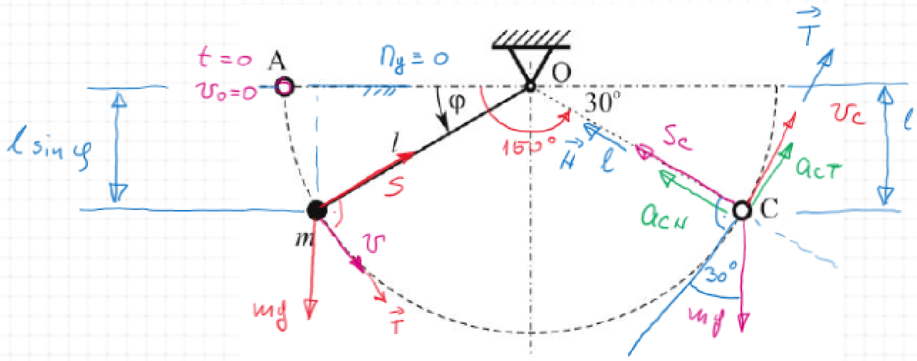
Odrediti:

a) brzinu materijalne tačke u funkciji ugla φ,

b) brzinu tačke u položaju C,

c) silu zatezanja užeta u položaju C.





$$E_k + \Pi = \cancel{E_{k0}}^{\,0} + \cancel{\Pi_0}^{\,0}$$

$$\frac{1}{2}\cancel{m}v^2 - \cancel{m}g\,l\sin\varphi = 0$$

$$v^2 = 2g\,l\sin\varphi$$

$$C \rightarrow \varphi = 150^\circ = 180^\circ - 30^\circ$$

$$v_C^2 = 2g\,l\,\underbrace{\cancel{\sin 150^\circ}}_{\sin 30^\circ = \frac{1}{2}}$$

$$v_C^2 = g\,l \rightarrow v_C = \sqrt{g\,l}$$

$$E_{kC} + \Pi_C = \cancel{E_{k0}}^{\,0} + \cancel{\Pi_0}^{\,0}$$

$$\frac{1}{2}\cancel{m}v_C^2 - \cancel{m}g\,l\sin 30^\circ = 0$$

$$v_C^2 = 2g\,l\,\cancel{\sin 30^\circ}^{\,1/2}$$

$$v_C^2 = g\,l$$

$$S_C = ?$$

$$\text{II} \quad m\vec{a} = m\vec{g} + \vec{S}$$

$$\boxed{m\vec{a}_C = m\vec{g} + \vec{S}_C}$$

$$m(\vec{a}_{CT} + \vec{a}_{CN}) = m\vec{g} + \vec{S}_C \quad / \cdot \vec{N}$$

$$m\,a_{CN} = -mg\sin 30^\circ + S_C$$

$$S_C = mg\,\cancel{\sin 30^\circ}^{\,\frac{1}{2}} + m\,a_{CN}$$

$$S_C = \frac{mg}{2} + m\cdot\frac{v_C^2}{l} = \frac{mg}{2} + m\cdot\frac{g\cancel{l}}{\cancel{l}} = \frac{3}{2}mg$$

Zadatak 5

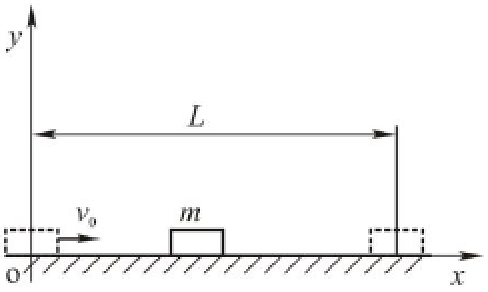
Materijalna tačka mase m započinje kretanje po glatkoj horizontalnoj ravni, u homogenom polju sile zemljine teže, početnom brzinom v_0. Na nju dejstvuje i sila otpora kretanju, proporcionalna kvadratu brzine $R=k\,v^2$ (k=$const$>0, v- brzina tačke). Odrediti:

a) diferencijalne jednačine kretanja materijalne tačke,

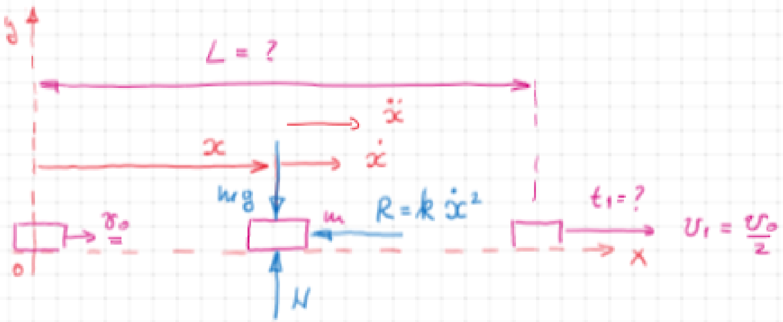
b) parametarsku jednačinu kretanja tačke ($x(t)$),

c) trenutak t_1 u kome je brzina tačke dva puta manja od početne,

d) pređeni put tačke L do trenutka t_1.



$R = k v^2$



$$m\vec{a} = m\vec{g} + \vec{N} + \vec{R}$$

$$m(\ddot{x}\,\vec{i}) = -mg\,\vec{j} + N\,\vec{j} + (-k\dot{x}^2\,\vec{i}) \quad / \cdot\vec{i} \;/ \cdot\vec{j}$$

$$(1)\quad m\ddot{x} = -k\dot{x}^2$$

$$(2)\quad 0 = -mg + N \rightarrow \boxed{N = mg}$$

$$(1)\quad \frac{d\dot{x}}{dt} = -\frac{k}{m}\dot{x}^2 \longrightarrow \int_{\dot{x}(0)=v_0}^{\dot{x}} \frac{d\dot{x}}{\dot{x}^2} = -\frac{k}{m}\int_0^t dt \longrightarrow -\frac{1}{\dot{x}}\Big|_{v_0}^{\dot{x}} = -\frac{k}{m}\,t\Big|_0^t$$

$$\int z^n dz = \frac{z^{n+1}}{\ldots}$$

$$\dot{x}(0)=v_0 \qquad 0 \qquad v_0$$

$$\int z^n \, dz = \frac{z^{n+1}}{n+1}$$

$$\int \frac{dz}{z^2} = \int z^{-2} \, dz = \frac{z^{-2+1}}{-2+1} = \frac{z^{-1}}{-1} = -\frac{1}{z}$$

$$\int \frac{dz}{a+bz} = \frac{1}{b}\int \frac{du}{u} = \frac{1}{b}\ln(a+bz)$$

$$u = a + bz \;/d$$

$$du = b\,dz$$

$$dz = \frac{du}{b}$$

$$-\left(\frac{1}{\dot{x}} - \frac{1}{v_0}\right) = -\frac{k}{m}(t-0)$$

$$-\frac{1}{\dot{x}} + \frac{1}{v_0} = -\frac{k}{m}t \quad /\cdot(-1) \quad \rightarrow \quad \frac{1}{\dot{x}} = \frac{1}{v_0} + \frac{k}{m}t \quad \rightarrow$$

$$* \; \boxed{\dot{x} = \frac{1}{\frac{1}{v_0} + \frac{k}{m}t}} \quad \rightarrow \quad \frac{dx}{dt} = \frac{1}{\frac{1}{v_0} + \frac{k}{m}t}$$

$$\int_{x(0)=0}^{x} dx = \int_0^t \frac{dt}{\frac{1}{v_0} + \frac{k}{m}t} \quad \rightarrow \quad x\Big|_0^x = \frac{m}{k}\ln\left(\frac{1}{v_0} + \frac{k}{m}t\right)\Big|_0^t$$

$$x = \frac{m}{k}\left(\ln\left(\frac{1}{v_0} + \frac{k}{m}t\right) - \ln\left(\frac{1}{v_0}\right)\right)$$

$$x = \frac{m}{k}\ln\frac{\frac{1}{v_0} + \frac{k}{m}t}{\frac{1}{v_0}}$$

$$\boxed{x = \frac{m}{k}\ln\left(1 + \frac{k v_0}{m}t\right)}$$

$$t_1 = ? \quad \rightarrow \quad \dot{x}(t_1) = \frac{v_0}{2}$$

$$* \rightarrow \dot{x}(t_1) = \frac{1}{\frac{1}{v_0} + \frac{k}{m} t_1} = \frac{v_0}{2} \quad \rightarrow \quad t_1$$

$$L = x(t_1) = \text{-----}$$

Zadatak 6

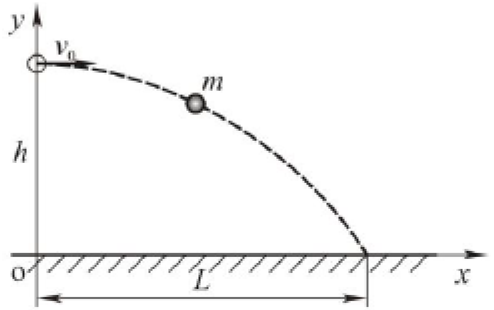
.. Materijalna tačka mase m započinje kretanje u vertikalnoj ravni, u homogenom polju sile zemljine teže, horizontalnom početnom brzinom v_0. Kretanje započinje sa visine h. Odrediti:

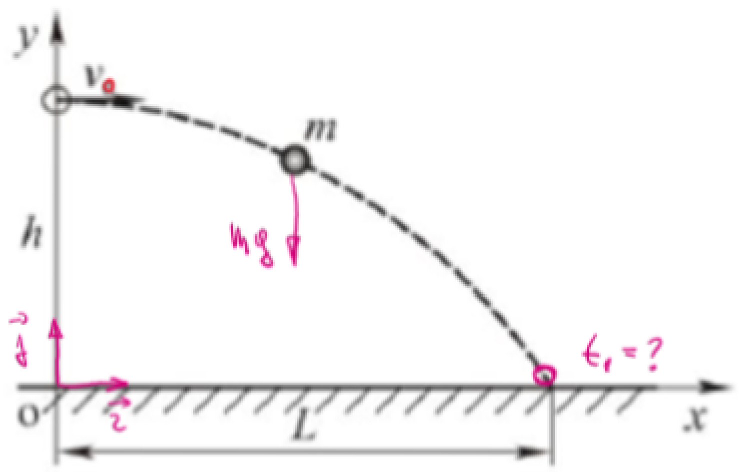
a) diferencijalne jednačine kretanja materijalne tačke,

b) parametarske jednačine kretanja tačke $(x(t),y(t))$,

c) trenutak t_1 u kome tačka padne na tlo,

d) brzinu tačke v_1 u trenutku t_1.





$$m\vec{a} = \vec{F}$$

$$m(\ddot{x}\vec{i} + \ddot{y}\vec{j}) = -mg\vec{j} \qquad /\cdot\vec{i} \quad /\cdot\vec{j}$$

$$(1) \quad m\ddot{x} = 0$$

$$(2) \quad m\ddot{y} = -mg$$

$$(1) \quad \ddot{x} = 0 \rightarrow \dot{x} = \text{const} = \dot{x}(0) = v_0 \rightarrow \underline{(a) \quad \dot{x} = v_0}$$

$$\frac{dx}{dt} = v_0 \rightarrow \int_{x(0)=0}^{x} dx = v_0 \int_0^t dt \rightarrow x \Big|_0^x = v_0 t \Big|_0^t \rightarrow \underline{(b) \quad x = v_0 t}$$

$$(2) \quad \ddot{y} = -g \rightarrow \frac{d\dot{y}}{dt} = -g \rightarrow \int_{\dot{y}(0)=0}^{\dot{y}} d\dot{y} = -g \int_0^t dt \rightarrow \underline{(c) \quad \dot{y} = -gt}$$

$$\frac{dy}{dt} = -gt \rightarrow \int_{y(0)=h}^{y} dy = -g \int_0^t t\, dt \rightarrow y \Big|_h^y = -g \frac{t^2}{2} \Big|_0^t \qquad \underline{(d) \quad y = h - g\frac{t^2}{2}}$$

$$t_1 = ? \rightarrow y(t_1) = 0 \;; \quad (d) \quad y(t_1) = \underline{h - g\frac{t_1^2}{2} = 0} \rightarrow t_1 = \sqrt{\frac{2h}{g}}$$

$$\vec{v}(t_1) = \dot{x}(t_1)\,\vec{i} + \dot{y}(t_1)\,\vec{j} \qquad \dot{x}(t_1) = v_0 \quad (a)$$

$$v(t_1) = \sqrt{\dot{x}^2(t_1) + \dot{y}^2(t_1)} \qquad \dot{y}(t_1) = -g t_1 = -g\sqrt{\frac{2h}{g}}$$

$$v(t_1) = \text{- - - -} \qquad = -\sqrt{2gh} \quad (c)$$

Zadatak 7

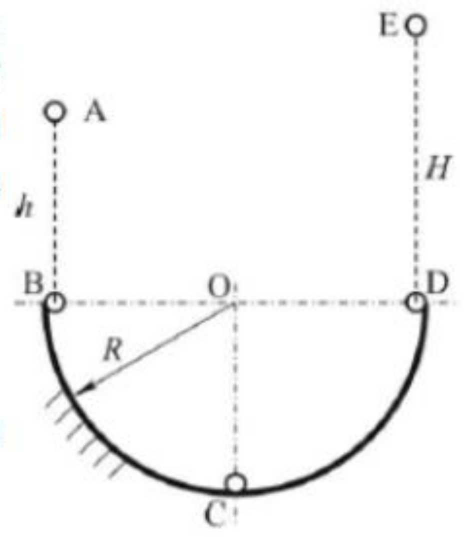
. Materijalna tačka, mase m, započinje kretanje naniže iz položaja A bez početne brzine. U položaju B ($\overline{AB} = h$) dospeva na glatku cilindričnu površinu, radijusa R. Vezu napušta u položaju D.

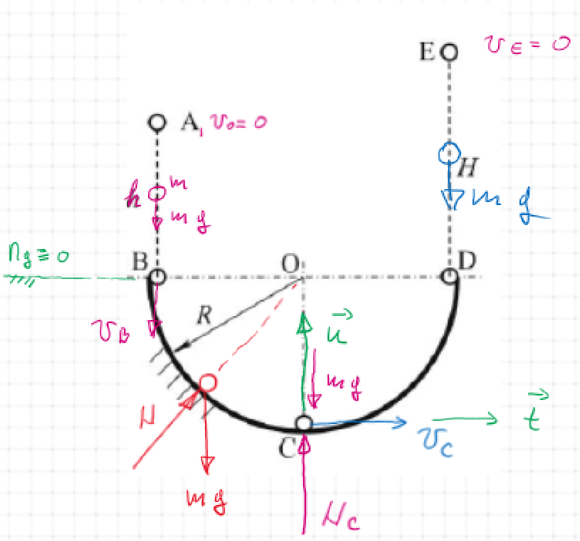
Odrediti:

a) brzinu tačke u položajima B, C i D,

b) reakciju veze u položaju C,

c) visinu H do koje će se tačka popeti nakon napuštanja veze.





„A – B"

$$E_{kB} + \cancel{\Pi_B}^{0} = \cancel{E_{kA}}^{0} + \Pi_A$$

$$\frac{1}{2} m v_B^2 = m g \cdot h$$

$$\underline{v_B^2 = 2 g h}$$

„B – D"

$$E_{kC} + \Pi_C = E_{kB} + \cancel{\Pi_B}^{0}$$

$$\frac{1}{2} m v_C^2 - m g \cdot R = \frac{1}{2} m v_B^2$$

$$\underline{v_C^2 = v_B^2 + 2 g R} = \underline{\cdots}$$

$$\cancel{E_{kE}}^{0} + \Pi_E = \cancel{E_{kA}}^{0} + \Pi_A$$

$$m g H = m g h$$

$$\underline{H = h}$$

„B – D"

$$\underline{m \vec{a} = m \vec{g} + \vec{N}}$$

$$m \vec{a}_C = m \vec{g} + \vec{N}_C \qquad / \cdot \vec{u}$$

$$m a_{NC} = - m g + N_C \quad \rightarrow \quad N_C = m g + m a_{NC}$$

$$N_C = m g + m \frac{v_C^2}{R} = m g + \frac{m}{R} \left(v_B^2 + 2 g R \right)$$

$$\underline{N_C = m g + \frac{m}{R} \left(2 g h + 2 g R \right)}$$

Dinamika – vežbe 3

Kinematika i dinamika

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