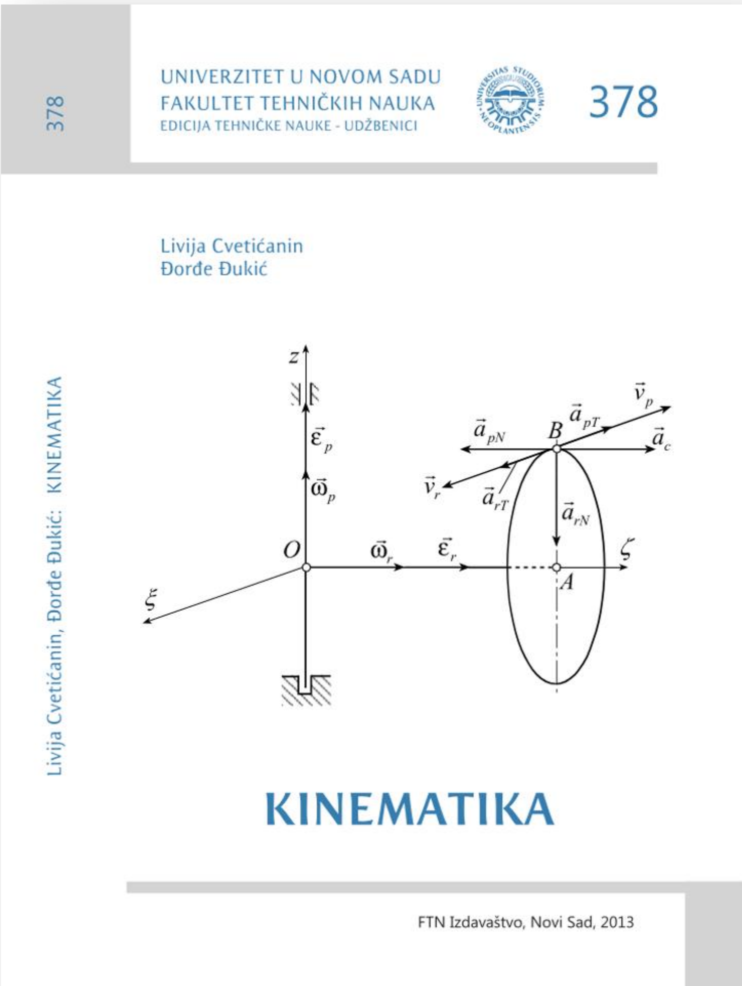
Mehanika 2 (Kinematika)
Predavanja 6

Miodrag Zuković

Novi Sad, 2023.

Literatura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FAKULTET TEHNIČKIH NAUKA
EDICIJA TEHNIČKE NAUKE - UDŽBENICI
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Livija Cvetićanin
Đorđe Đukić
Livija Cvetićanin, Đorđe Đukić: KINEMATIKA
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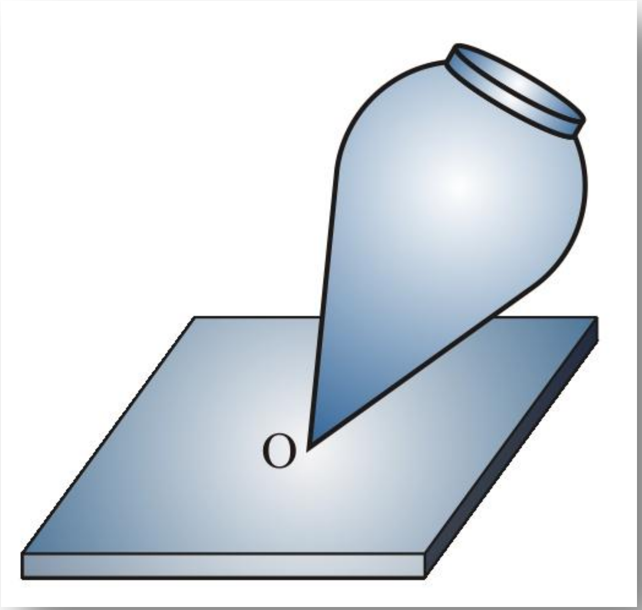
ИЗ КИНЕМАТИКЕ
ФТН Издаваштво, Нови Сад, 2013

Šta ćemo naučiti?

17. Sferno kretanje krutog tela (broj stepeni slobode kretanja, Ojler - D'Alamberova teorema)
18. Sferno kretanje - Ojlerovi uglovi
19. Ugaona brzina i ugaono ubrzanje tela pri sfernom kretanju
20. Projekcije ugaone brzine sfernog kretanja na koordinatne ose (Ojlerove kinematičke jednačine)
21. Projekcije ugaonog ubrzanja sfernog kretanja na koordinatne ose
22. Brzine i ubrzanja tačaka tela pri sfernom kretanju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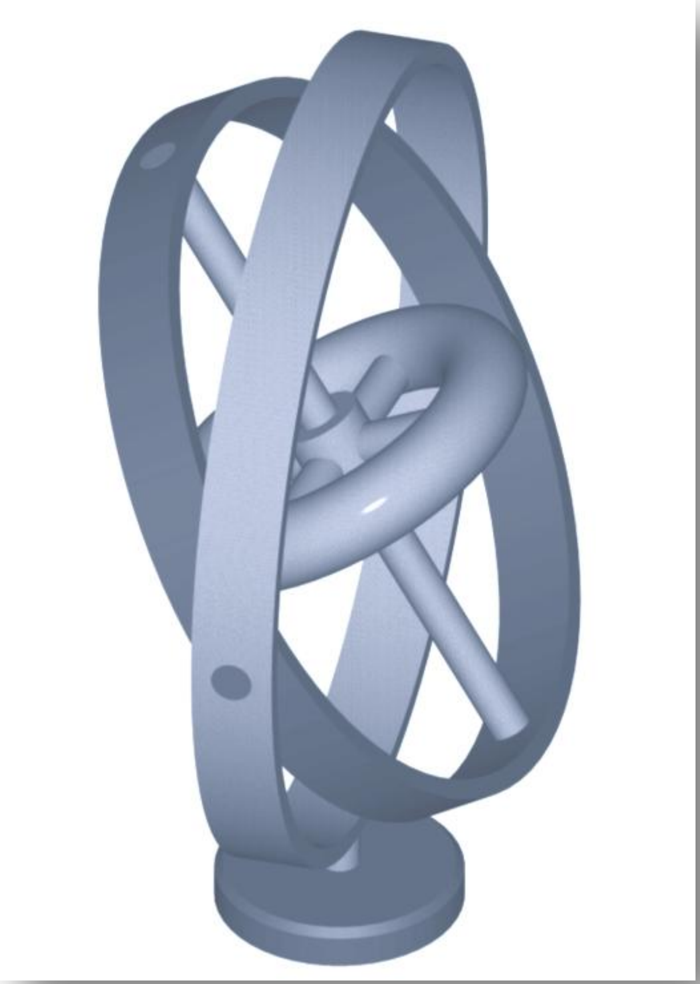
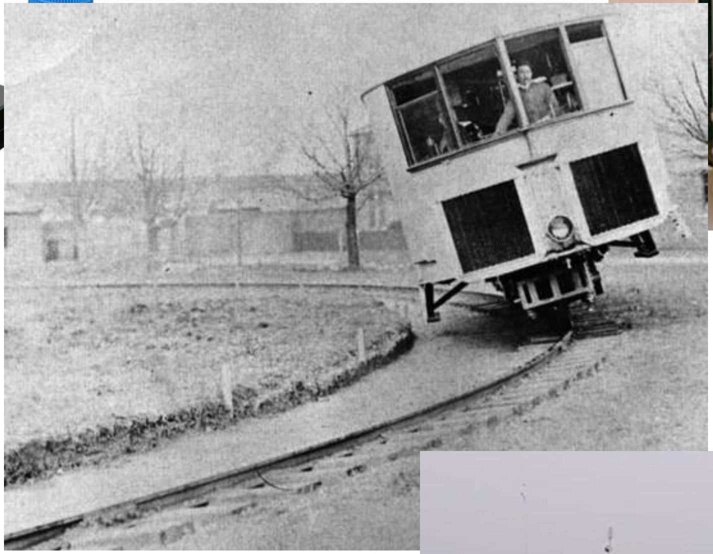
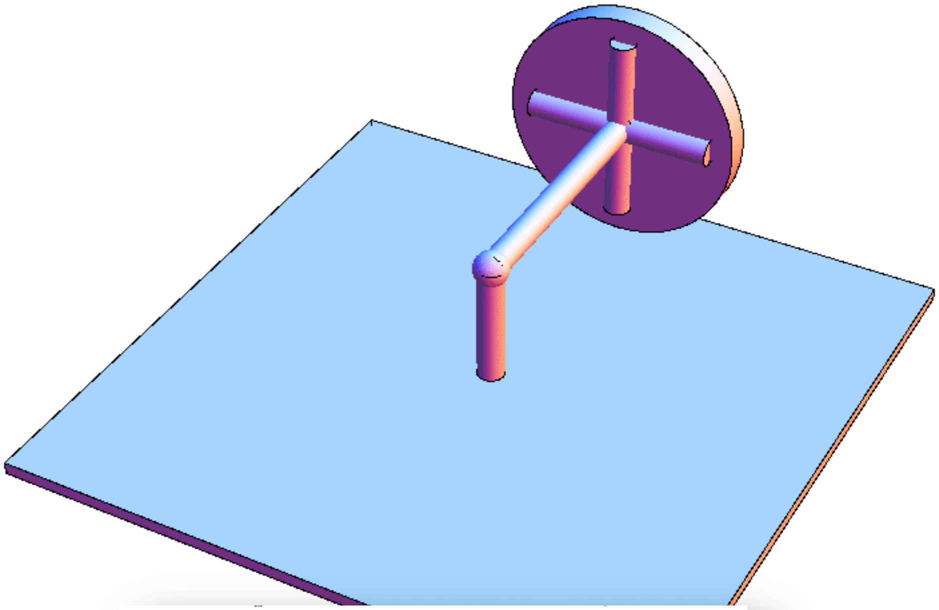
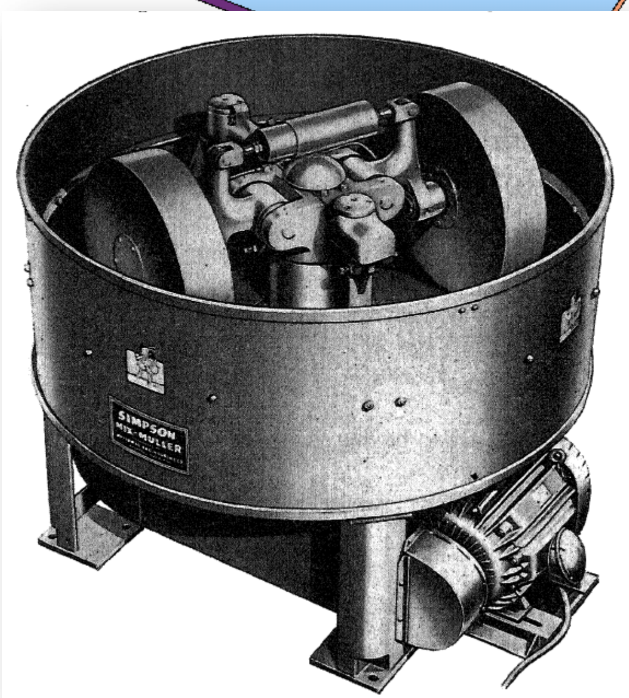
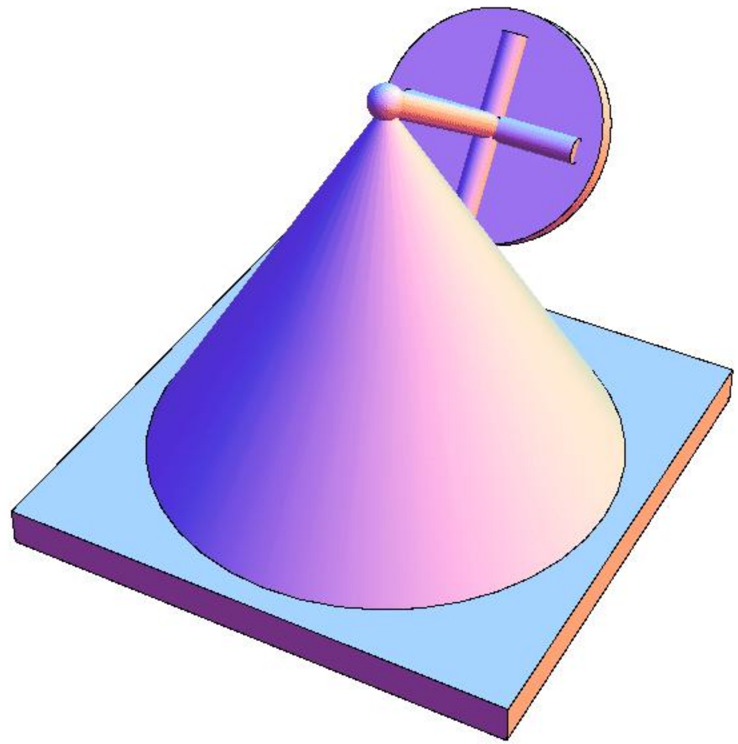




(od st. grčkog *γυρο* „okret“ i drev. grčkog *σκοπεω* „posmatrati")

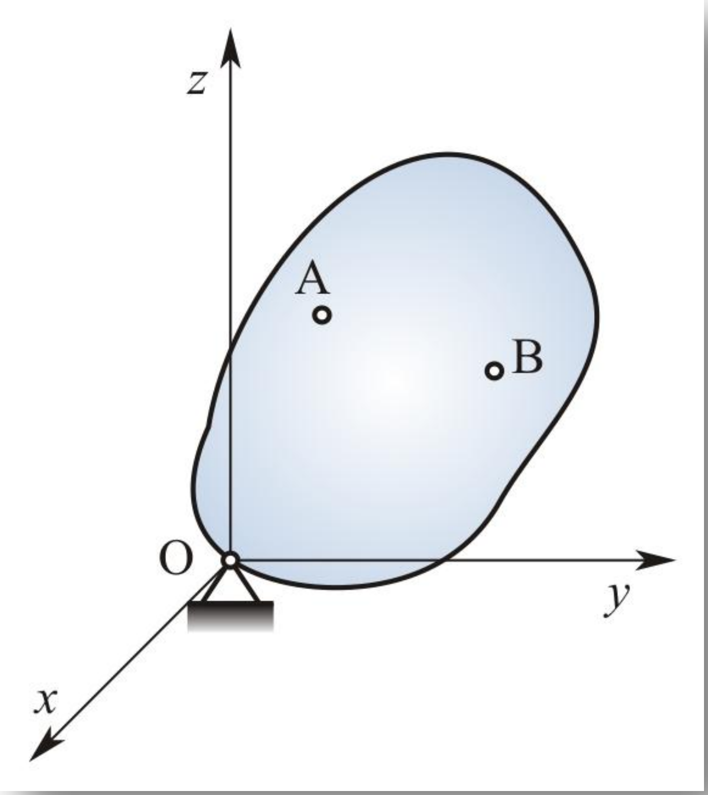



SIMPSON
MIX-MULLER



17. Sferno kretanje krutog tela (broj stepeni slobode kretanja, Ojler - D'Alamberova teorema)

Sferno kretanje krutog tela (obrtanje oko nepokretne tačke) – broj stepeni slobode kretanja



$$x_O, y_0, z_O, x_A, y_A, z_A, x_B, y_B, z_B$$

$$x_O = 0$$

$$y_0 = 0$$

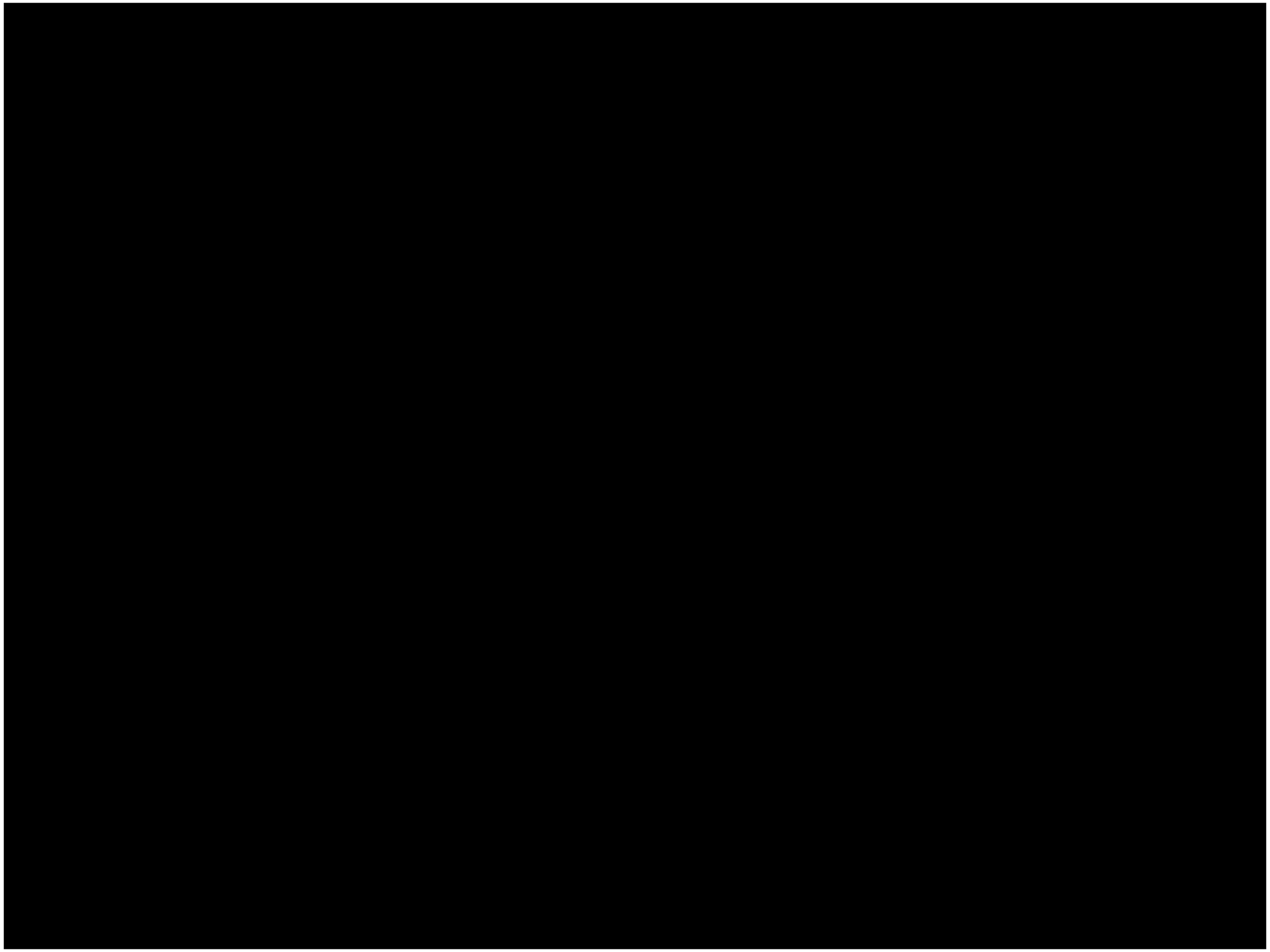
$$z_O = 0$$

$$\overline{\mathrm{OA}} = \sqrt{x_A^2 + y_A^2 + z_A^2} = const$$

$$\overline{\mathrm{OB}} = \sqrt{x_B^2 + y_B^2 + z_B^2} = const$$

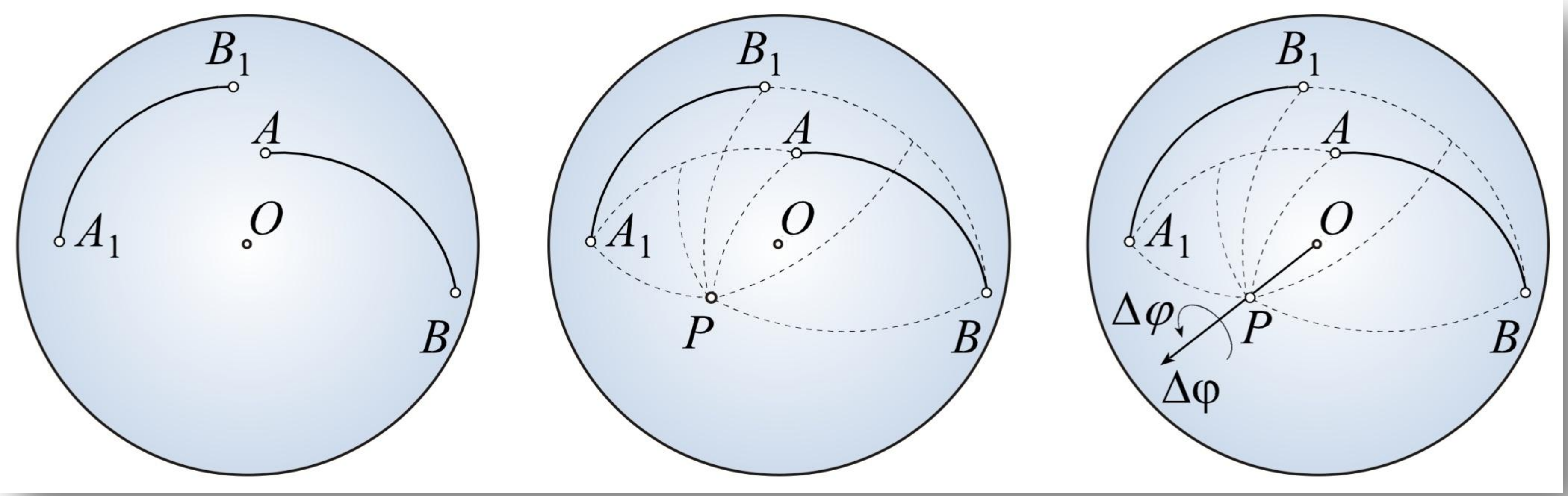
$$\overline{\mathrm{AB}} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2} = const$$

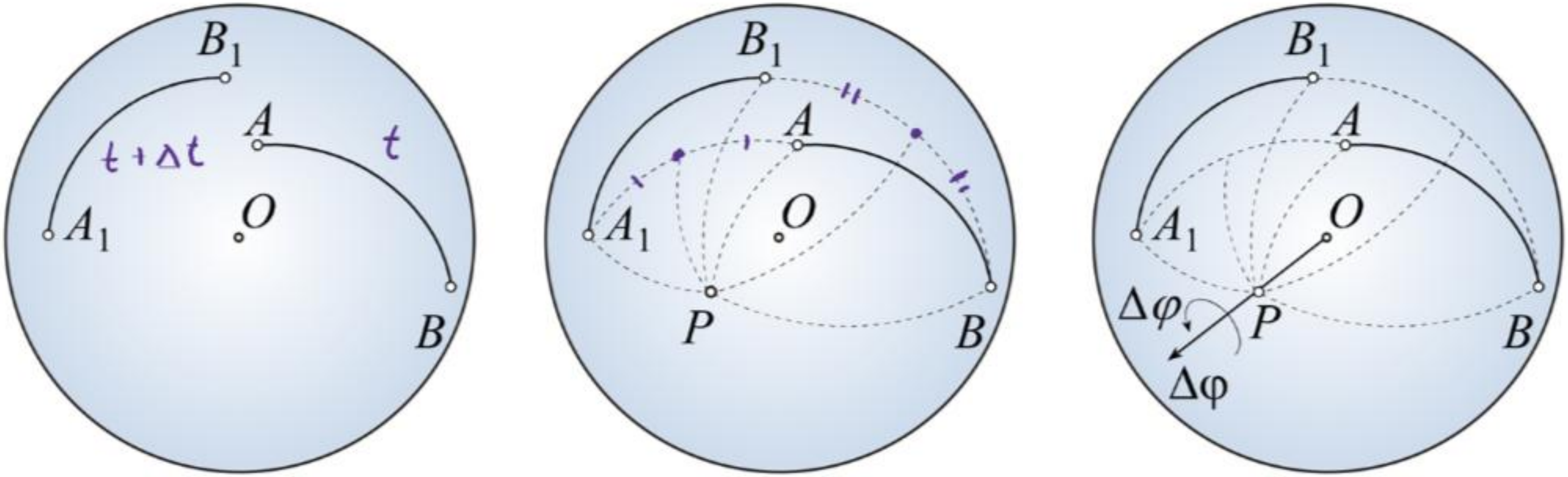
Tri nezavisne koordinate (parametra) – tri stepena slobode kretanja (maksimalno)



http://www.mech-in-ns.ftn.uns.ac.rs/projects/18-obrtanje-oko-nepokretne-tacke-1-stepen-slobode/

Ojler - D'Alamberova teorema





$$\overline{OA} = \overline{OB} = \overline{OA_1} = \overline{OB_1}$$

$$\overset{\frown}{AB} = \overset{\frown}{A_1B_1}$$

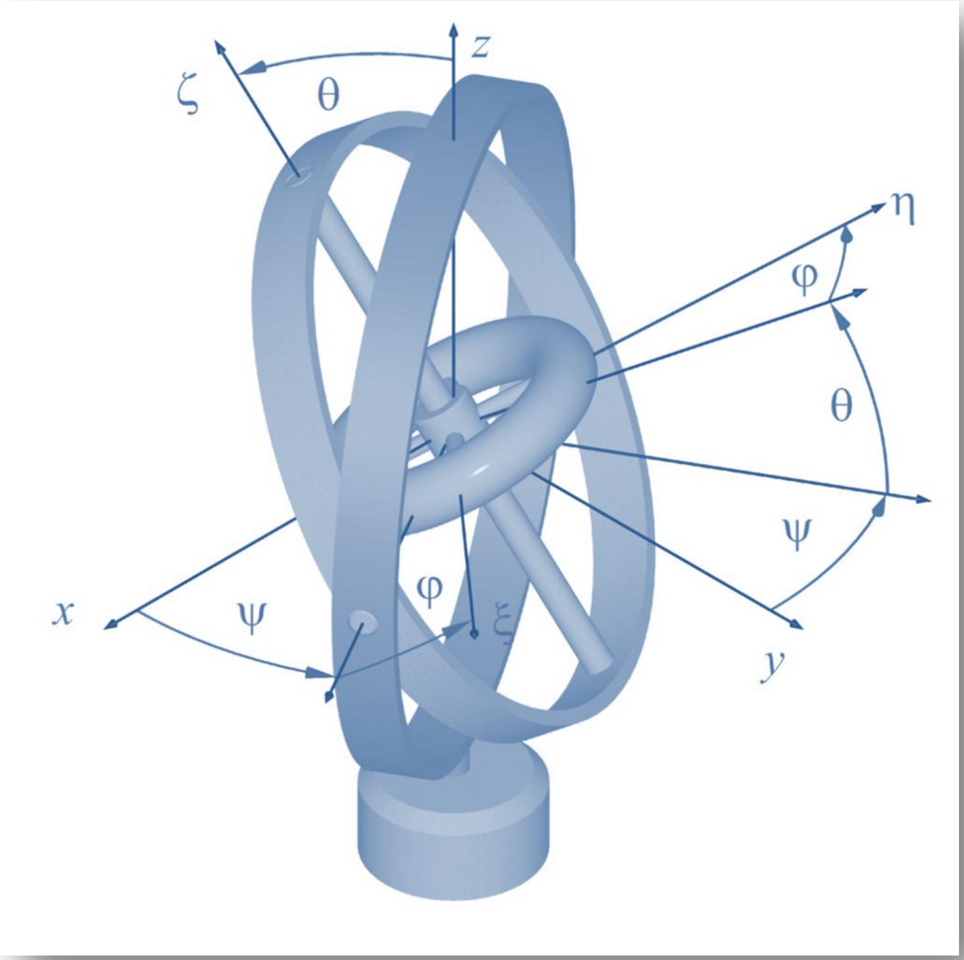
$$\overset{\frown}{AP} = \overset{\frown}{A_1P}$$

$$\overset{\frown}{BP} = \overset{\frown}{B_1P}$$

$$\omega_{sr} = \frac{\Delta\varphi}{\Delta t}$$

$$\omega = \frac{d\varphi}{dt}$$

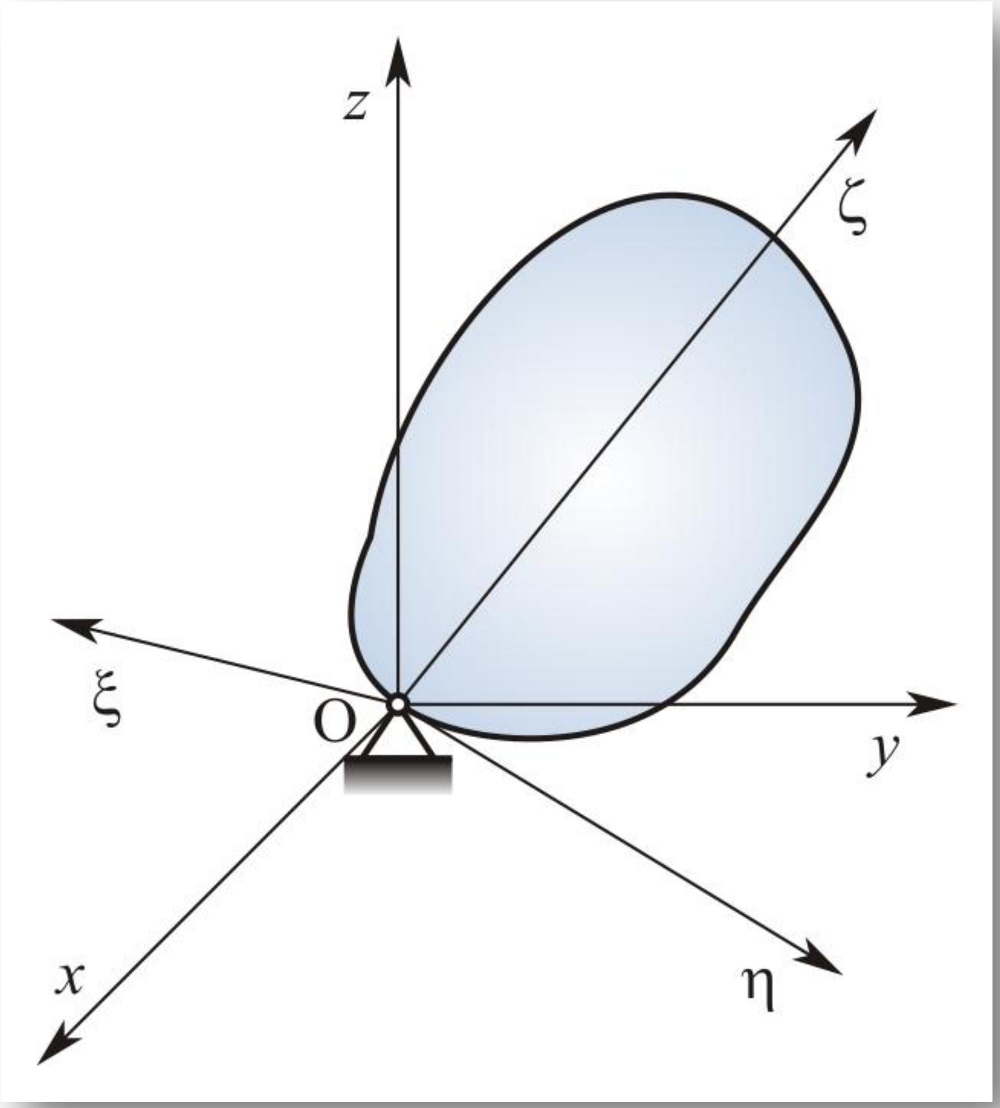
18. Sferno kretanje - Ojlerovi uglovi



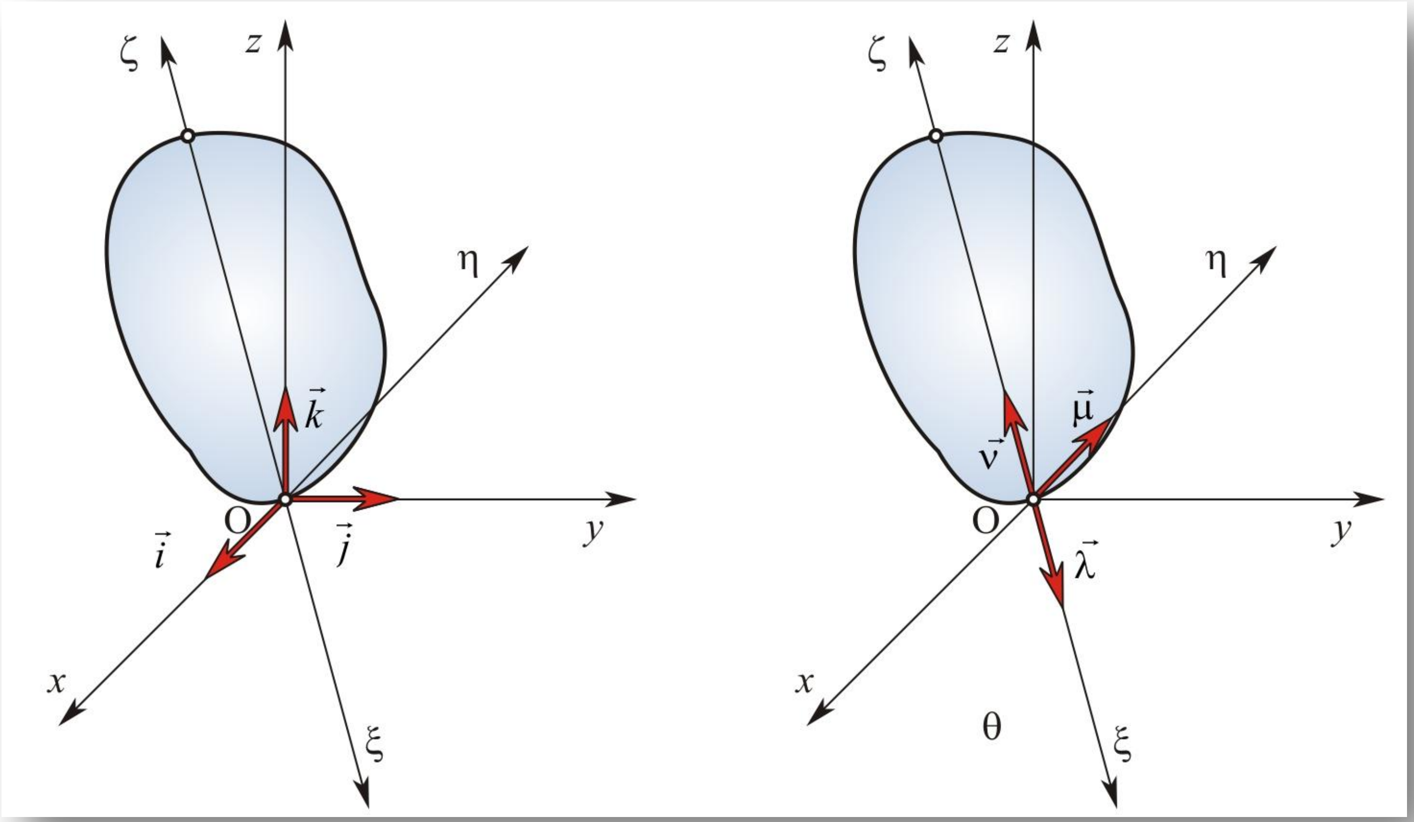
Poznate zavisnosti uglova **precesije, nutacije i sopstvene rotacije** u funkciji od vremena:

$$\boxed{\psi = \psi(t);\ \ \theta = \theta(t);\ \varphi = \varphi(t)}$$

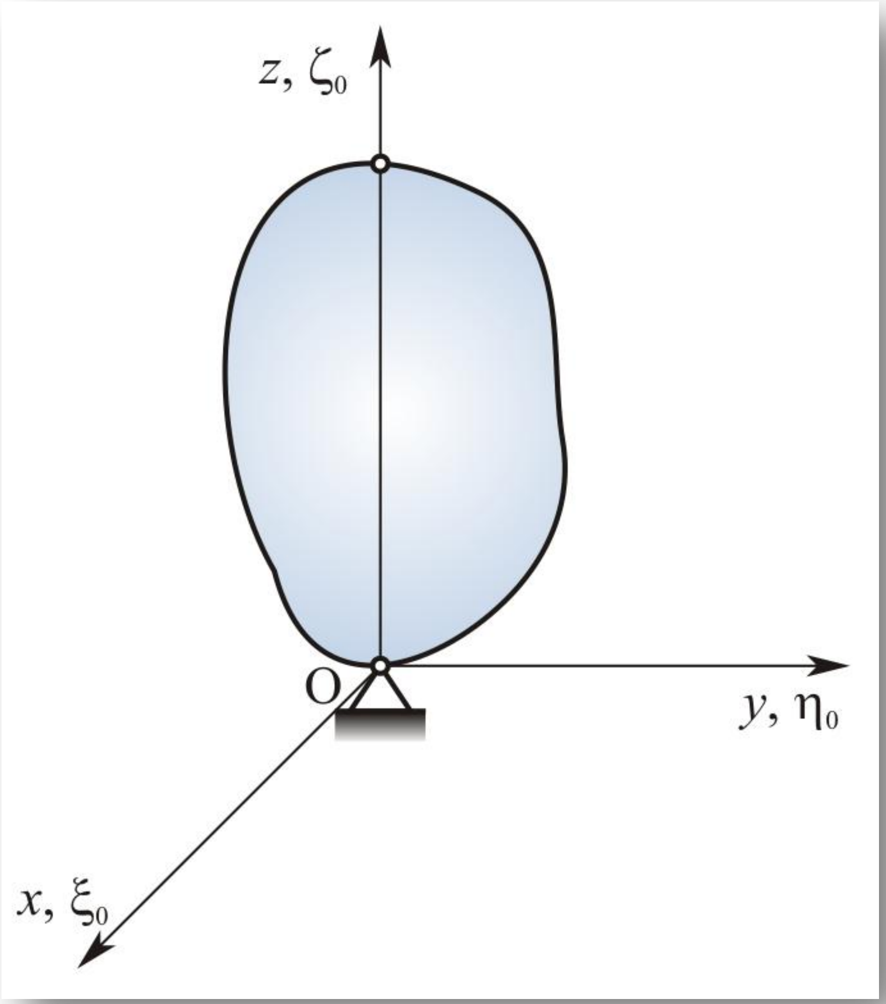
su zakoni sfernog kretanja tela.

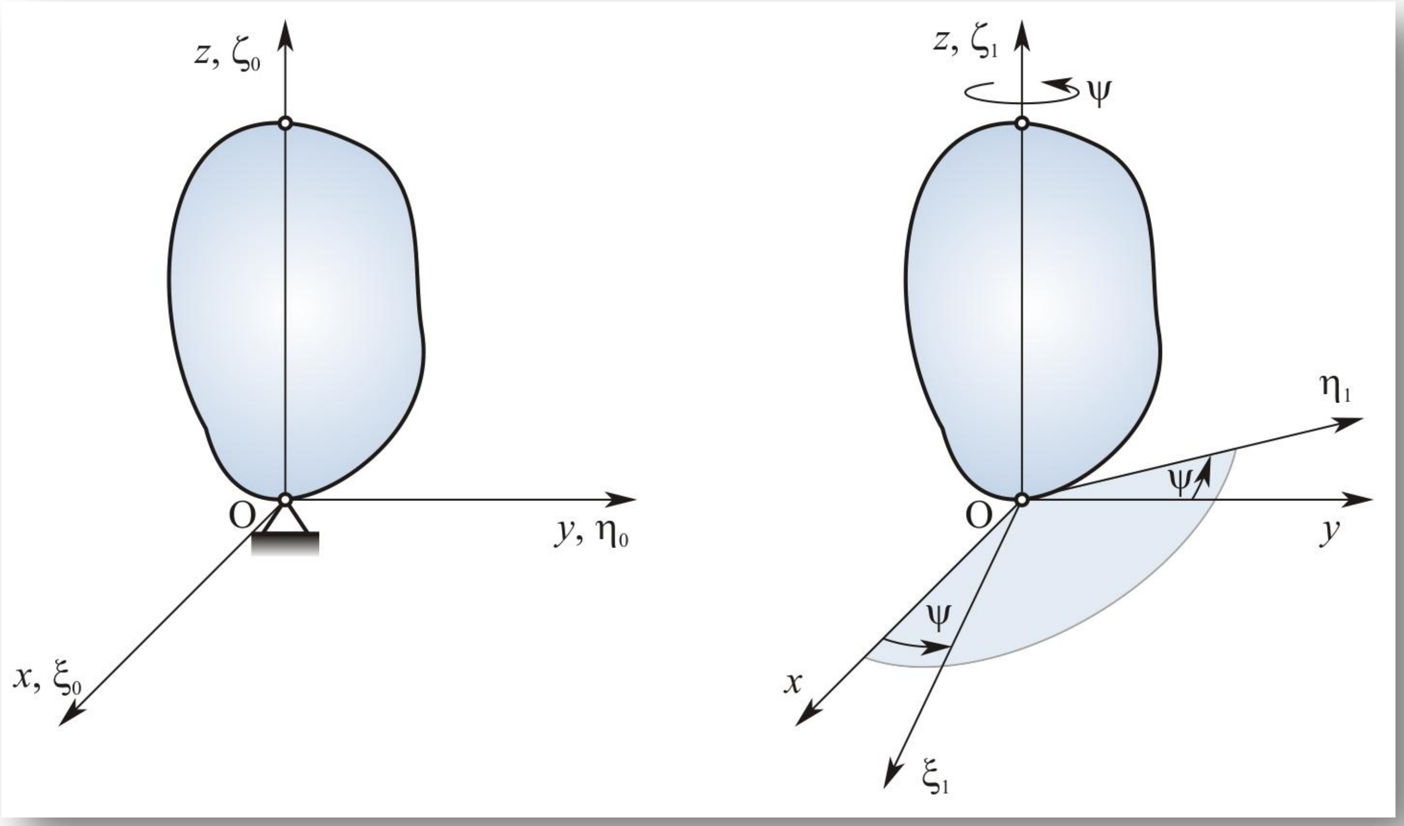


Oxyz – nepokretni koordinatni sistem
Oξηζ – pokretni koordinatni sistem (vezan za telo)

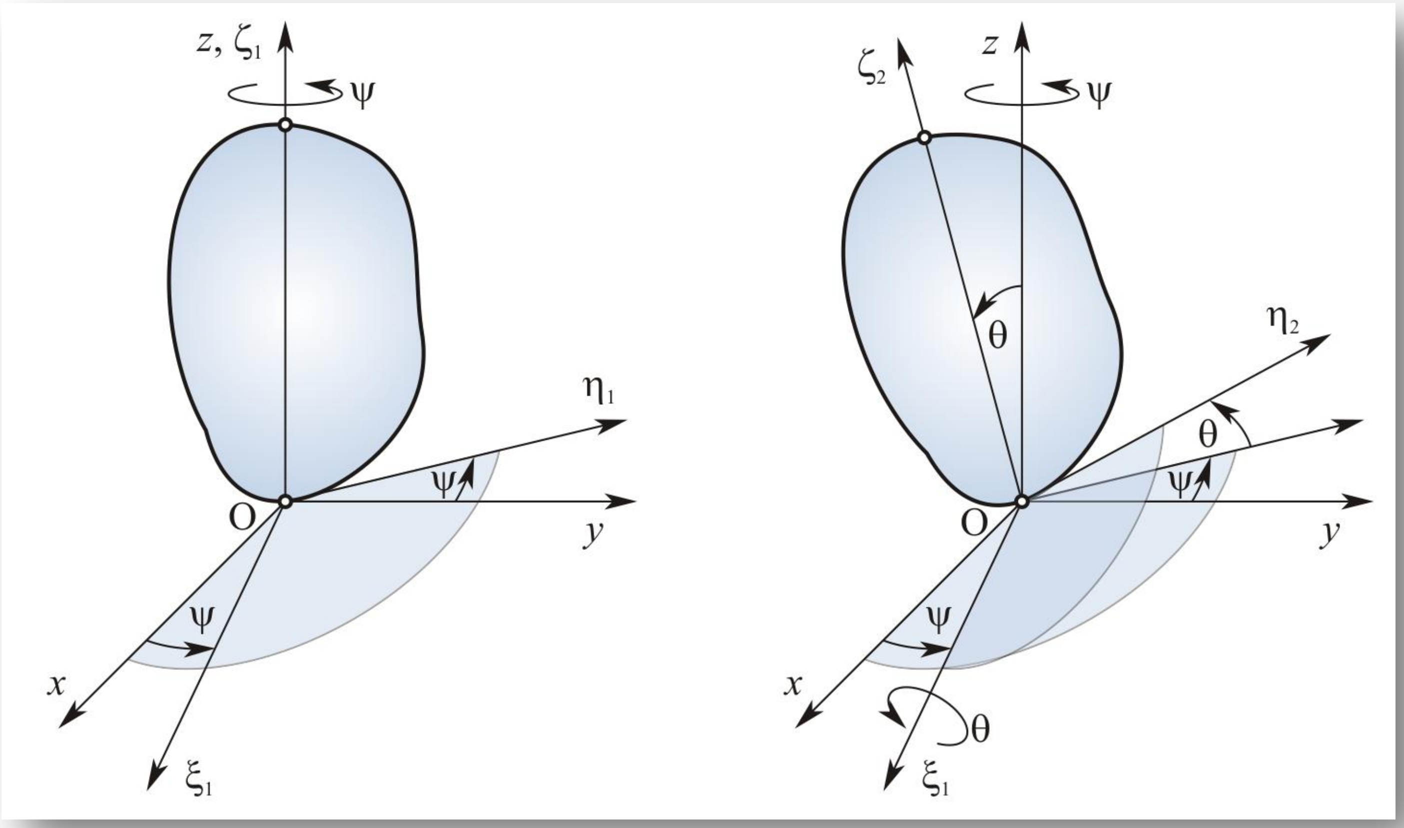


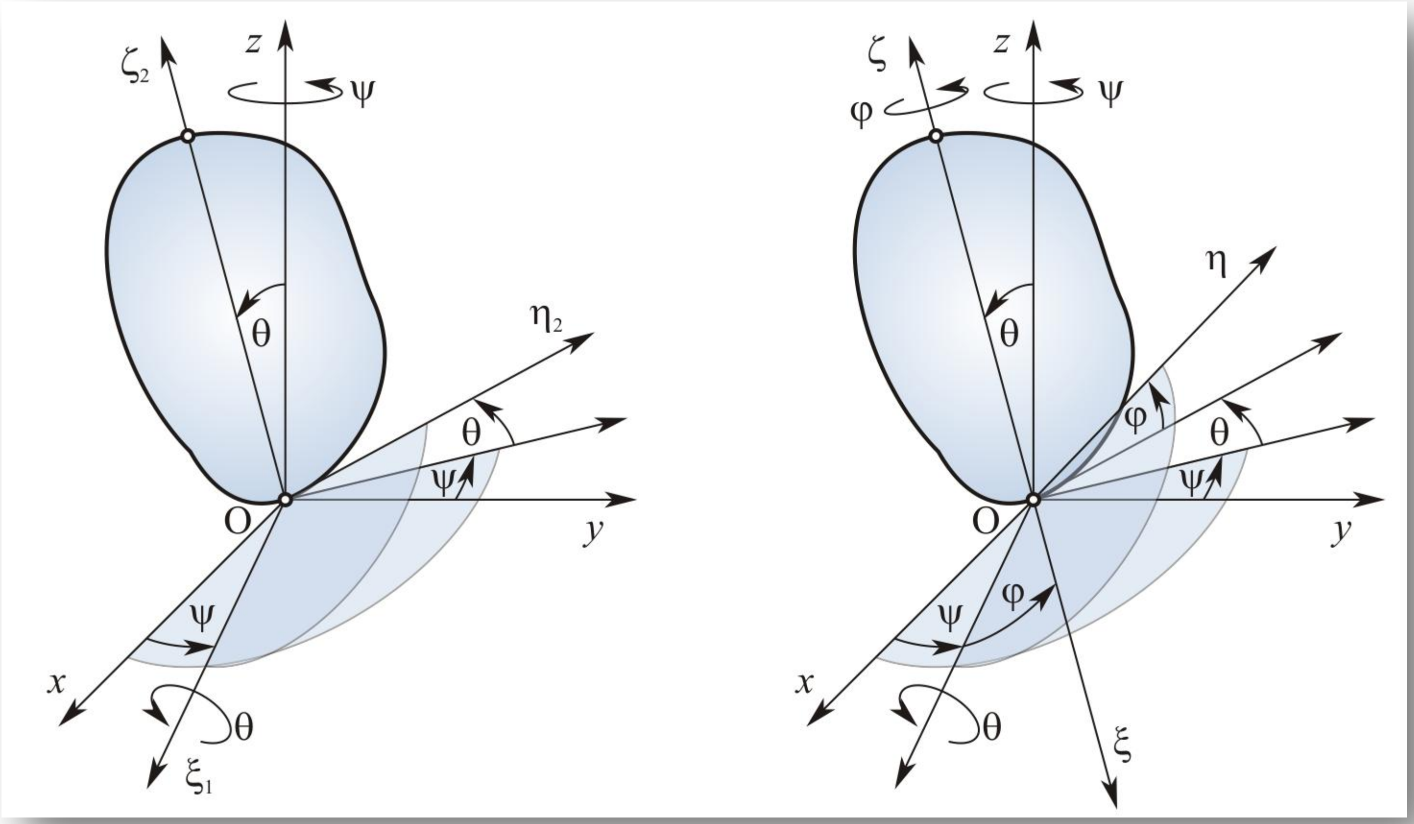
ζ
z
η
$\vec{k}$
O
$\vec{i}$
$\vec{j}$
y
x
ξ
ζ
z
η
$\vec{\mu}$
$\vec{\nu}$
O
$\vec{\lambda}$
y
x
θ
ξ


z, ζ0
O
y, η0
x, ξ0

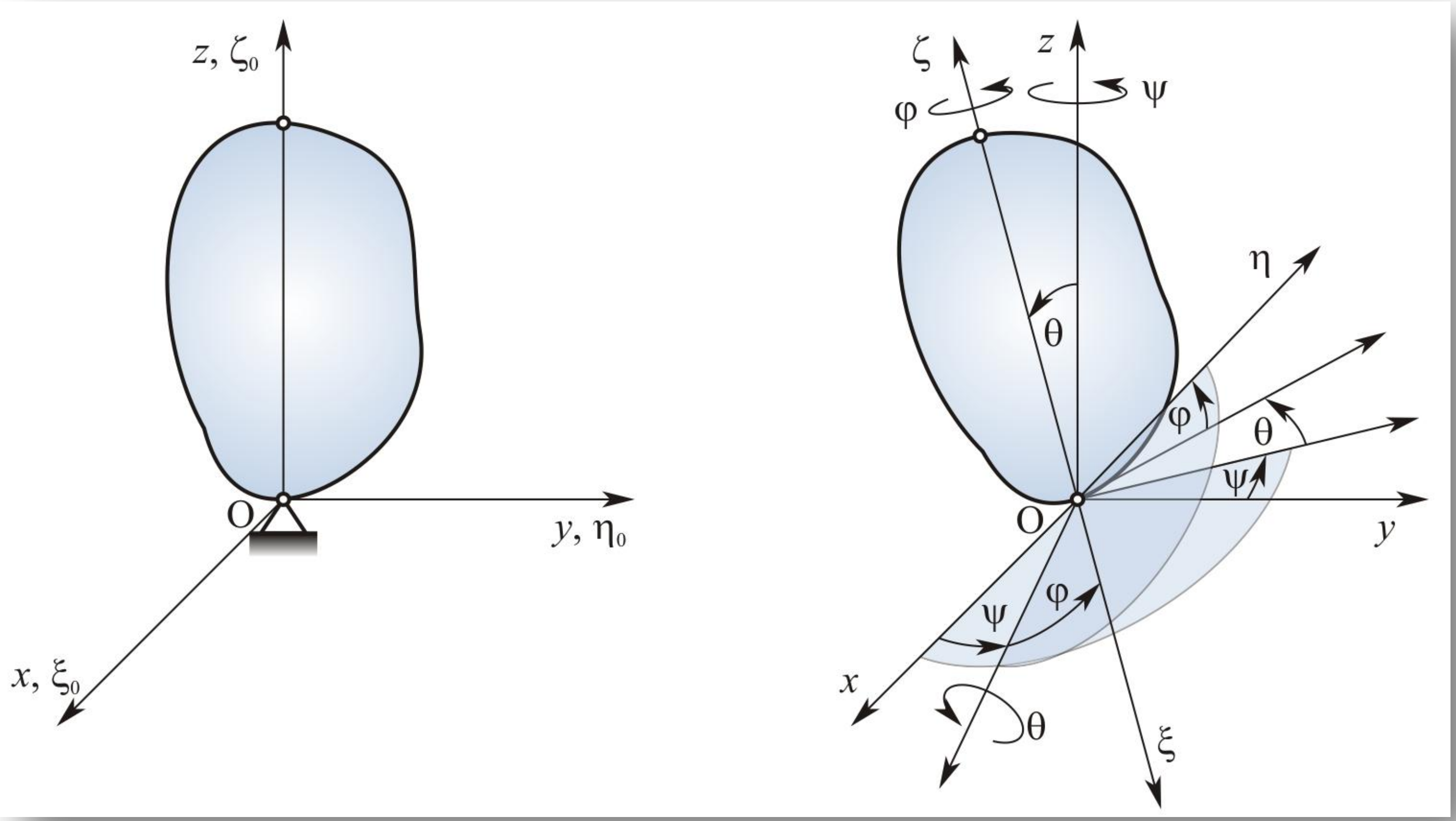


z, ζ0
O
y, η0
x, ξ0
z, ζ1
ψ
η1
ψ
O
y
ψ
x
ξ1

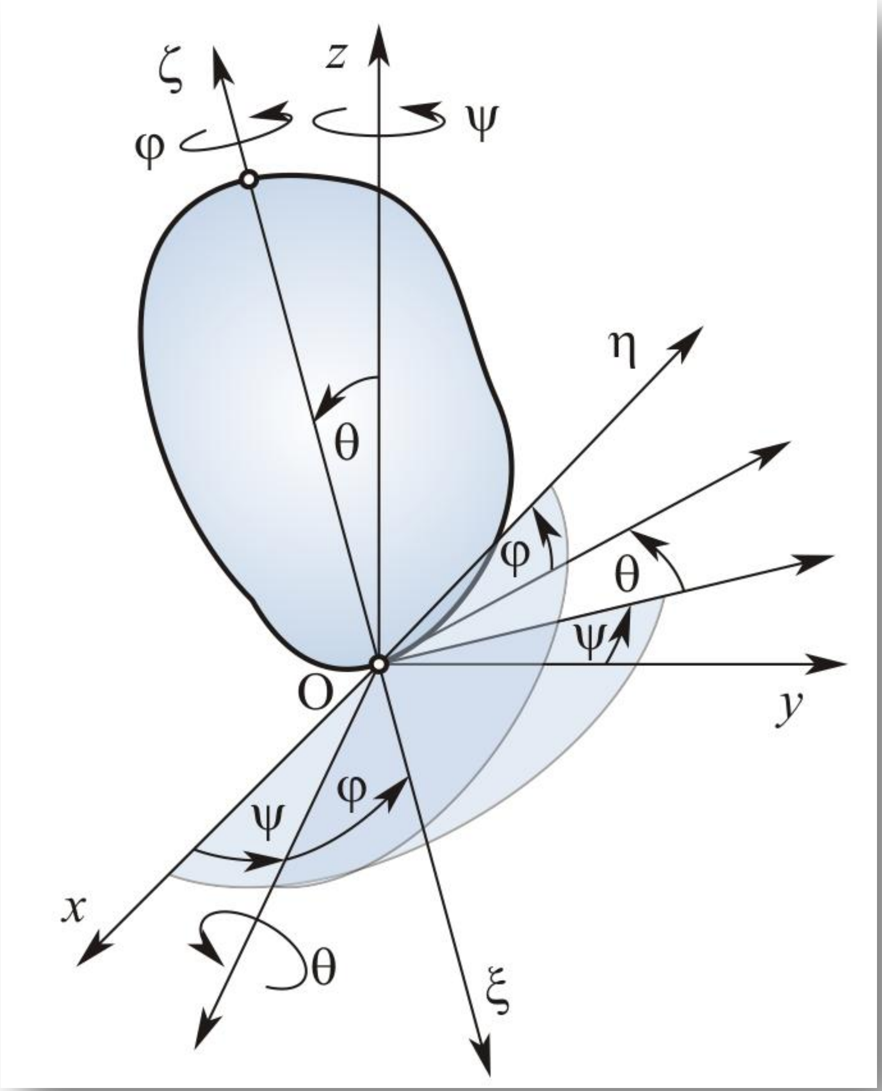

z, ζ1
ψ
η1
ψ
O
y
ψ
x
ξ1
ζ2
z
ψ
θ
η2
θ
ψ
O
y
ψ
x
θ
ξ1

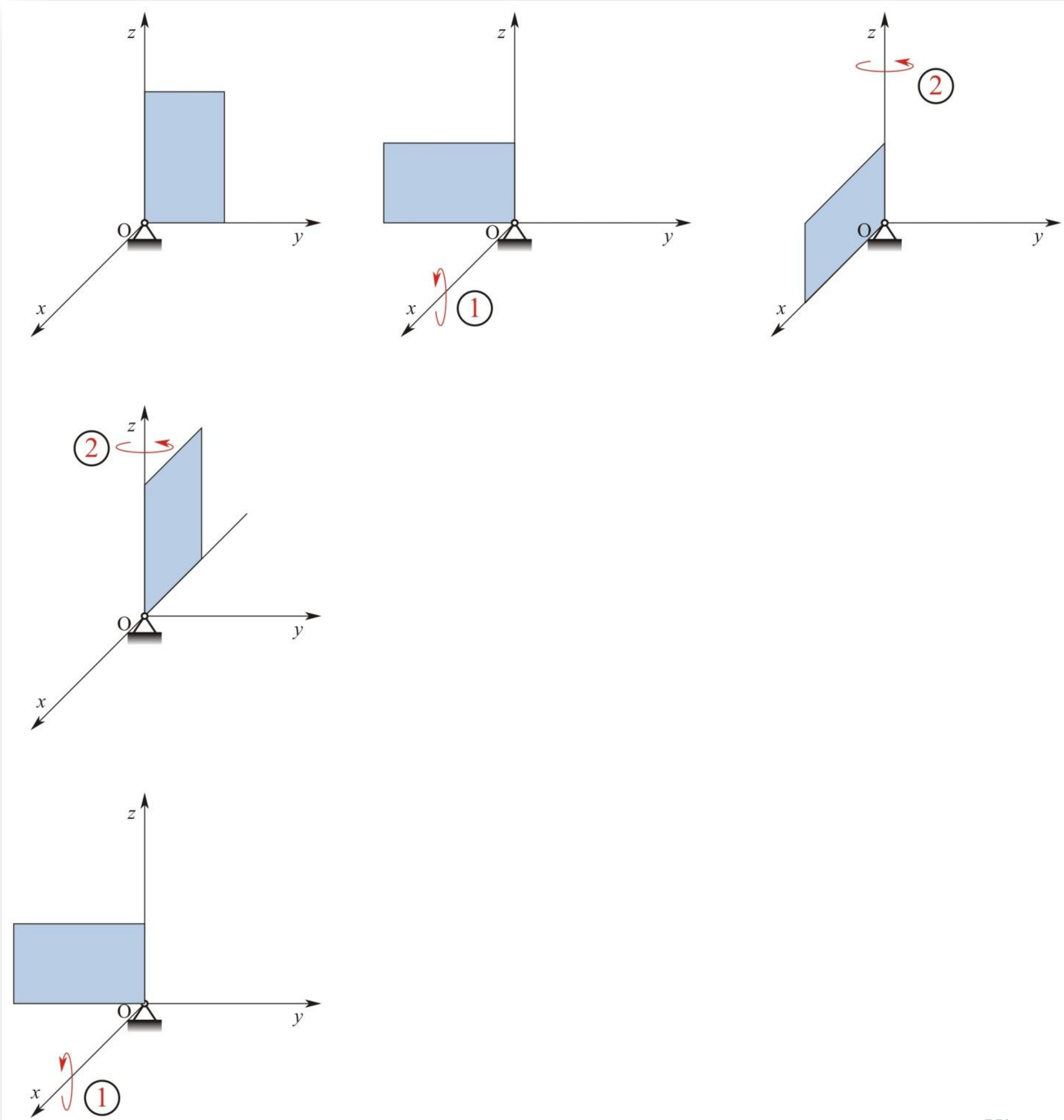


ζ₂
z
ψ
θ
η₂
θ
ψ
O
y
ψ
x
θ
ξ₁
ζ
z
φ
ψ
η
θ
φ
θ
ψ
O
y
φ
ψ
x
θ
ξ

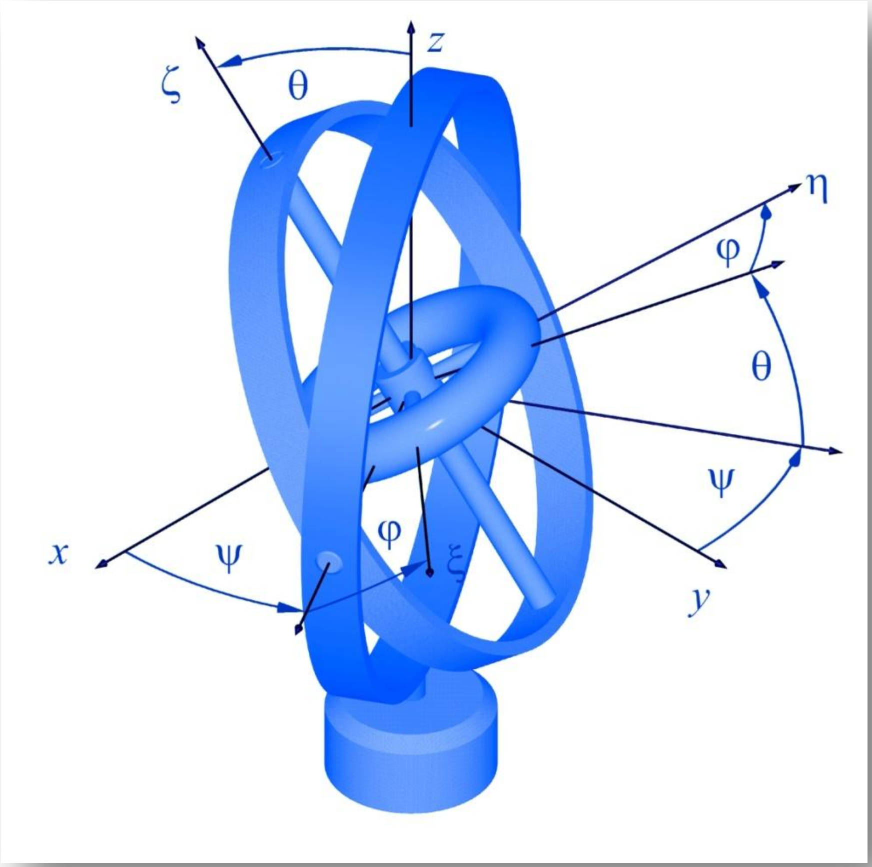


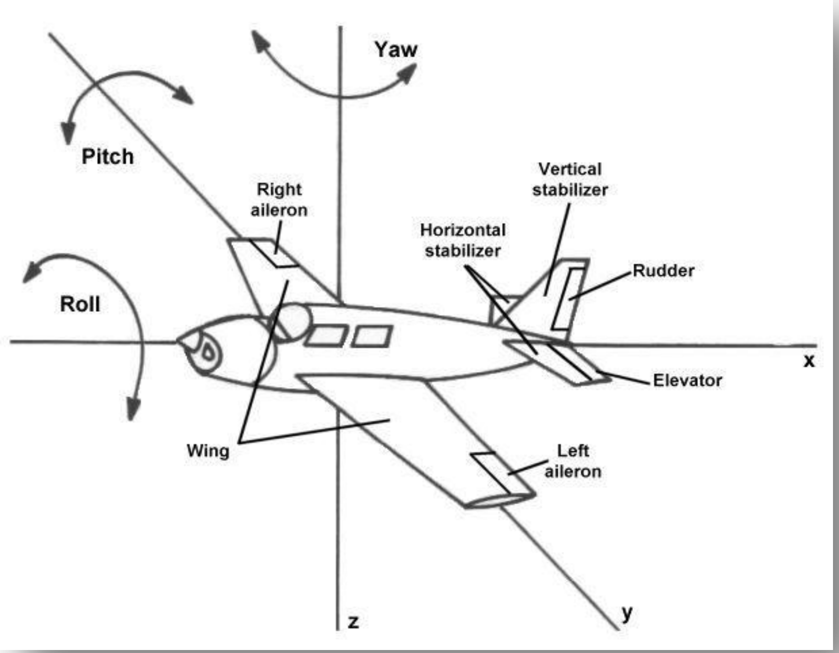
z, ζ0
O
y, η0
x, ξ0
ζ
z
φ
ψ
η
θ
O
y
x
ξ


ζ
z
φ
ψ
η
θ
φ
θ
ψ
O
y
φ
ψ
x
θ
ξ


z
O
y
x
z
O
y
x
1
z
2
O
y
x
z
2
O
y
x
z
O
y
x
1

Ojlerovi uglovi

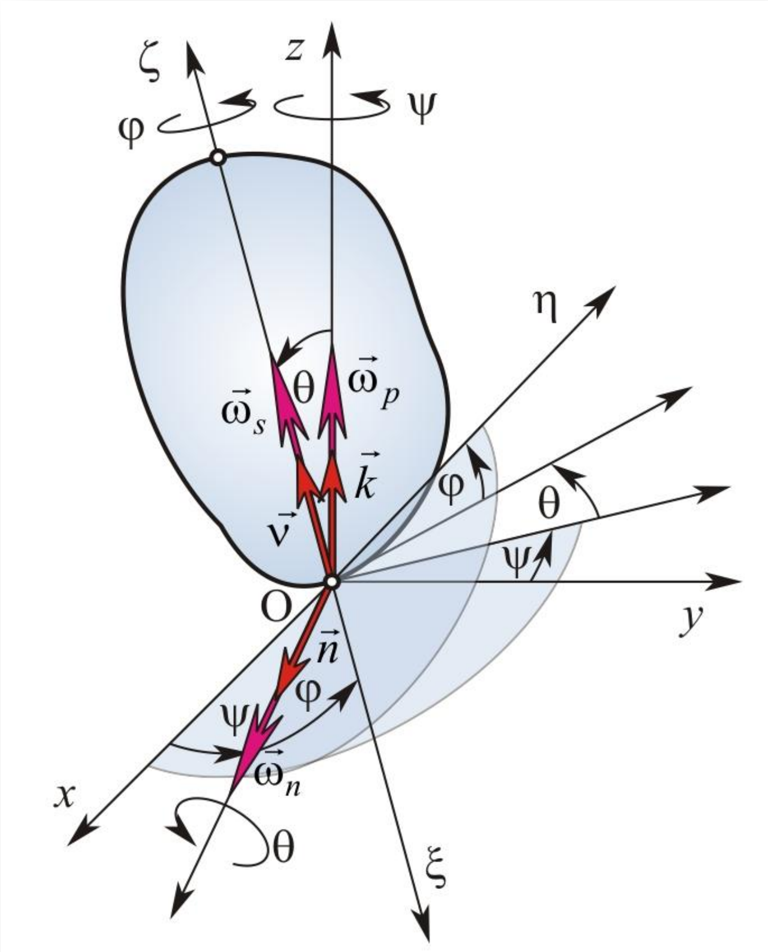




Avionski uglovi:

Yaw – skretanje (sa pravca)
Roll – uvrtanje
Pitch - naginjanje

19. Ugaona brzina i ugaono ubrzanje tela pri sfernom kretanju

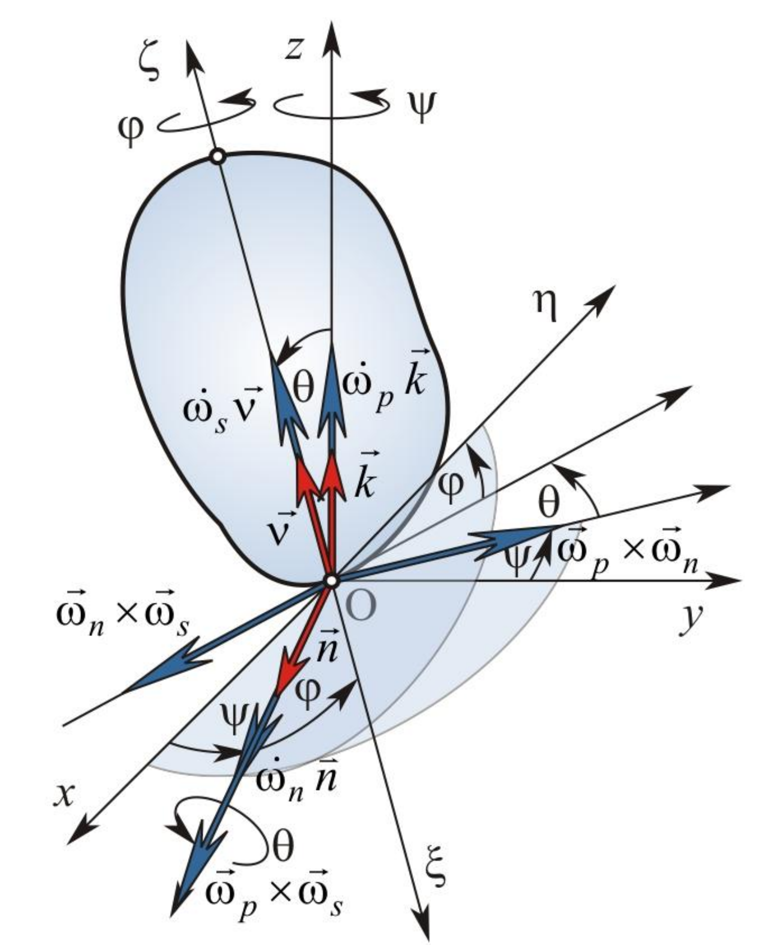


$$\vec{\omega} = \vec{\omega}_p + \vec{\omega}_n + \vec{\omega}_s$$

$$\vec{\omega} = \omega_p \, \vec{k} + \omega_n \, \vec{n} + \omega_s \, \vec{\nu}$$

$$\vec{\omega} = \dot{\psi} \, \vec{k} + \dot{\theta} \, \vec{n} + \dot{\varphi} \, \vec{\nu}$$

$$\vec{\omega} = \omega \vec{\omega}_0$$



$$\vec{\omega} = \omega_p \vec{k} + \omega_n \vec{n} + \omega_s \vec{v}$$

$$\vec{\varepsilon} = \dot{\vec{\omega}} = \dot{\omega}_p \vec{k} + \dot{\omega}_n \vec{n} + \dot{\omega}_s \vec{v} + \omega_p \dot{\vec{k}} + \omega_n \dot{\vec{n}} + \omega_s \dot{\vec{v}}$$

$$\dot{\vec{k}} = 0$$

$$\dot{\vec{n}} = \vec{\omega}_p \times \vec{n} \rightarrow$$

$$\omega_n \dot{\vec{n}} = \omega_n (\vec{\omega}_p \times \vec{n}) = (\vec{\omega}_p \times \omega_n \vec{n}) = \vec{\omega}_p \times \vec{\omega}_n$$

$$\dot{\vec{v}} = \vec{\omega} \times \vec{v} = (\vec{\omega}_p + \vec{\omega}_n + \vec{\omega}_s) \times \vec{v} = \vec{\omega}_p \times \vec{v} + \vec{\omega}_n \times \vec{v} \rightarrow$$

$$\omega_s \dot{\vec{v}} = \vec{\omega}_p \times \vec{\omega}_s + \vec{\omega}_n \times \vec{\omega}_s$$

$$\vec{\varepsilon} = \vec{\varepsilon}_1 + \vec{\varepsilon}_2 + \vec{\varepsilon}_3$$

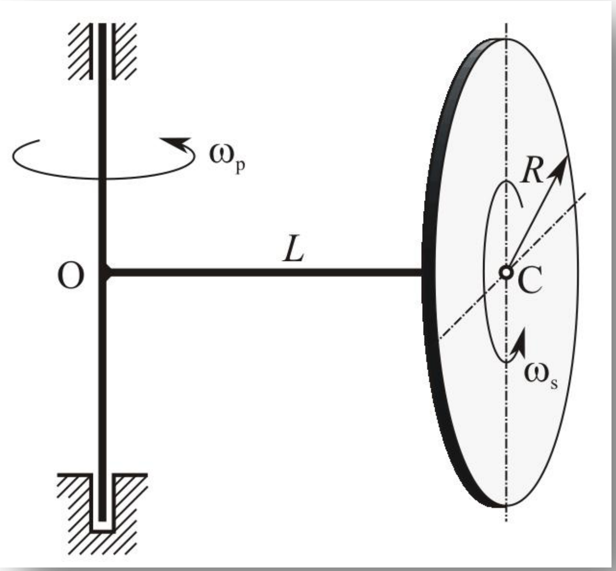
$$\vec{\varepsilon}_1 = \dot{\omega}_p \vec{k} + \dot{\omega}_n \vec{n} + \dot{\omega}_s \vec{v}$$

$$\vec{\varepsilon}_2 = \vec{\omega}_p \times \vec{\omega}_s$$

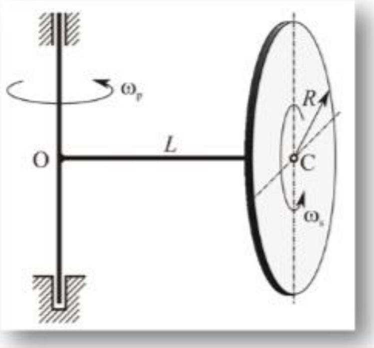
$$\vec{\varepsilon}_3 = \vec{\omega}_p \times \vec{\omega}_n + \vec{\omega}_n \times \vec{\omega}_s$$

Primer 1

Za vertikalno kruto vratilo vezana je horizontalna osovina OC, dužine $L=\sqrt{3}$m, na čijem kraju je cilindričnim zglobom vezan disk, poluprečnika R=1m. Vratilo se obrće oko vertikalne ose ugaonom brzinom $\omega_p=3t$ [s^{-1}] . Disk se, dodatno, obrće oko osovine OC ugaonom brzinom $\omega_s=2t^2$ [s^{-1}] . Odrediti ugaonu brzinu i ugaono ubrzanje diska u trenutku t_1=1s.



Za vertikalno kruto vratilo vezana je horizontalna osovina OC, dužine $L=\sqrt{3}$m, na čijem kraju je cilindričnim zglobom vezan disk, poluprečnika R=1m. Vratilo se obrće oko vertikalne ose ugaonom brzinom $\omega_p=3t$ [s^{-1}] . Disk se, dodatno, obrće oko osovine OC ugaonom brzinom $\omega_s=2t^2$ [s^{-1}] . Odrediti ugaonu brzinu i ugaono ubrzanje diska u trenutku t_1=1s.

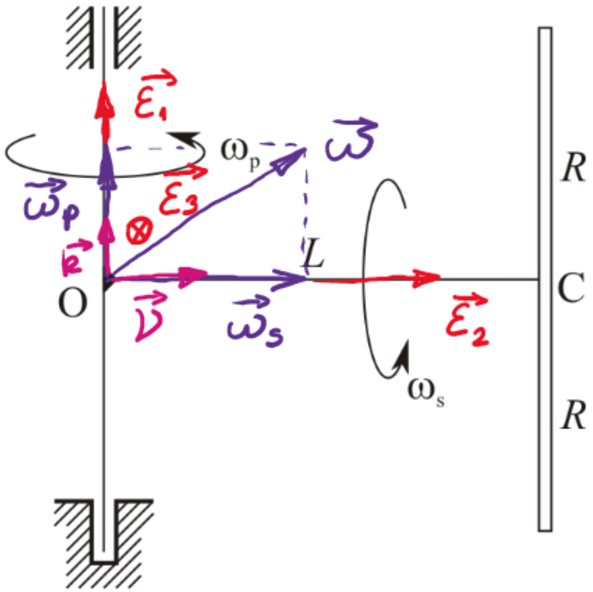


$\omega_p = 3t$ $\qquad$ $\omega_p(1) = 3$

$\omega_s = 2t^2$ $\qquad$ $\omega_s(1) = 2$

$\dot{\omega}_p = 3$ $\qquad$ $\dot{\omega}_p(1) = 3$

$\dot{\omega}_s = 4t$ $\qquad$ $\dot{\omega}_s(1) = 4$



$$\vec{\omega} = \vec{\omega}_p + \vec{\omega}_s$$

$$\vec{\omega}_p \perp \vec{\omega}_s \rightarrow \omega = \sqrt{\omega_p^2 + \omega_s^2}$$

$$\vec{\omega} = \omega_p \vec{k} + \omega_s \vec{\nu}$$

$$\vec{\varepsilon} = \dot{\vec{\omega}} = \underbrace{\dot{\omega}_p \vec{k}}_{\vec{\varepsilon}_1} + \omega_p \cancel{\dot{\vec{k}}}^{\,0} + \underbrace{\dot{\omega}_s \vec{\nu}}_{\vec{\varepsilon}_2} + \underbrace{\omega_s \dot{\vec{\nu}}}_{\vec{\varepsilon}_3}$$

$$\vec{\varepsilon}_3 = \omega_s \dot{\vec{\nu}} = \omega_s (\vec{\omega}_p \times \vec{\nu})$$

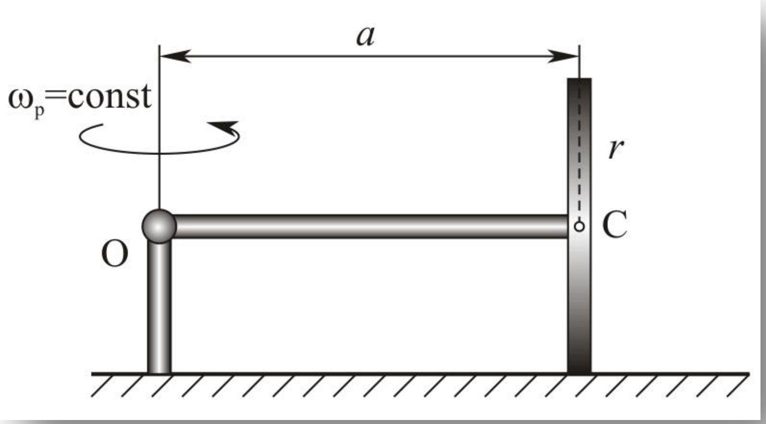
$$= \vec{\omega}_p \times \omega_s \vec{\nu} = \vec{\omega}_p \times \vec{\omega}_s$$

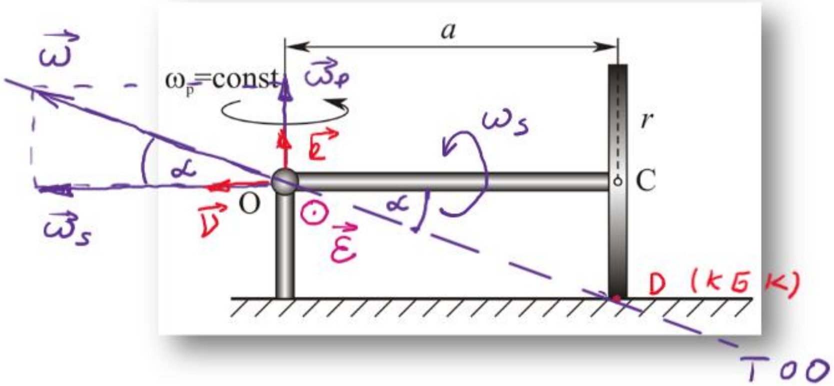
$$\varepsilon_3 = \omega_p \omega_s \sin\left(\frac{\pi}{2}\right) = \omega_p \omega_s$$

$$\varepsilon = \sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2}$$

Primer 2

Disk, poluprečnika r, kruto je vezan za štap OC, dužine a, koji je sfernim zglobom O vezan za podlogu. Disk se kotrlja bez klizanja po nepokretnoj horizontalnoj podlozi. Štap OC se oko vertikalne ose obrće konstantnom ugaonom brzinom ω_p=const [s^{-1}]. . Odrediti ugaonu brzinu i ugaono ubrzanje diska.





$$\vec{\omega} = \vec{\omega}_p + \vec{\omega}_s$$

$$\sin\alpha = \frac{\omega_p}{\omega} \rightarrow \omega = \frac{\omega_p}{\sin\alpha} = const$$

$$tg\,\alpha = \frac{\omega_p}{\omega_s} \rightarrow \omega_s = \omega_p\, ctg\,\alpha = const$$

$$ctg\,\alpha = \frac{a}{r}\,,\quad \sin\alpha = \frac{r}{\sqrt{a^2 + r^2}}$$

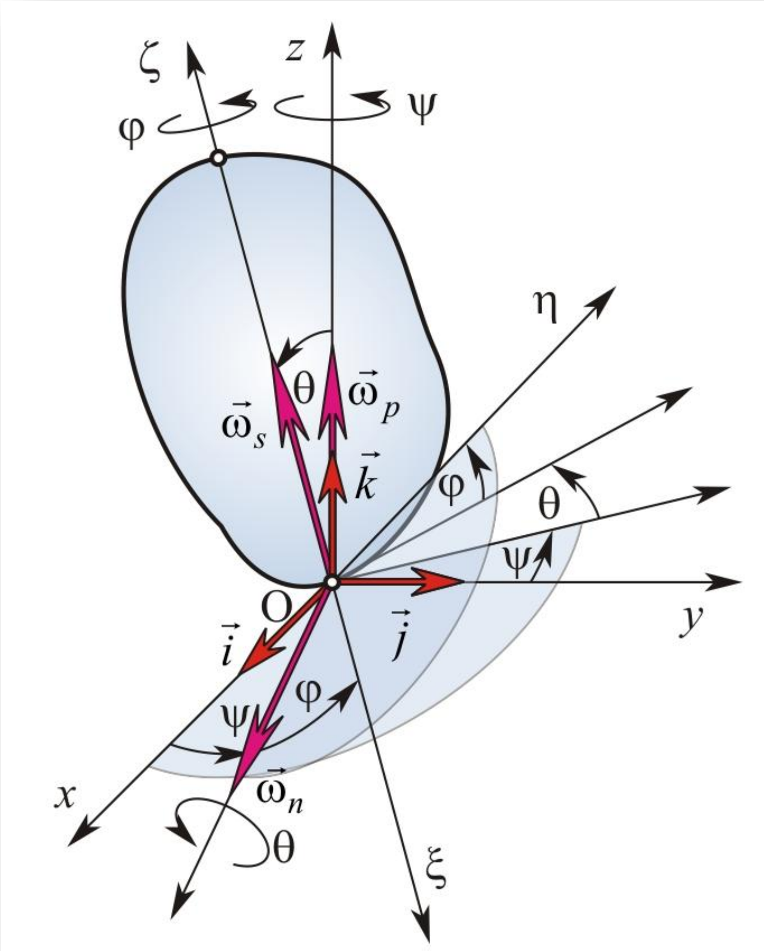
$$\vec{\omega} = \omega_p \vec{k} + \omega_s \vec{\nu}$$

$$\vec{\varepsilon} = \dot{\vec{\omega}} = \cancel{\dot{\omega}_p}^{\,0} \vec{k} + \omega_p \cancel{\dot{\vec{k}}}^{\,0} + \cancel{\dot{\omega}_s}^{\,0} \vec{\nu} + \omega_s \dot{\vec{\nu}}$$

$$\vec{\varepsilon} = \omega_s \dot{\vec{\nu}} = \omega_s (\vec{\omega}_p \times \vec{\nu}) = \vec{\omega}_p \times \omega_s \vec{\nu} = \vec{\omega}_p \times \vec{\omega}_s$$

$$\varepsilon = \omega_p \omega_s \cancel{\sin\frac{\pi}{2}}^{\,1} = \omega_p \omega_s = \omega_p^2\, ctg\,\alpha$$

20. Projekcije ugaone brzine sfernog kretanja na koordinatne ose (Ojlerove kinematičke jednačine)



$$\vec{\omega} = \omega_p \vec{k} + \omega_n \vec{n} + \omega_s \vec{\nu} = \dot{\psi} \vec{k} + \dot{\theta} \vec{n} + \dot{\varphi} \vec{\nu}$$

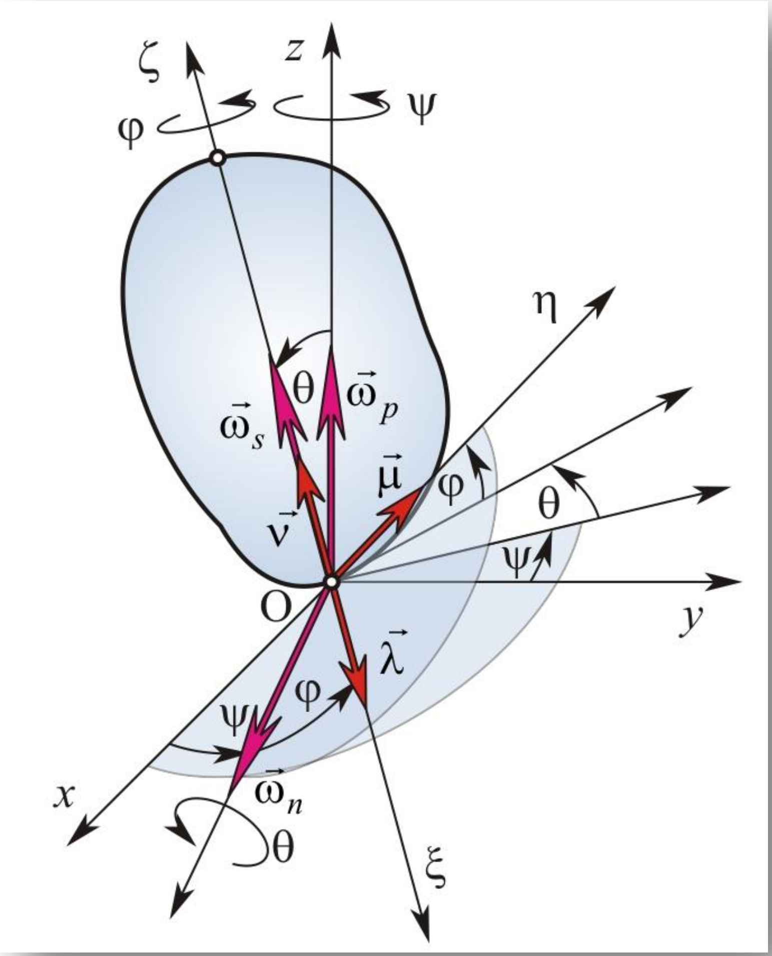
$$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$

$$\omega_x = \dot{\varphi} \sin\theta \sin\psi + \dot{\theta} \cos\psi$$

$$\omega_y = -\dot{\varphi} \sin\theta \cos\psi + \dot{\theta} \sin\psi$$

$$\omega_z = \dot{\psi} + \dot{\varphi} \cos\theta$$

$$\omega = \sqrt{\omega_x^2 + \omega_y^2 + \omega_z^2} = \sqrt{\dot{\psi}^2 + \dot{\theta}^2 + \dot{\varphi}^2 + 2\dot{\psi}\dot{\varphi}\cos\theta}$$



$$\vec{\omega} = \omega_p \vec{k} + \omega_n \vec{n} + \omega_s \vec{v} = \dot{\psi} \vec{k} + \dot{\theta} \vec{n} + \dot{\varphi} \vec{v}$$

$$\vec{\omega} = \omega_\xi \vec{\lambda} + \omega_\eta \vec{\mu} + \omega_\zeta \vec{v}$$

$$\omega_\xi = \dot{\psi} \sin\theta \sin\varphi + \dot{\theta} \cos\varphi$$

$$\omega_\eta = \dot{\psi} \sin\theta \cos\varphi - \dot{\theta} \sin\varphi$$

$$\omega_\zeta = \dot{\psi} \cos\theta + \dot{\varphi}$$

$$\omega = \sqrt{\omega_\xi^2 + \omega_\eta^2 + \omega_\zeta^2}$$

21. Projekcije ugaonog ubrzanja sfernog kretanja na koordinatne ose

$$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$

$$\omega_x = \dot{\varphi}\sin\theta\sin\psi + \dot{\theta}\cos\psi$$

$$\omega_y = -\dot{\varphi}\sin\theta\cos\psi + \dot{\theta}\sin\psi$$

$$\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta$$

$$\vec{\varepsilon} = \dot{\vec{\omega}} = \dot{\omega}_x \vec{i} + \dot{\omega}_y \vec{j} + \dot{\omega}_z \vec{k} = \ldots$$

$$\vec{\omega} = \omega_\xi \, \vec{\lambda} + \omega_\eta \, \vec{\mu} + \omega_\zeta \, \vec{\nu}$$

$$\omega_\xi = \dot{\psi} \sin\theta \sin\varphi + \dot{\theta} \cos\varphi$$

$$\omega_\eta = \dot{\psi} \sin\theta \cos\varphi - \dot{\theta} \sin\varphi$$

$$\omega_\zeta = \dot{\psi} \cos\theta + \dot{\varphi}$$

$$\vec{\varepsilon} = \dot{\vec{\omega}} = \dot{\omega}_\xi \, \vec{\lambda} + \dot{\omega}_\eta \, \vec{\mu} + \dot{\omega}_\zeta \, \vec{\nu} + \omega_\xi \, \dot{\vec{\lambda}} + \omega_\eta \, \dot{\vec{\mu}} + \omega_\zeta \, \dot{\vec{\nu}}$$

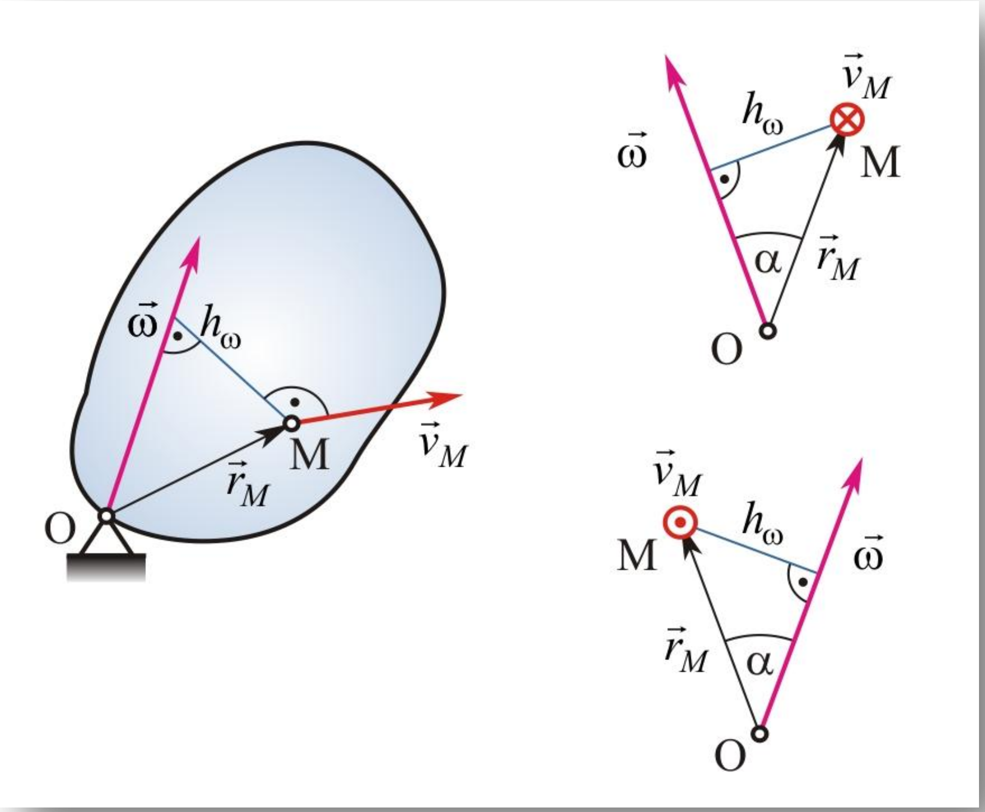
$$\dot{\vec{\lambda}} = \vec{\omega} \times \vec{\lambda} \, , \; \dot{\vec{\mu}} = \vec{\omega} \times \vec{\mu} \, , \; \dot{\vec{\nu}} = \vec{\omega} \times \vec{\nu}$$

$$\omega_\xi \, \dot{\vec{\lambda}} + \omega_\eta \, \dot{\vec{\mu}} + \omega_\zeta \, \dot{\vec{\nu}} = \vec{\omega} \times \omega_\xi \vec{\lambda} + \vec{\omega} \times \omega_\eta \vec{\mu} + \vec{\omega} \times \omega_\zeta \vec{\nu} =$$

$$= \vec{\omega} \times (\omega_\xi \vec{\lambda} + \omega_\eta \vec{\mu} + \omega_\zeta \vec{\nu}) = \vec{\omega} \times \vec{\omega} = 0$$

$$\vec{\varepsilon} = \dot{\omega}_\xi \, \vec{\lambda} + \dot{\omega}_\eta \, \vec{\mu} + \dot{\omega}_\zeta \, \vec{\nu} = \dots$$

22. Brzine i ubrzanja tačaka tela pri sfernom kretanju

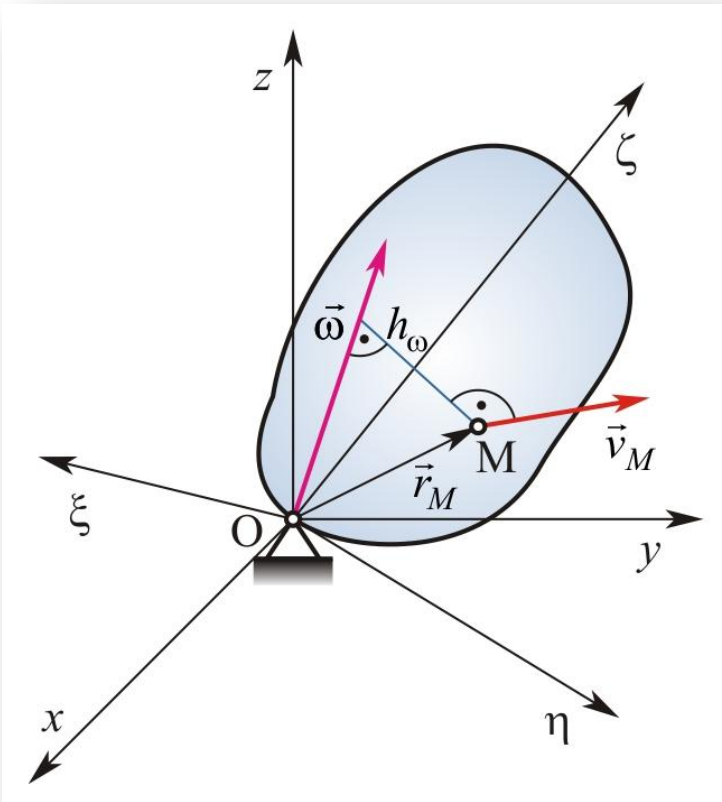


$$\vec{v}_M = \vec{\omega} \times \vec{r}_M$$

$$v_M = \omega \cdot r_M \sin \angle(\vec{\omega}, \vec{r}_M)$$

$$h_\omega = r_M \sin \angle(\vec{\omega}, \vec{r}_M)$$

$$v_M = \omega \cdot h_\omega$$



$$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k}$$

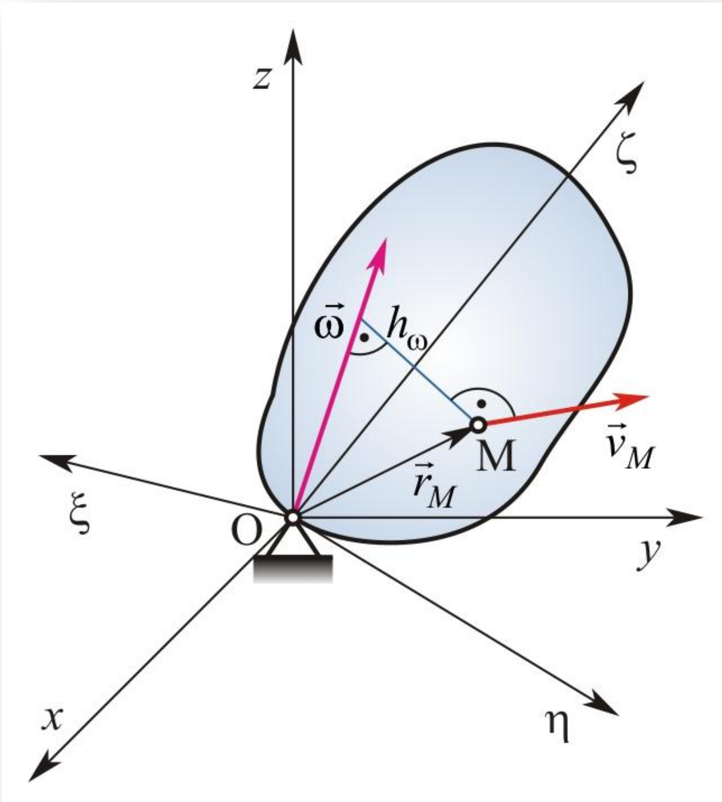
$$\vec{r}_M = x_M \vec{i} + y_M \vec{j} + z_M \vec{k}$$

$$\vec{v}_M = \vec{\omega} \times \vec{r}_M = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \omega_x & \omega_y & \omega_z \\ x_M & y_M & z_M \end{vmatrix}$$

$$v_{Mx} = \omega_y z_M - \omega_z y_M$$

$$v_{My} = \omega_z x_M - \omega_x z_M$$

$$v_{Mz} = \omega_x y_M - \omega_y x_M$$



$$\vec{\omega} = \omega_\xi \vec{\lambda} + \omega_\eta \vec{\mu} + \omega_\zeta \vec{\nu}$$

$$\vec{r}_M = \xi_M \vec{\lambda} + \eta_M \vec{\mu} + \zeta_M \vec{\nu}$$

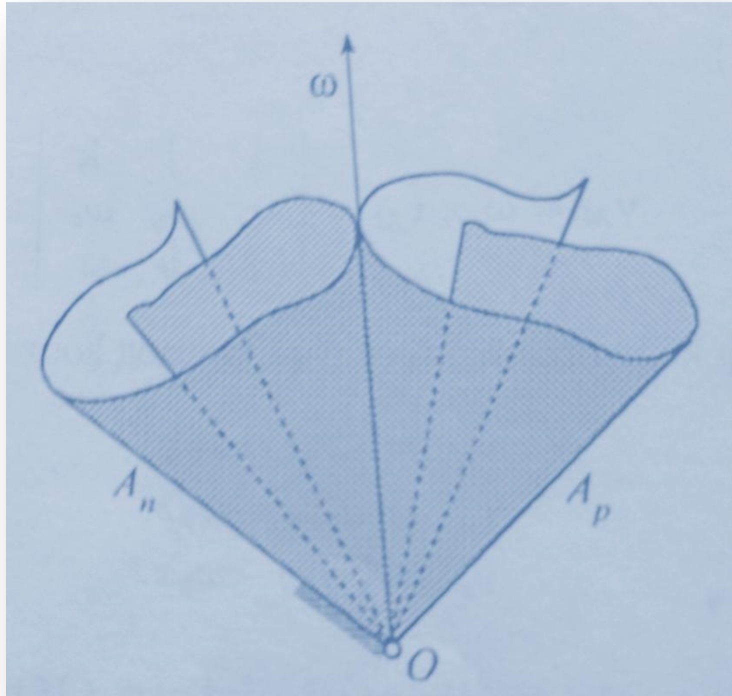
$$\vec{v}_M = \vec{\omega} \times \vec{r}_M = \begin{vmatrix} \vec{\lambda} & \vec{\mu} & \vec{\nu} \\ \omega_\xi & \omega_\eta & \omega_\zeta \\ \xi_M & \eta_M & \zeta_M \end{vmatrix}$$

$$v_{M\xi} = \omega_\eta \zeta_M - \omega_\zeta \eta_M$$

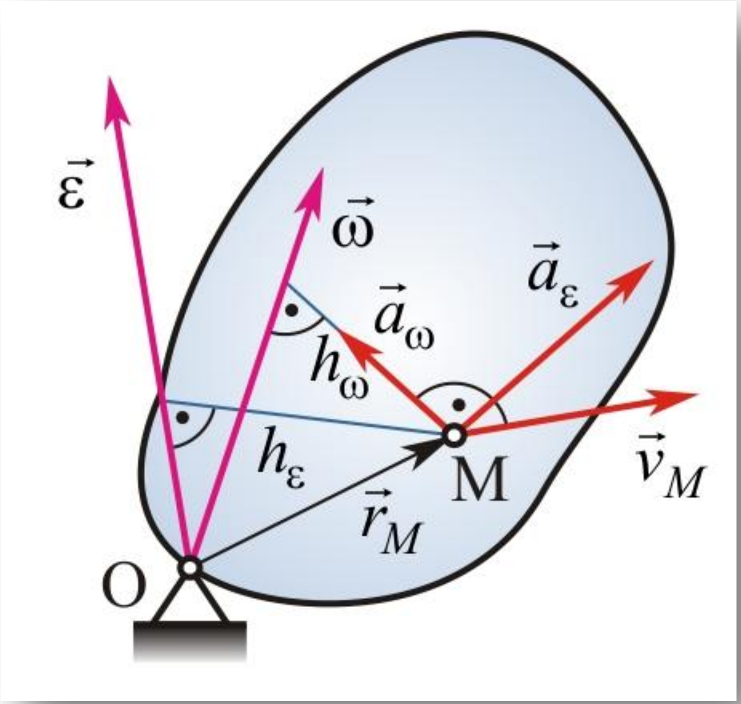
$$v_{M\eta} = \omega_\zeta \xi_M - \omega_\xi \zeta_M$$

$$v_{M\zeta} = \omega_\xi \eta_M - \omega_\eta \xi_M$$

Aksoidi



Ako se napravi telo oblika nepokretnog aksoida i čvrsto veže za zemlju, i telo oblika pokretnog aksoida i čvrsto veže za telo koje vrši sferno kretanje, onda se sferno kretanje tela podudara sa kretanjem istog tela koje je vezano za pokretan aksoid pri kotrljanju bez klizanja pokretnog aksoida po nepokretnom. U svakom trenutku vremena pokretan i nepokretan aksoid se dodiruju duž trenutne ose obrtanja, pa vektor trenutne ugaone brzine $\boldsymbol{\omega}$ tokom kretanja opisiju površinu u koordinatnom sistemu $Oxyz$ koja se poklapa sa nepokretnim aksoidom.



$$\vec{v}_M = \vec{\omega} \times \vec{r}_M$$

$$\vec{a}_M = \dot{\vec{v}}_M = \dot{\vec{\omega}} \times \vec{r}_M + \vec{\omega} \times \dot{\vec{r}}_M$$

$$\vec{a}_M = \vec{\varepsilon} \times \vec{r}_M + \vec{\omega} \times \vec{v}_M$$

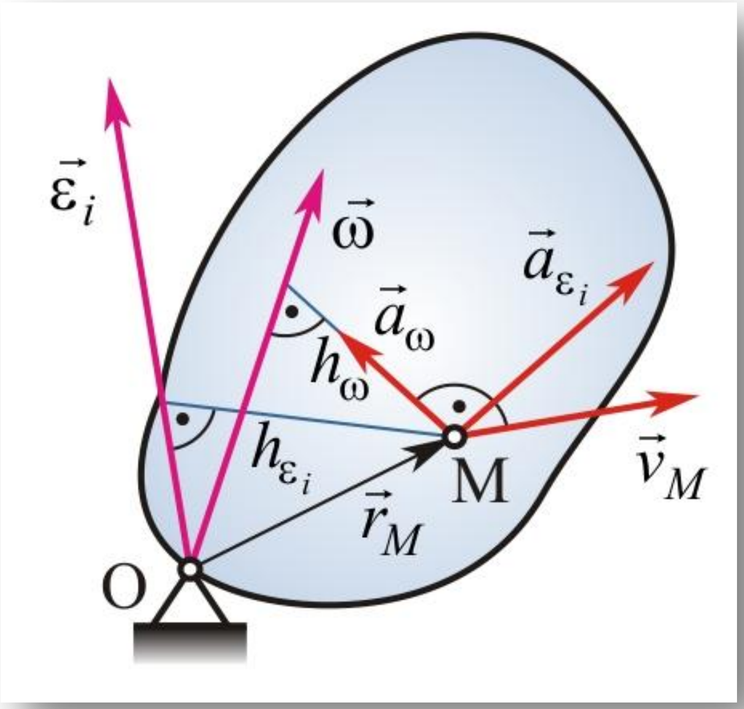
$$\vec{a}_M = \vec{a}_\varepsilon + \vec{a}_\omega$$

$$\vec{a}_\varepsilon = \vec{\varepsilon} \times \vec{r}_M$$

$$\vec{a}_\omega = \vec{\omega} \times \vec{v}_M$$

$$a_\varepsilon = \varepsilon \cdot r_M \cdot \sin \angle(\vec{\varepsilon}, \vec{r}_M) = \varepsilon \cdot h_\varepsilon$$

$$a_\omega = \omega \cdot v_M \cdot \sin \angle(\vec{\omega}, \vec{v}_M) = \omega \cdot h_\omega \cdot \omega \cdot \sin \frac{\pi}{2} = h_\omega \omega^2$$

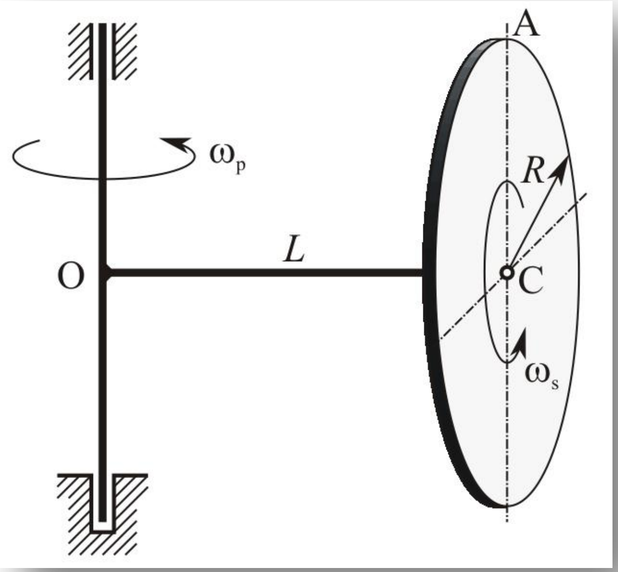


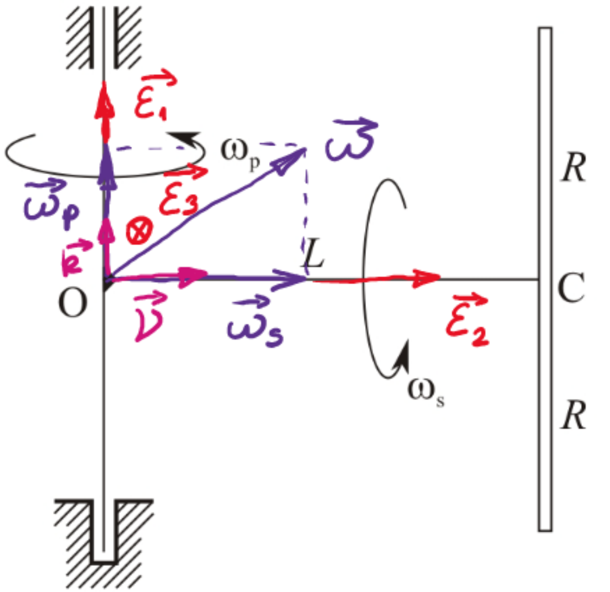
$$\vec{a}_{\varepsilon} = \vec{\varepsilon} \times \vec{r}_M = \vec{\varepsilon}_1 \times \vec{r}_M + \vec{\varepsilon}_2 \times \vec{r}_M + \vec{\varepsilon}_3 \times \vec{r}_M$$

$$a_{\varepsilon_i} = \varepsilon_i \cdot r_M \cdot \sin \angle(\vec{\varepsilon}_i, \vec{r}_M) = \varepsilon_i \cdot h_{\varepsilon_i}$$

Primer 3

Za vertikalno kruto vratilo vezana je horizontalna osovina OC, dužine $L=\sqrt{3}$m, na čijem kraju je cilindričnim zglobom vezan disk, poluprečnika R=1m. Vratilo se obrće oko vertikalne ose ugaonom brzinom $\omega_p=3t$ [s^{-1}] . Disk se, dodatno, obrće oko osovine OC ugaonom brzinom $\omega_s=2t^2$ [s^{-1}] . Odrediti ugaonu brzinu i ugaono ubrzanje diska, kao i brzinu i ubrzanje tačke A na obodu diska u trenutku t_1=1s (u datom trenuktku tačka A se nalazi u najvišem položaju).





$$\vec{\omega} = \vec{\omega}_p + \vec{\omega}_s$$

$$\vec{\omega}_p \perp \vec{\omega}_s \rightarrow \omega = \sqrt{\omega_p^2 + \omega_s^2}$$

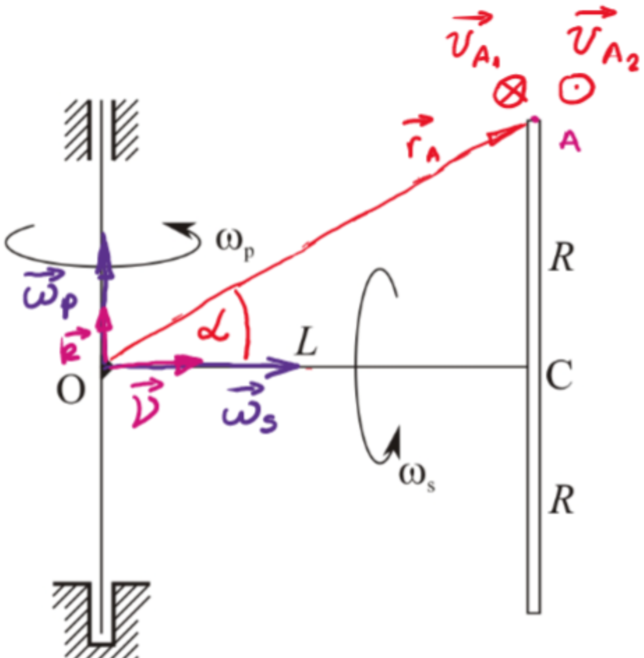
$$\vec{\omega} = \omega_p \vec{k} + \omega_s \vec{\nu}$$

$$\vec{\varepsilon} = \dot{\vec{\omega}} = \underbrace{\dot{\omega}_p \vec{k}}_{\vec{\varepsilon}_1} + \omega_p \cancel{\dot{\vec{k}}}^{\,0} + \underbrace{\dot{\omega}_s \vec{\nu}}_{\vec{\varepsilon}_2} + \underbrace{\omega_s \dot{\vec{\nu}}}_{\vec{\varepsilon}_3}$$

$$\vec{\varepsilon}_3 = \omega_s \dot{\vec{\nu}} = \omega_s (\vec{\omega}_p \times \vec{\nu})$$
$$= \vec{\omega}_p \times \omega_s \vec{\nu} = \vec{\omega}_p \times \vec{\omega}_s$$

$$\varepsilon_3 = \omega_p \omega_s \sin\left(\frac{\pi}{2}\right) = \omega_p \omega_s$$

$$\varepsilon = \sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2}$$

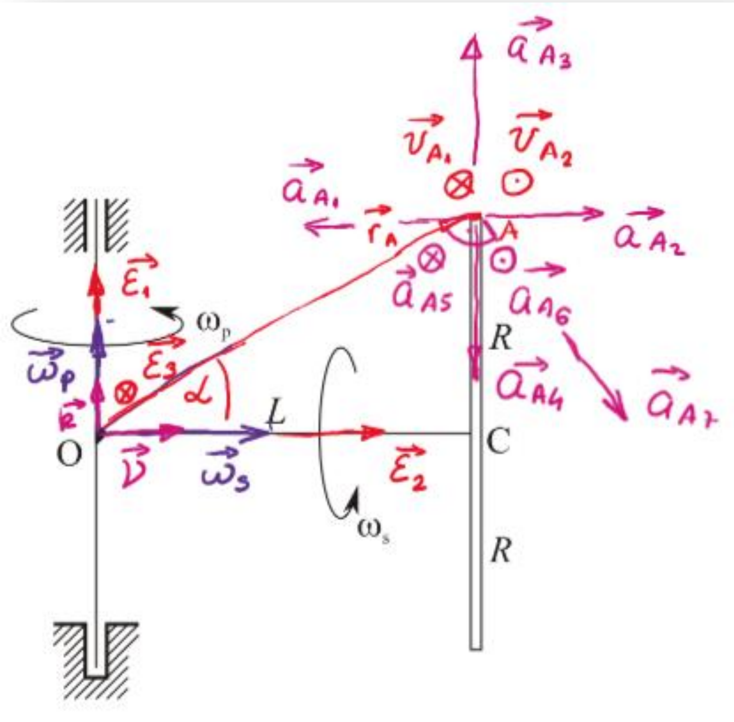


$$\vec{v}_A = \vec{\omega} \times \vec{r}_A = \underbrace{\vec{\omega}_p \times \vec{r}_A}_{\vec{v}_{A_1}} + \underbrace{\vec{\omega}_s \times \vec{r}_A}_{\vec{v}_{A_2}}$$

$$v_{A_1} = \omega_p r_A \sin\left(\frac{\pi}{2} - \alpha\right) = \omega_p (r_A \cos\alpha)$$

$$= L\,\omega_p$$

$$v_{A_2} = \omega_s (r_A \sin\alpha) = R\,\omega_s$$



$$\vec a_A = \underbrace{\vec\omega \times \vec v_A}_{\vec a_\omega} + \underbrace{\vec\varepsilon \times \vec r_A}_{\vec a_\varepsilon}$$

$$\vec a_\omega = \underbrace{\vec\omega_p \times \vec v_{A_1}}_{\vec a_{A_1}} + \underbrace{\vec\omega_p \times \vec v_{A_2}}_{\vec a_{A_2}} + \underbrace{\vec\omega_s \times \vec v_{A_1}}_{\vec a_{A_3}} + \underbrace{\vec\omega_s \times \vec v_{A_2}}_{\vec a_{A_4}}$$

$$a_{A_1} = \omega_p v_{A_1} \sin\frac{\pi}{2} = \omega_p v_{A_1} =$$

$$a_{A_2} = \omega_p v_{A_2} \sin\frac{\pi}{2} = \omega_p v_{A_2} =$$

$$a_{A_3} = \omega_s v_{A_1} \sin\frac{\pi}{2} = \omega_s v_{A_1} =$$

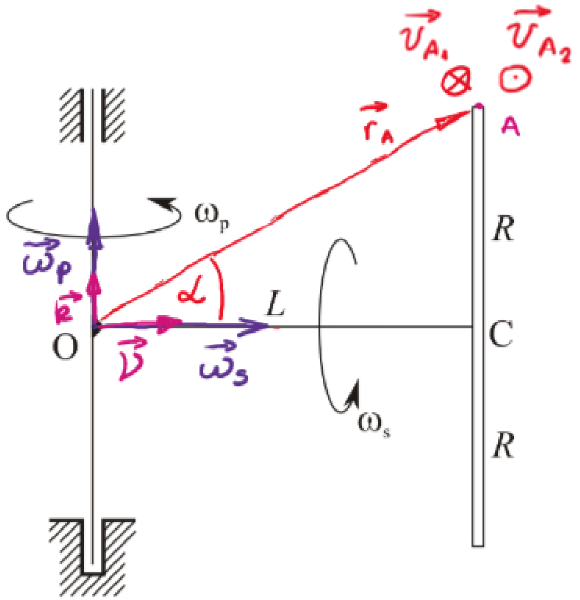
$$a_{A_4} = \omega_s v_{A_2} \sin\frac{\pi}{2} = \omega_s v_{A_2} =$$

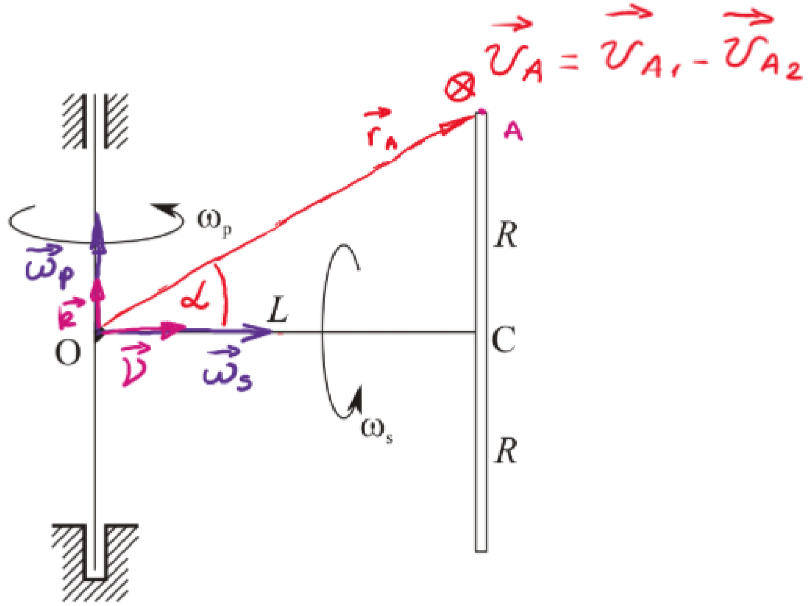
$$\vec a_\varepsilon = \underbrace{\vec\varepsilon_1 \times \vec r_A}_{\vec a_{A5}} + \underbrace{\vec\varepsilon_2 \times \vec r_A}_{\vec a_{A6}} + \underbrace{\vec\varepsilon_3 \times \vec r_A}_{\vec a_{A7}}$$

$$a_{A5} = \varepsilon_1 r_A \sin\left(\frac{\pi}{2} - \alpha\right) = \varepsilon_1 (r_A \cos\alpha) = L\,\varepsilon_1 =$$

$$a_{A6} = \varepsilon_2 (r_A \sin\alpha) = R\,\varepsilon_2 =$$

$$a_{A7} = \varepsilon_3 r_A \sin\frac{\pi}{2} = r_A \varepsilon_3 =$$



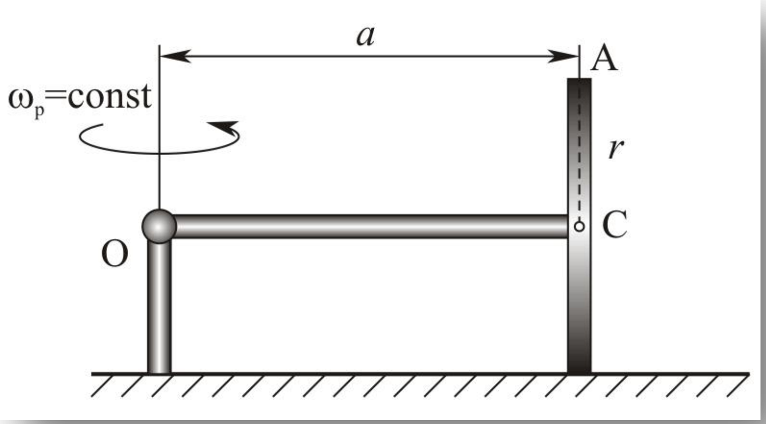


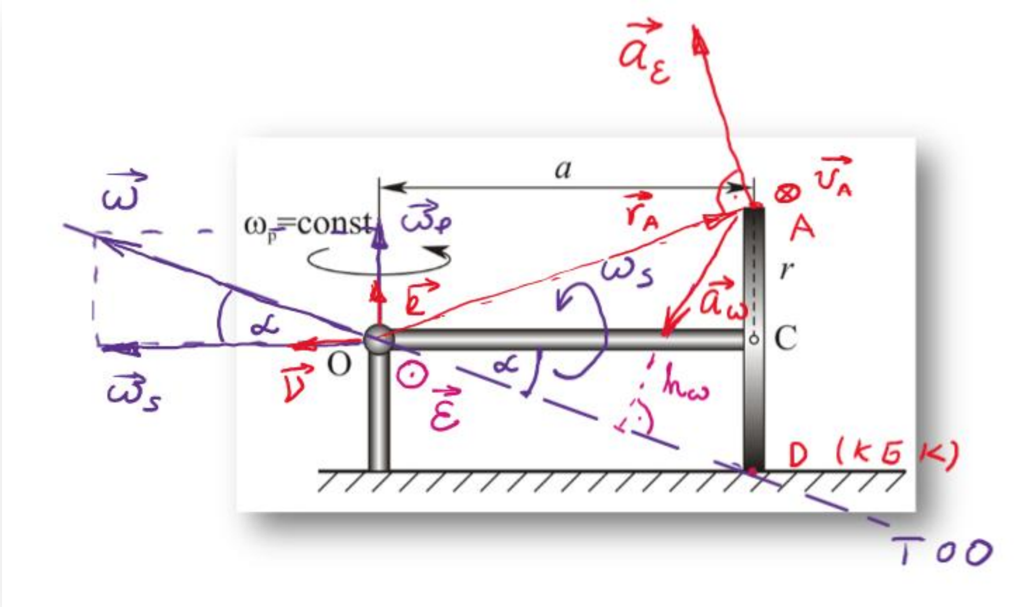
$$v_A = v_{A1} - v_{A2} = L\omega_p - R\omega_s$$

$$\vec{a}_\omega = \vec{\omega}_p \times \vec{v}_A + \vec{\omega}_s \times \vec{v}_A$$

Primer 4

Disk, poluprečnika *r*, kruto je vezan za štap OC, dužine *a*, koji je sfernim zglobom O vezan za podlogu. Disk se kotrlja bez klizanja po nepokretnoj horizontalnoj podlozi. Štap OC se oko vertikalne ose obrće konstantnom ugaonom brzinom ω_p=const [s^{-1}]. . Odrediti ugaonu brzinu i ugaono ubrzanje diska, kao i brzinu i ubrzanje tačke A na obodu diska u trenutku kada se nalazi u najvišem položaju.





$$\vec{v}_A = \vec{\omega} \times \vec{r}_A$$

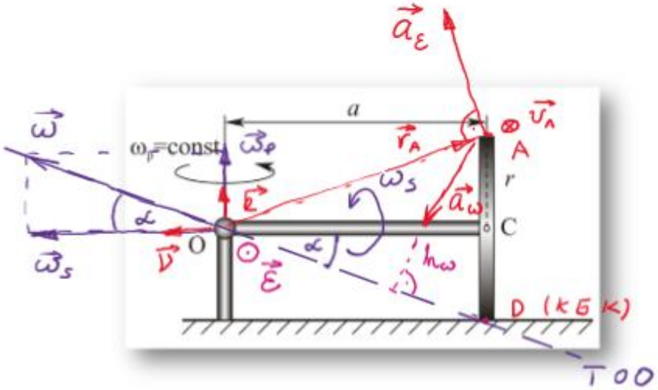
$$v_A = \omega\, r_A \sin(\pi - 2\alpha)$$
$$= \omega\, r_A \sin(2\alpha)$$
$$= \omega \frac{r}{\cancel{\sin\alpha}} \cdot 2 \cancel{\sin\alpha} \cos\alpha$$
$$= \omega \cdot (2r\cos\alpha) = \omega \cdot h_\omega$$
$$= \frac{\omega_p}{\sin\alpha} \cdot 2r\cos\alpha$$
$$= \omega_p \cdot 2r\, ctg\,\alpha = \omega_p \cdot 2r \frac{a}{r}$$
$$v_A = 2a\,\omega_p$$

$$\vec{a}_A = \dot{\vec{v}}_A = \dot{\vec{\omega}} \times \vec{r}_A + \vec{\omega} \times \dot{\vec{r}}_A$$

$$\vec{a}_A = \underbrace{\vec{\varepsilon} \times \vec{r}_A}_{\vec{a}_\varepsilon} + \underbrace{\vec{\omega} \times \vec{v}_A}_{\vec{a}_\omega}$$

$$\vec{a}_\omega = \vec{\omega} \times \vec{v}_A \;; \quad a_\omega = \omega\, v_A \sin\frac{\pi}{2} = \omega\, v_A = \dots$$

$$\vec{a}_\varepsilon = \vec{\varepsilon} \times \vec{r}_A \;; \quad a_\varepsilon = \varepsilon\, r_A \sin\frac{\pi}{2} = \varepsilon\, r_A = \dots$$



$$\vec{\omega} = \vec{\omega}_p + \vec{\omega}_s$$

$$\sin\alpha = \frac{\omega_p}{\omega} \rightarrow \omega = \frac{\omega_p}{\sin\alpha} = const$$

$$tg\,\alpha = \frac{\omega_p}{\omega_s} \rightarrow \omega_s = \omega_p\, ctg\alpha = const$$

$$ctg\alpha = \frac{a}{r}\,, \quad \sin\alpha = \frac{r}{\sqrt{a^2+r^2}}$$

$$\vec{\omega} = \omega_p \vec{k} + \omega_s \vec{\nu}$$

$$\vec{\varepsilon} = \dot{\vec{\omega}} = \cancel{\dot{\omega}_p}^{0} \vec{k} + \omega_p \cancel{\dot{\vec{k}}}^{0} + \cancel{\dot{\omega}_s}^{0} \vec{\nu} + \omega_s \dot{\vec{\nu}}$$

$$\vec{\varepsilon} = \omega_s \dot{\vec{\nu}} = \omega_s (\vec{\omega}_p \times \vec{\nu}) = \vec{\omega}_p \times \omega_s \vec{\nu} = \vec{\omega}_p \times \vec{\omega}_s$$

$$\varepsilon = \omega_p \omega_s \cancel{\sin\frac{\pi}{2}}^{1} = \omega_p \omega_s = \omega_p^2\, ctg\alpha$$

$$\vec{v}_A = \vec{\omega} \times \vec{r}_A$$

$$\begin{aligned}
v_A &= \omega r_A \sin(\pi - 2\alpha) \\
&= \omega r_A \sin(2\alpha) \\
&= \omega \frac{r}{\cancel{\sin\alpha}} \cdot 2 \cancel{\sin\alpha} \cos\alpha \\
&= \omega \cdot (2r\cos\alpha) = \omega \cdot h_\omega \\
&= \frac{\omega_p}{\sin\alpha} \cdot 2r\cos\alpha \\
&= \omega_p \cdot 2r\, ctg\alpha = \omega_p \cdot 2r \frac{a}{r}
\end{aligned}$$

$$v_A = 2a\omega_p$$

$$\vec{a}_A = \dot{\vec{v}}_A = \dot{\vec{\omega}} \times \vec{r}_A + \vec{\omega} \times \dot{\vec{r}}_A$$

$$\vec{a}_A = \underbrace{\vec{\varepsilon} \times \vec{r}_A}_{\vec{a}_\varepsilon} + \underbrace{\vec{\omega} \times \vec{v}_A}_{\vec{a}_\omega}$$

$$\vec{a}_\omega = \vec{\omega} \times \vec{v}_A \;; \quad a_\omega = \omega v_A \sin\frac{\pi}{2} = \omega v_A = \ldots$$

$$\vec{a}_\varepsilon = \vec{\varepsilon} \times \vec{r}_A \;; \quad a_\varepsilon = \varepsilon r_A \sin\frac{\pi}{2} = \varepsilon r_A = \ldots$$

Šta smo naučili?

17. **Sferno kretanje krutog tela (broj stepeni slobode kretanja, Ojler - D'Alamberova teorema)**
18. **Sferno kretanje - Ojlerovi uglovi**
19. **Ugaona brzina i ugaono ubrzanje tela pri sfernom kretanju**
20. **Projekcije ugaone brzine sfernog kretanja na koordinatne ose (Ojlerove kinematičke jednačine)**
21. **Projekcije ugaonog ubrzanja sfernog kretanja na koordinatne ose**
22. **Brzine i ubrzanja tačaka tela pri sfernom kretanju**

Mehanika 2 (Kinematika)
Predavanja 5

Miodrag Zuković

Novi Sad, 2023.