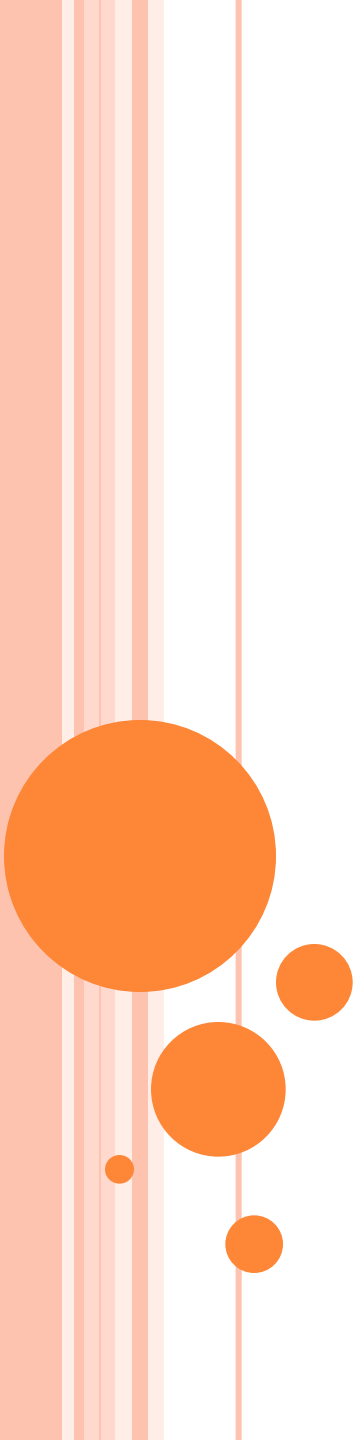




# **KINEMATIKA**

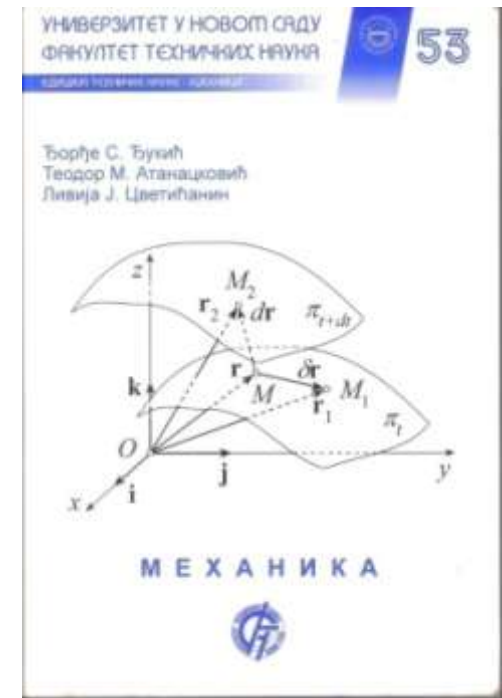
## **KINEMATIKA KRUTOG TELA**

**Mehanika**

**Miodrag Zuković**

# LITERATURA

- Đorđe S. Đukić, Teodor M. Atanacković, Livija J. Cvetićanin: **Mehanika**, Fakultet tehničkih nauka u Novom Sadu, Novi Sad, 2003.



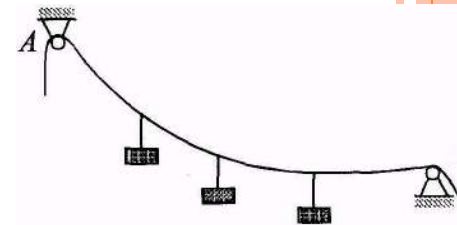
# ŠTE ĆEMO NAUČITI?

- Translatorno kretanje tela,
- Obrtanje tela oko nepokretne ose,
- Ravansko kretanje krutog tela.



# TRANSLATORNO KRETANJE KRUTOG TELA

- Kretanje tela je translatorno, ako svaki zamišljeni pravac u telu za vreme kretanja ostaje sam sebi paralelan,
- Pri translatorsnom kretanju tela putanje, brzine i ubrzanja svih tačaka tela su u bilo kom trenutku vremena iste. Kretanje ima tri stepena slobode.



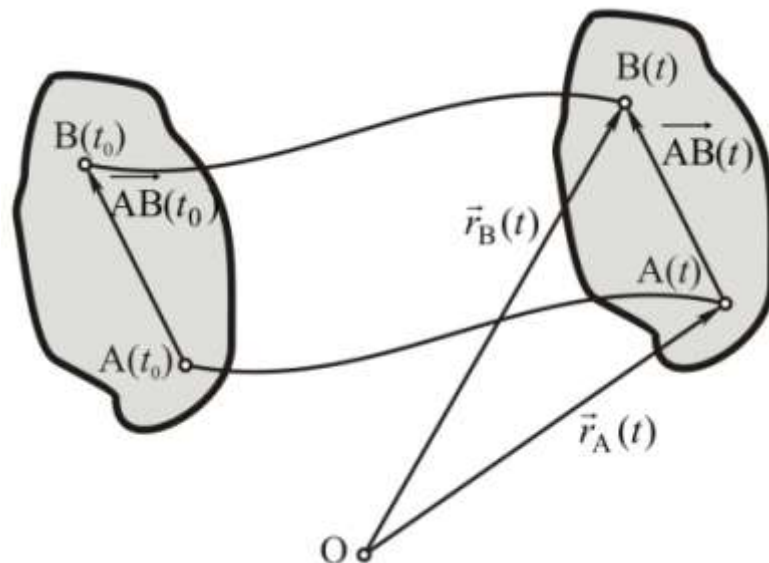
$$\overrightarrow{AB}(t) = \overrightarrow{AB}(t_0)$$

$$\dot{\overrightarrow{AB}}(t) = 0$$

$$\vec{r}_B(t) = \vec{r}_A(t) + \overrightarrow{AB}(t)$$

$$\dot{\vec{r}}_B(t) = \dot{\vec{r}}_A(t) \rightarrow \vec{v}_B(t) = \vec{v}_A(t)$$

$$\ddot{\vec{r}}_B(t) = \ddot{\vec{r}}_A(t) \rightarrow \vec{a}_B(t) = \vec{a}_A(t)$$



# OBRTANJE TELA OKO NEPOKRETNE OSE

- Kretanje tela je pri kome se tačke tela kreću po kružnicama čiji se centar nalazi na nepokretnoj osi - osa obrtanja.
- ugao obrtanja (parametarska jednačina kretanja)

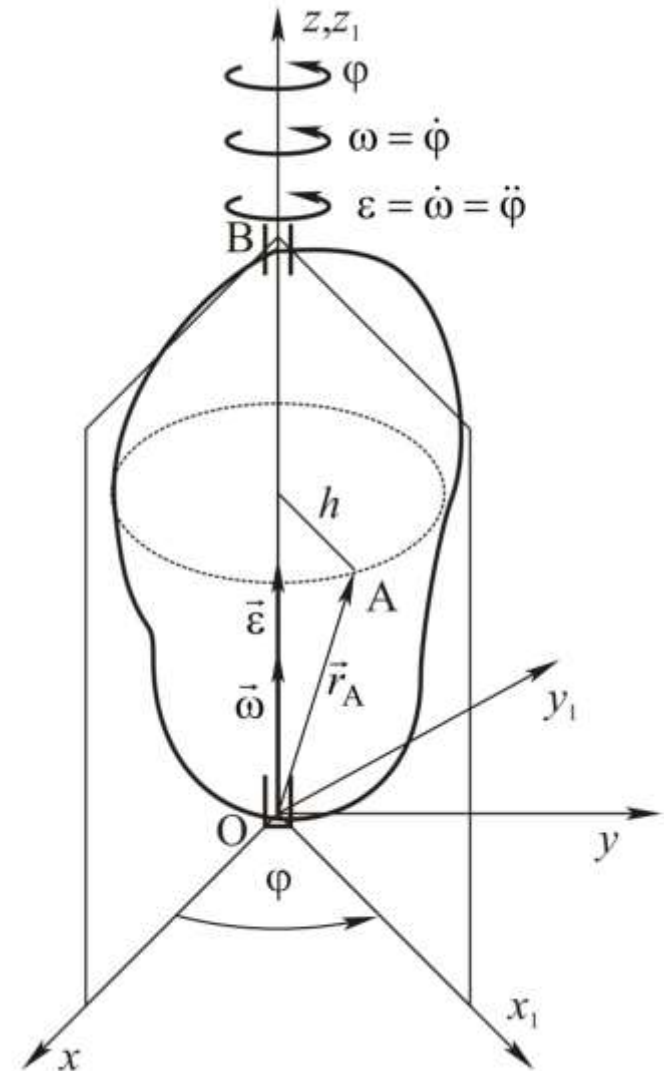
$$\varphi = \varphi(t)$$

- ugaona brzina

$$\vec{\omega}(t) = \omega(t)\vec{k}, \omega(t) = \dot{\varphi}(t)$$

- ugaono ubrzanje

$$\vec{\varepsilon}(t) = \dot{\vec{\omega}} = \varepsilon(t)\vec{k}, \varepsilon(t) = \dot{\omega}(t) = \ddot{\varphi}(t)$$



# OBRTANJE TELA OKO NEPOKRETNE OSE

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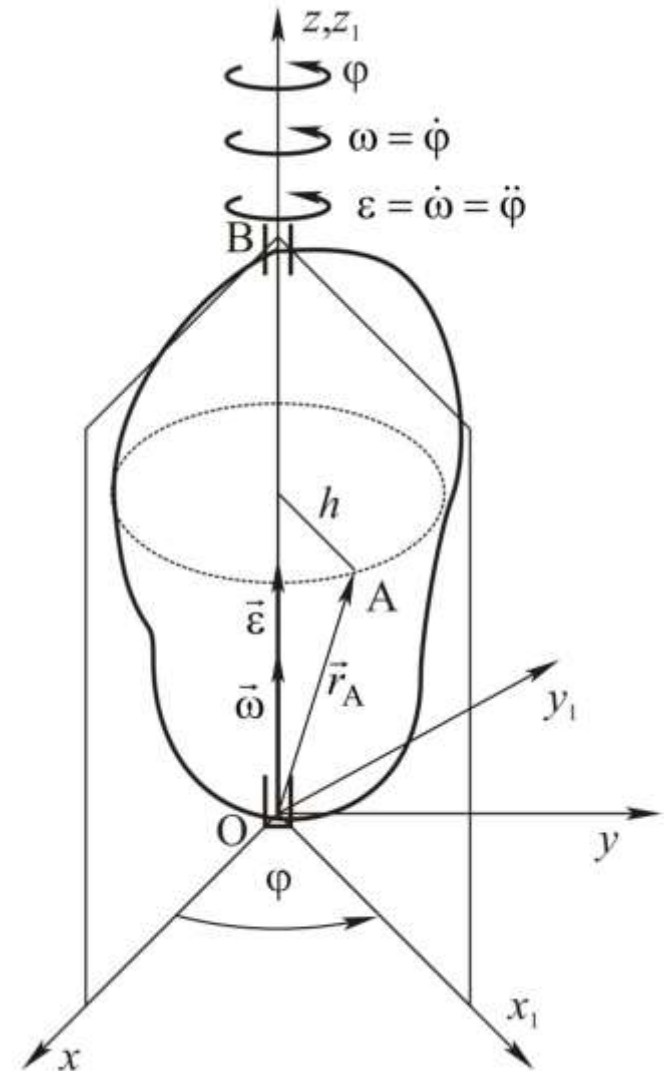
$$\varphi = \varphi(t)$$

- ugaona brzina

$$\vec{\omega}(t) = \omega(t)\vec{k}, \omega(t) = \dot{\varphi}(t)$$

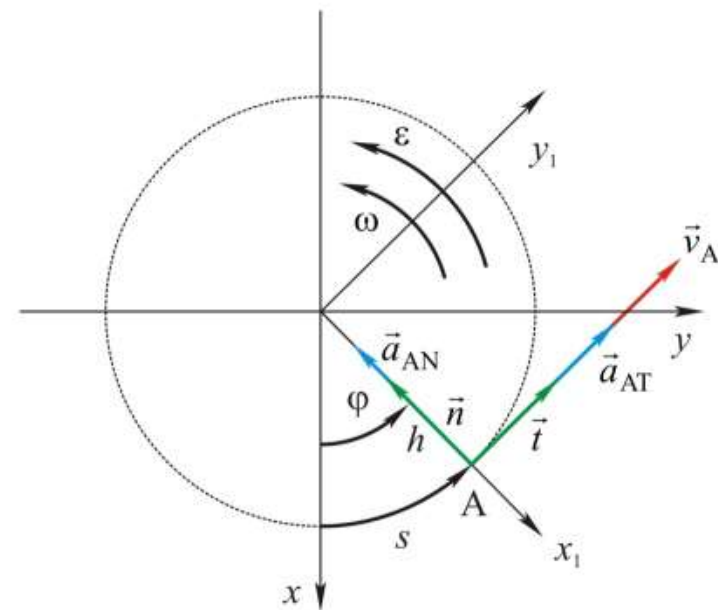
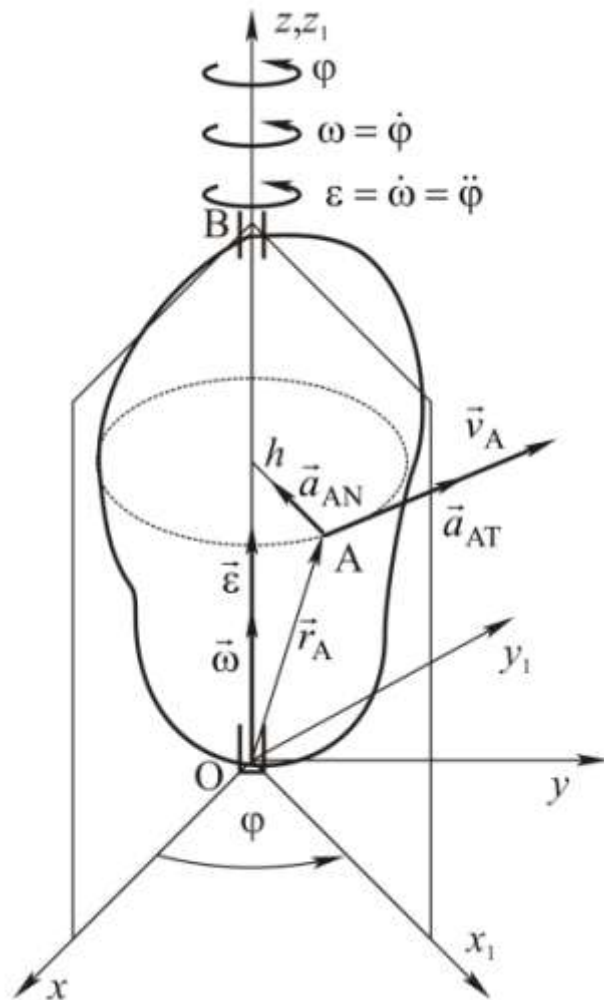
- ugaono ubrzanje

$$\vec{\varepsilon}(t) = \dot{\vec{\omega}} = \varepsilon(t)\vec{k}, \varepsilon(t) = \dot{\omega}(t) = \ddot{\varphi}(t)$$



# OBRTANJE TELA OKO NEPOKRETNE OSE

- Brzina i ubrzanje tačke tela koje se obrće oko nepokretne ose



# OBRTANJE TELA OKO NEPOKRETNE OSE

## Brzina tačke

$$\vec{v}(t) = \dot{s}(t)\vec{t}$$

$$\vec{v}(t) = h\dot{\phi}(t)\vec{t} = h\omega(t)\vec{t}$$

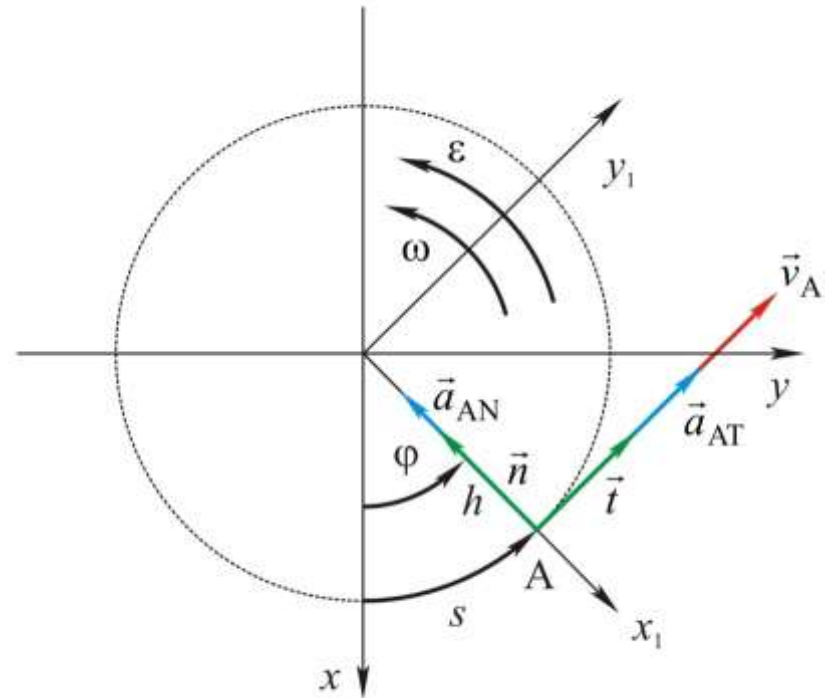
## Ubrzanje tačke

$$\vec{a}(t) = a_T(t)\vec{t} + a_N(t)\vec{n}$$

$$= \ddot{s}(t)\vec{t} + \frac{v^2(t)}{h}\vec{n}$$

$$= h\ddot{\phi}(t)\vec{t} + h\dot{\phi}^2(t)\vec{n}$$

$$= h\varepsilon(t)\vec{t} + h\omega^2(t)\vec{n}$$



$$\vec{v} = \vec{\omega} \times \vec{r}$$

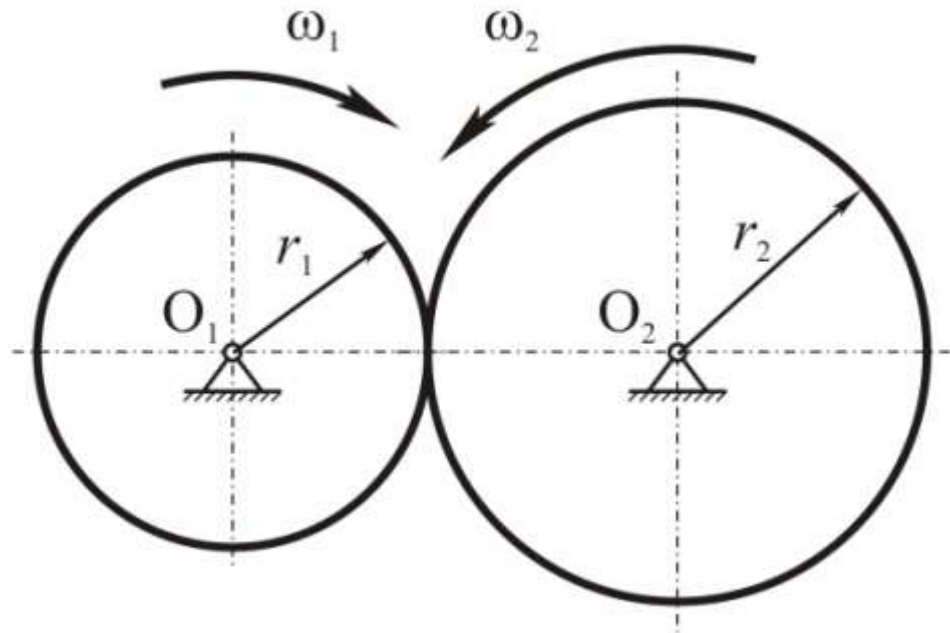
$$\vec{a} = \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times \vec{v}$$



# OBRTANJE TELA OKO NEPOKRETNE OSE

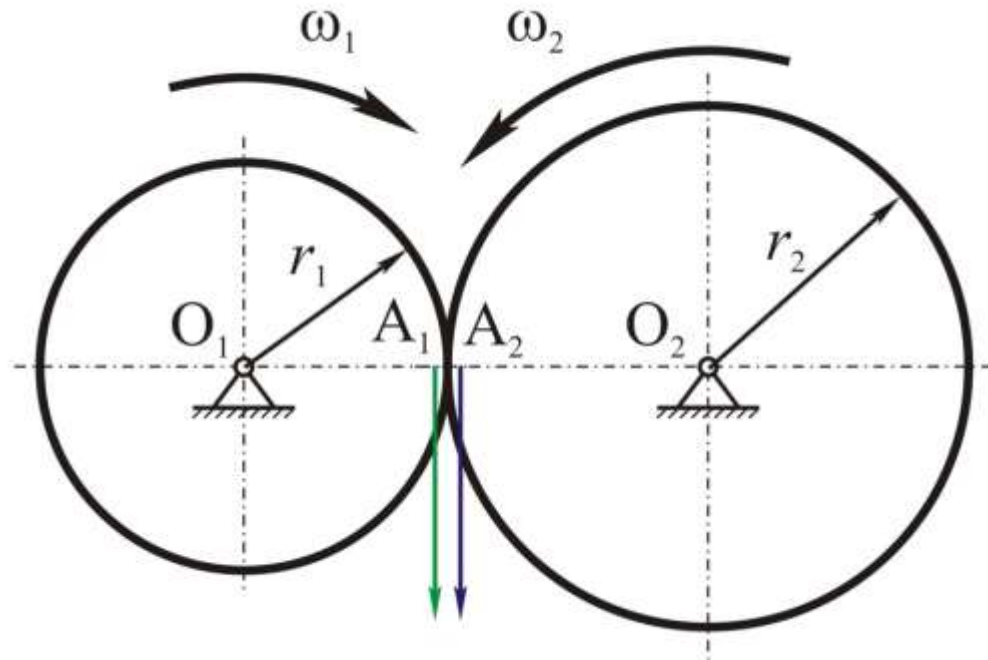
## ○ Primer

Ugaona brzina pogonskog zupčanika, poluprečnika  $r_1$ , iznosi  $\omega_1$ . Odrediti ugaonu brzinu  $\omega_2$  gonjenog zupčanika, poluprečnika  $r_2$ .



# OBRTANJE TELA OKO NEPOKRETNE OSE

## ○ Primer-rešenje

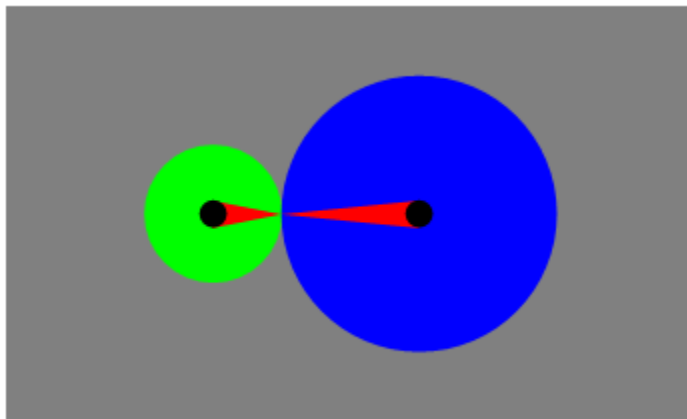


$$v_{A_1} = v_{A_2} \rightarrow r_1 \omega_1 = r_2 \omega_2 \rightarrow \omega_2 = \frac{r_1}{r_2} \omega_1$$



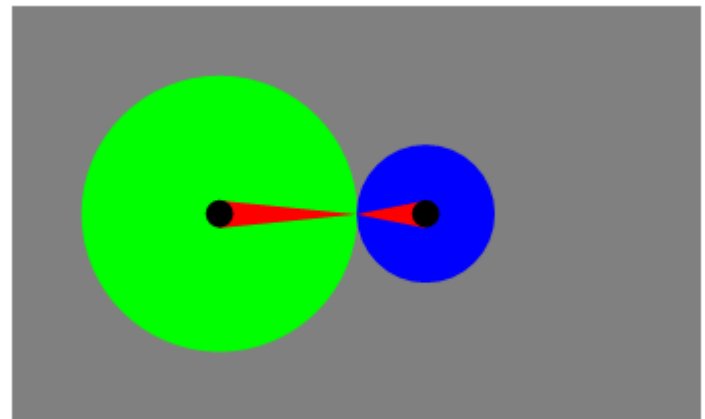
# OBRTANJE TELA OKO NEPOKRETNE OSE

## ○ Primer-rešenje



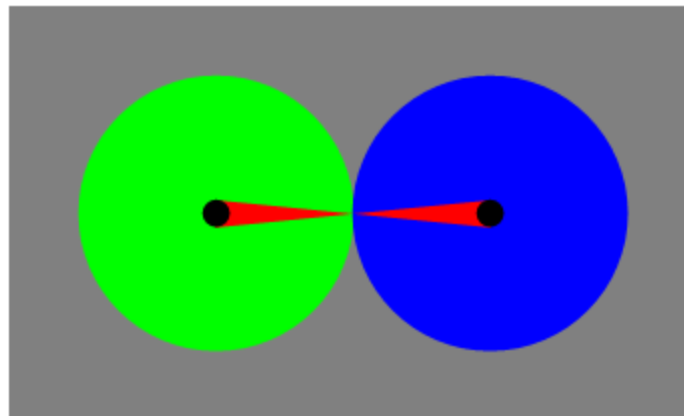
$$r_2 = 2r_1$$

$$\omega_2 = \frac{1}{2} \omega_1$$



$$r_2 = \frac{r_1}{2}$$

$$\omega_2 = 2\omega_1$$



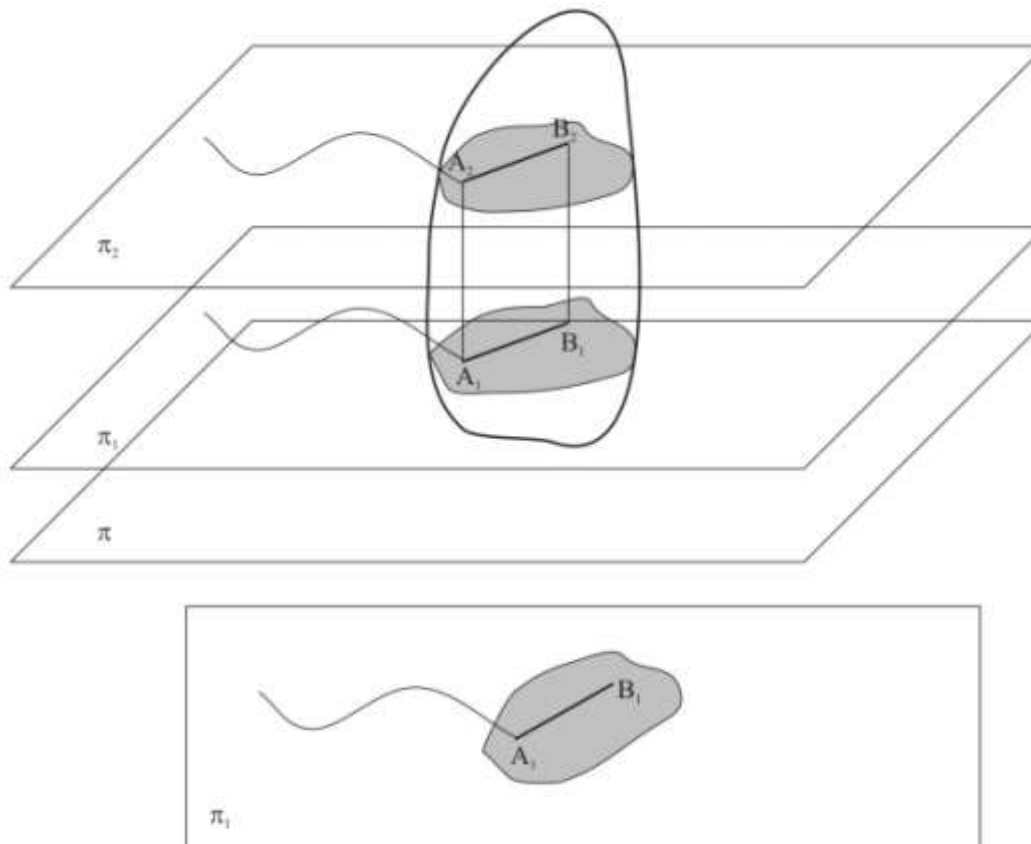
$$r_2 = r_1$$

$$\omega_2 = \omega_1$$



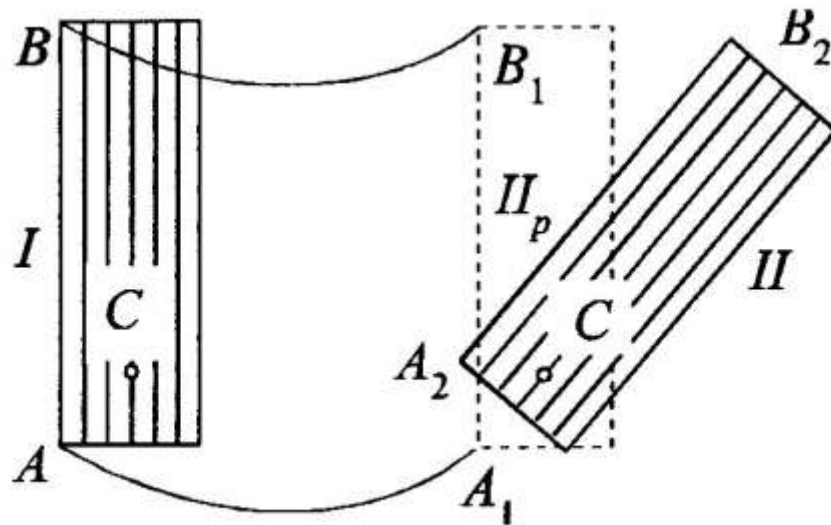
# RAVANSKO KRETANJE KRUTOG TELA

- Pri ravanskom kretanju sve tačke tela kreću se u ravnima koje su paralelne nekoj nepokretnoj ravni ( $\pi$ ).



# RAVANSKO KRETANJE KRUTOG TELA

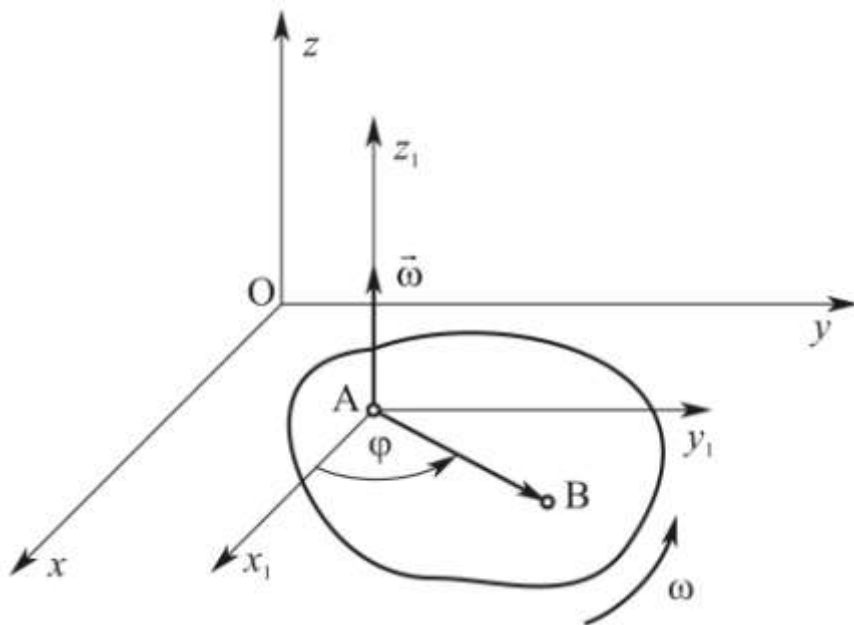
- Ravansko kretanje ima tri stepena slobode.
- Pri ravanskom kretanju tela ono se može dovesti iz jednog položaja u drugi jednom translacijom (dva stepena slobode) i jednim obrtanjem oko ose (jedan stepen slobode).



# RAVANSKO KRETANJE KRUTOG TELA

- Parametarske jednačine ravnanskog kretanja tela

$$x_A = x_A(t), y_A = y_A(t), \varphi = \varphi(t)$$



- Ugaona brzina i ugaono ubrzanje

$$\vec{\omega} = \omega \vec{k}, \quad \omega = \dot{\varphi}$$

$$\vec{\varepsilon} = \dot{\vec{\omega}} = \varepsilon \vec{k}, \quad \varepsilon = \dot{\omega} = \ddot{\varphi}$$

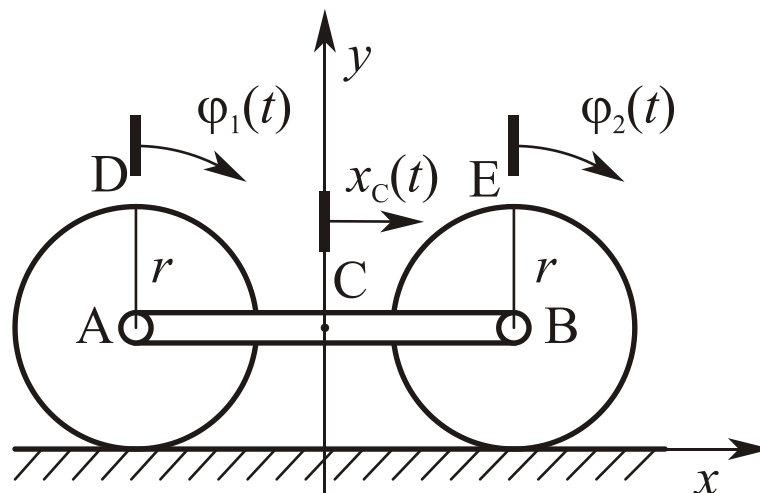


# RAVANSKO KRETANJE KRUTOG TELA

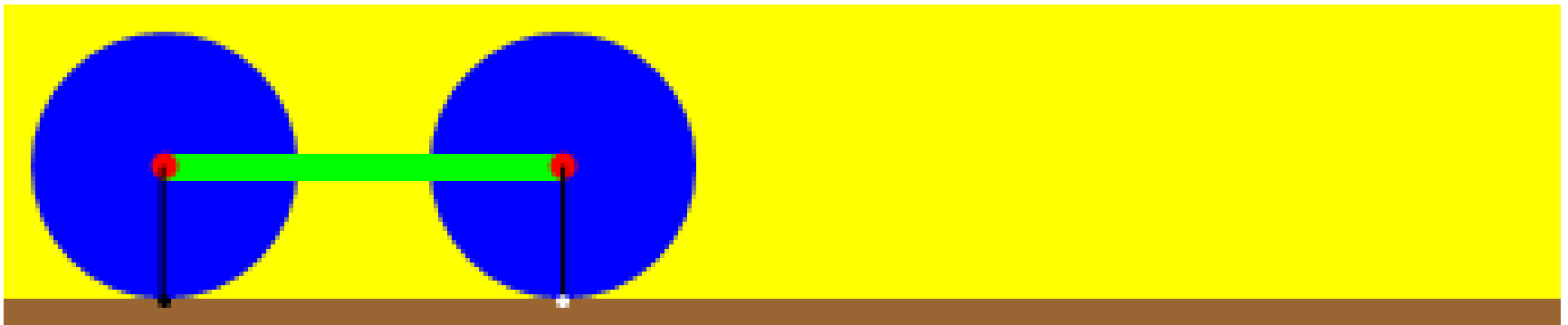
- Primer: Date su jednačine kretanja prikazanog sistema krutih tela:

$$x_c(t) = v_0 t, \quad \varphi_1(t) = k \frac{v_0}{r} t, \quad \varphi_2(t) = \frac{v_0}{r} t.$$

Odrediti trajektorije kretanja tačaka D i E na obodima diskova za  $k=1/2$  i  $k=2$ .



# RAVANSKO KRETANJE KRUTOG TELA



# RAVANSKO KRETANJE KRUTOG TELA

- Veza između brzina tačaka tela pri ravanskom kretanju

$$\vec{r}_B = \vec{r}_A + \vec{\rho}$$

$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{\rho}}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$$

$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{\rho}}$$

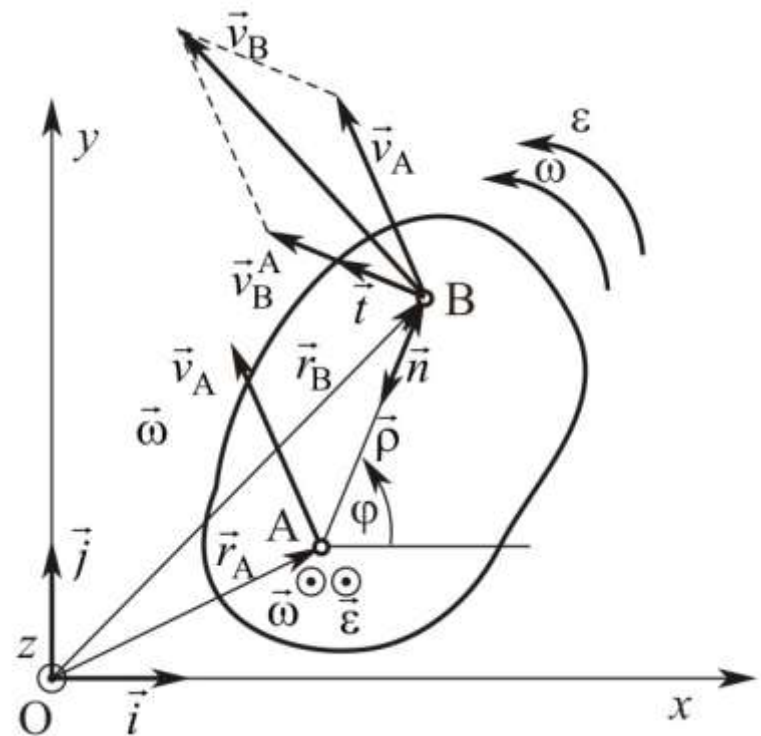
$$\vec{a}_B = \vec{a}_A + \vec{a}_B^A$$

$$\vec{\rho} = \overline{AB} \cos \varphi \vec{i} + \overline{AB} \sin \varphi \vec{j} + 0 \vec{k}$$

$$\vec{v}_B^A = \dot{\vec{\rho}} = -\overline{AB} \dot{\varphi} \sin \varphi \vec{i} + \overline{AB} \dot{\varphi} \cos \varphi \vec{j} = \overline{AB} \dot{\varphi} \vec{t}$$

$$\vec{a}_B^A = \ddot{\vec{\rho}} = -\overline{AB} \ddot{\varphi} \sin \varphi \vec{i} - \overline{AB} \dot{\varphi}^2 \cos \varphi \vec{i} + \overline{AB} \ddot{\varphi} \cos \varphi \vec{j} - \overline{AB} \dot{\varphi}^2 \sin \varphi \vec{j}$$

$$\vec{a}_B^A = \overline{AB} \ddot{\varphi} \vec{t} + \overline{AB} \dot{\varphi}^2 \vec{n} = \vec{a}_{BT}^A + \vec{a}_{BN}^A$$

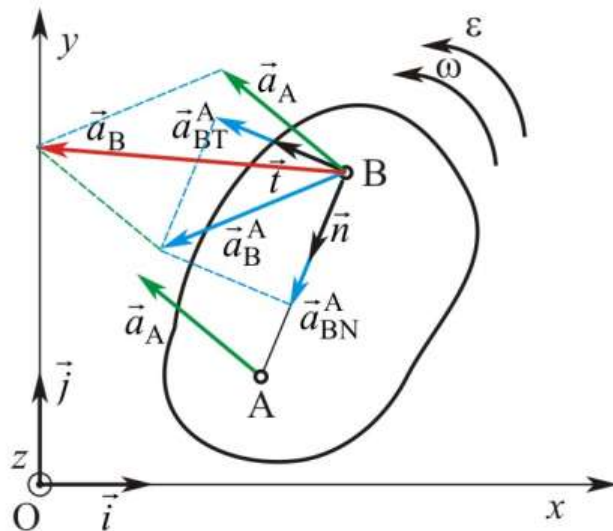


# RAVANSKO KRETANJE KRUTOG TELA

○ ...

$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BT}^A + \vec{a}_{BN}^A$$



$$\vec{v}_B^A = \vec{\omega} \times \vec{\rho} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \dot{\phi} \\ \overline{AB} \cos \varphi & \overline{AB} \sin \varphi & 0 \end{vmatrix}$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{\rho}$$

$$\vec{a}_B = \vec{a}_A + \dot{\vec{\omega}} \times \vec{\rho} + \vec{\omega} \times \dot{\vec{\rho}}$$

$$\vec{a}_{BT}^A = \vec{\varepsilon} \times \vec{\rho} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \ddot{\phi} \\ \overline{AB} \cos \varphi & \overline{AB} \sin \varphi & 0 \end{vmatrix}$$

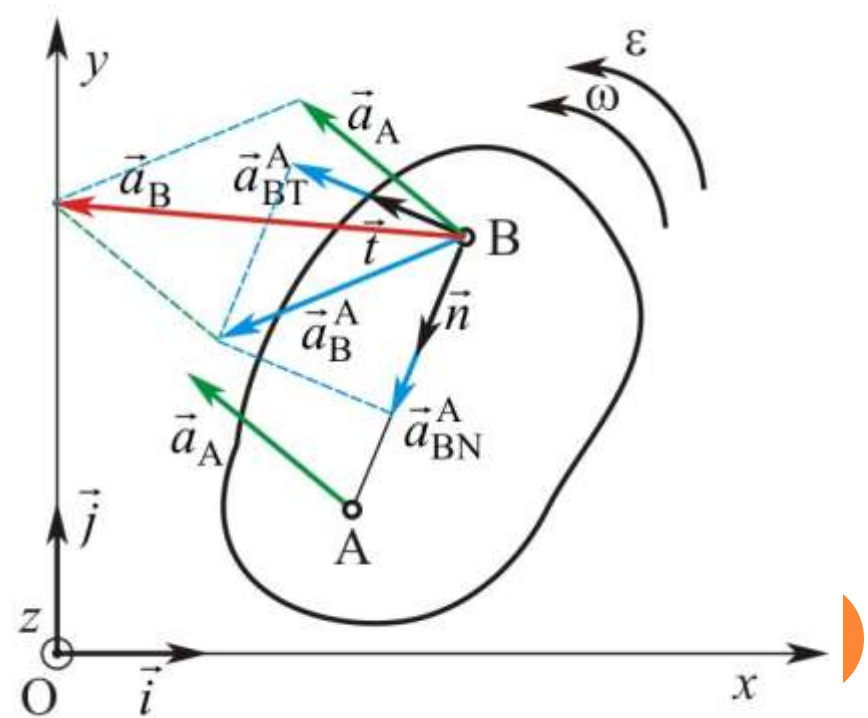
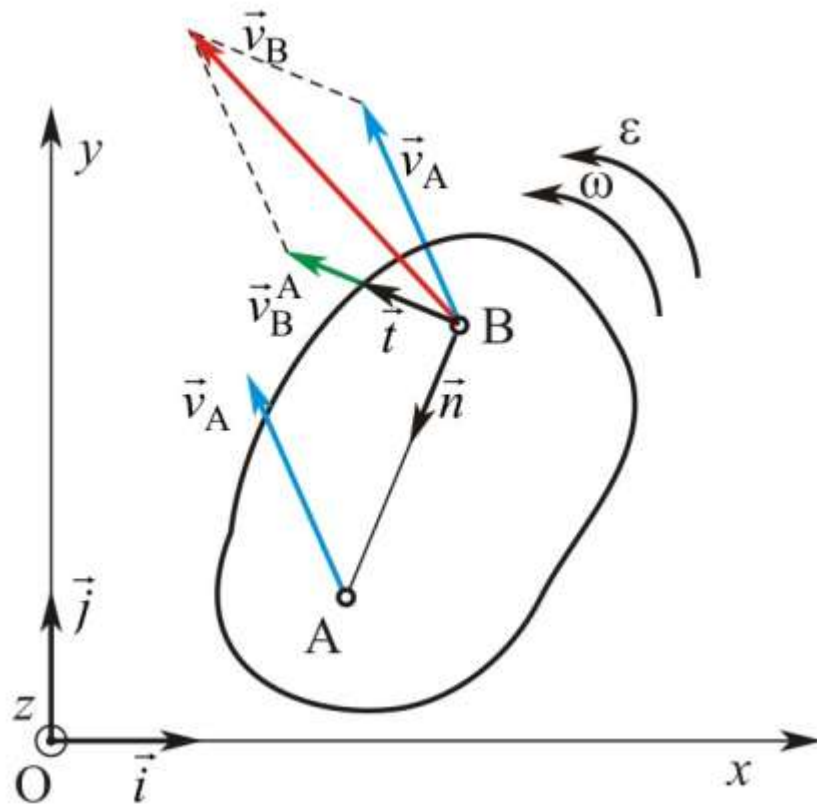
$$\vec{a}_{BN}^A = \vec{\omega} \times \vec{v}_B^A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \dot{\phi} \\ -\overline{AB} \dot{\phi} \sin \varphi & \overline{AB} \dot{\phi} \cos \varphi & 0 \end{vmatrix}$$

# RAVANSKO KRETANJE KRUTOG TELA

○ ...

$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$$

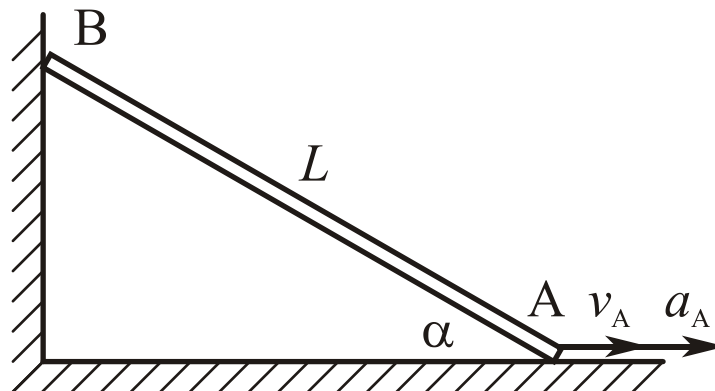
$$\vec{a}_B = \vec{a}_A + \vec{a}_{BT}^A + \vec{a}_{BN}^A$$



# RAVANSKO KRETANJE KRUTOG TELA

## ○ Primer:

Štap AB, dužine  $L$ , oslanja se krajem A o horizontalni pod, a krajem B o vertikalni zid. U položaju u kome štap gradi ugao  $\alpha$  sa horizontalom brzina i ubrzanje tačke A iznose  $v_A$  i  $a_A$ . Odrediti brzinu i ubrzanje tačke B i ugaonu brzinu i ugaono ubrzanje štapa u datom položaju.



## ŠTE SMO NAUČILI?

- Translatorno kretanje tela,
- Obrtanje tela oko nepokretne ose,
- Ravansko kretanje krutog tela.

