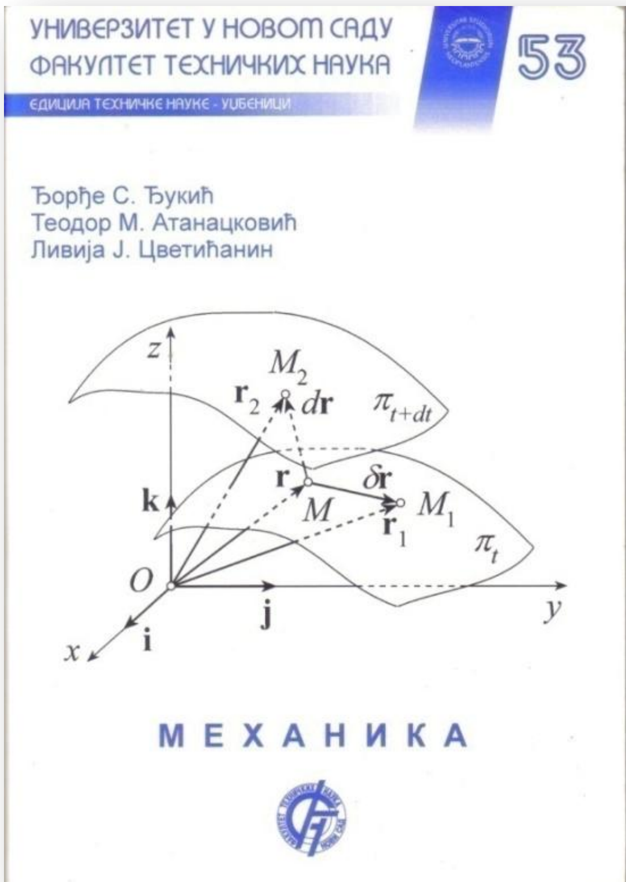
Kinematika – vežbe 4

Kinematika i dinamika

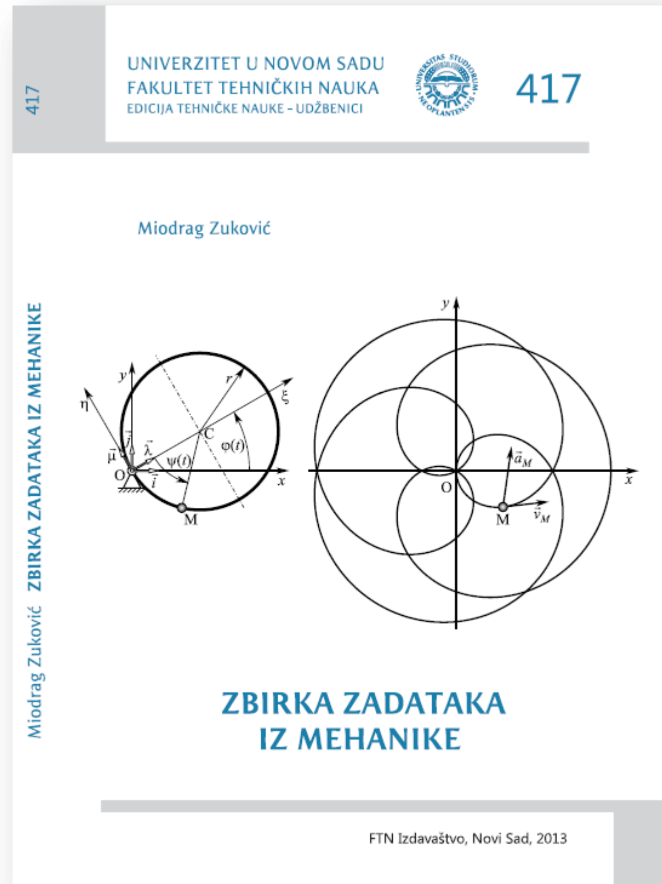
Miodrag Zuković

Novi Sad, 2021.

Literatura



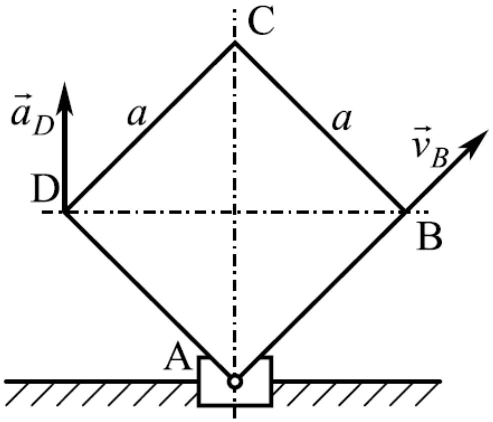
УНИВЕРЗИТЕТ У НОВОМ САДУ
ФАКУЛТЕТ ТЕХНИЧКИХ НАУКА
ЕДИЦИЈА ТЕХНИЧКЕ НАУКЕ - УЏБЕНИЦИ
53
Ђорђе С. Ђукић
Теодор М. Атанацковић
Ливија Ј. Цветићанин
МЕХАНИКА



417
UNIVERZITET U NOVOM SADU
FAKULTET TEHNIČKIH NAUKA
EDICIJA TEHNIČKE NAUKE – UDŽBENICI
417
Miodrag Zuković ZBIRKA ZADATAKA IZ MEHANIKE
Miodrag Zuković
ZBIRKA ZADATAKA
IZ MEHANIKE
FTN Izdavaštvo, Novi Sad, 2013

Zadatak 1

Zadatak 2.20 Kvadratna ploča, stranica dužine $a = 1\text{m}$, vezana je zglobno, temenom A, za klizač koji se nalazi na pravolinijskoj vođici. U prikazanom položaju ploče poznati su brzina temena B, $v_B = 1\frac{\text{m}}{\text{s}}$, i ubrzanje temena D, $a_D = \frac{\sqrt{2}}{2}\frac{\text{m}}{s^2}$. Odrediti brzinu i ubrzanje temena C u datom položaju ploče.



Trajektorija temena C ploče je nepoznata. Da bi se odredila brzina tačke C mora se prvo odrediti ugaona brzina ploče.

Prvo će se uspostaviti veza između brzina tačaka A i B ploče:

$$\vec{v}_B = \vec{v}_A + \vec{v}_B^A \tag{a}$$

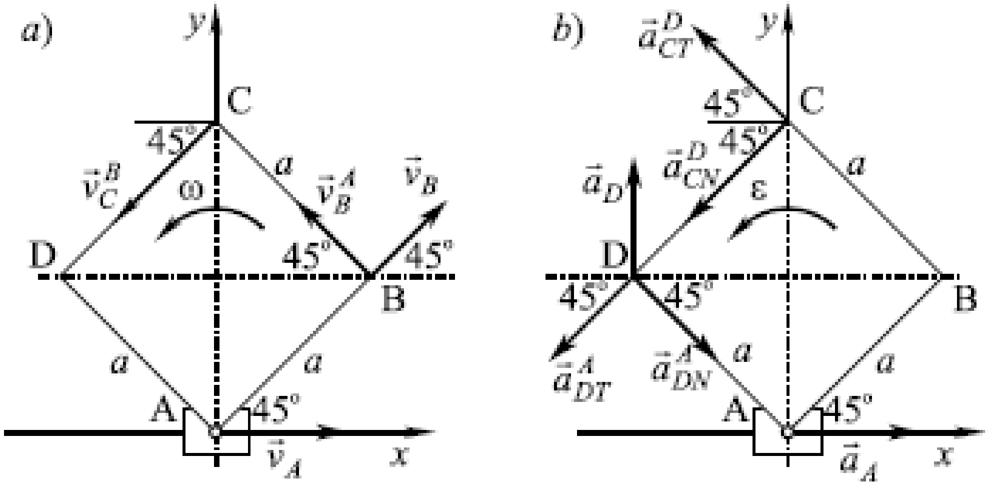
Brzina tačke B je poznata. Tačka A pripada i klizaču koji može da se kreće duž horizontalne vođice, pa je pravac brzine ove tačke horizontalan, Slika 2.26a. Smer je pretpostavljen. Brzina $\vec{v}_B^A$ je upravna na pravac AB, usmerena je u skladu sa pretpostavljenim smerom ugaone brzine ploče ω, a intenzitet joj je $v_B^A = \overline{AB}\omega = a\omega$.

Projektovanjem jednačine (a) na osu y dobija se jednačina

$$v_B \sin 45° = 0 + v_B^A \sin 45°$$

iz koje sledi $v_B^A = v_B$, a odatle i

$$\omega = \frac{v_B^A}{a} = \frac{v_B}{a} = 1[\mathrm{s}^{-1}]$$



Slika 2.26

Brzina tačke C sada može da se izrazi na sledeći način

$$\vec{v}_C = \vec{v}_B + \vec{v}_C^B \tag{b}$$

gde je brzina $\vec{v}_C^B$ upravna na BC i ima intenzitet $v_C^B = \overline{BC}\omega = a\omega = 1[\mathrm{m/s}]$.

Projektovanjem jednačine (b) na x i y osu dobijaju se skalarne jednačine

$$v_{Cx} = v_B \cos 45° - v_C^B \cos 45°$$

$$v_{Cy} = v_B \sin 45° - v_C^B \sin 45°$$

iz kojih sledi $v_{Cx} = 0$ i $v_{Cy} = 0$, pa je u datom položaju ploče

$$v_C = 0$$

Da bi se odredilo ubrzanje tačke C mora se prvo odrediti ugaono ubrzanje ploče.

Prvo će se uspostaviti veza između ubrzanja tačaka D i A ploče:

$$\vec{a}_D = \vec{a}_A + \vec{a}_{DT}^A + \vec{a}_{DN}^A \qquad \text{(c)}$$

Tačka D ima poznato ubrzanje. Ubrzanje tačke A ima horizontalan pravac i pretpostavljeni smer, Slika 2.26*b*. Ubrzanje $\vec{a}_{DT}^A$ je upravno na AD, usmereno u skladu sa pretpostavljenim smerom ugaonog ubrzanja ploče ε, a intenzitet mu je nepoznat, $a_{DT}^A = \overline{AD}\varepsilon = a\varepsilon$. Vektor ubrzanja $\vec{a}_{DN}^A$ usmeren je od tačke D ka tački A, a intenzitet mu je $a_{DN}^A = \overline{AD}\omega^2 = a\omega^2 = 1\frac{\mathrm{m}}{s^2}$.

Projektovanjem jednačine (c) na osu y, , dobija se jednačina

$$a_D = -a_{DT}^A \sin 45° - a_{DN}^A \sin 45°$$

iz koje sledi

$$a_{DT}^{A} = -\frac{1}{\sin 45°}(a_D + a_{DN}^{A} \sin 45°) = -2\left[\frac{\mathrm{m}}{s^2}\right]$$

$$\varepsilon = \frac{a_{DT}^{A}}{a} = -\frac{1}{a \sin 45°}(a_D + a_{DN}^{A} \sin 45°) = -2[\mathrm{s}^{-2}]$$

Ubrzanje tačke C može da se izrazi jednačinom

$$\vec{a}_C = \vec{a}_D + \vec{a}_{CT}^{D} + \vec{a}_{CN}^{D} \qquad \text{(d)}$$

Ubrzanje $\vec{a}_{CT}^{D}$ je upravno na CD i ima intenzitet $a_{CT}^{D} = \overline{CD}\varepsilon = a\varepsilon = -2\left[\frac{\mathrm{m}}{s^2}\right]$. Intenzitet ubrzanja $\vec{a}_{CN}^{D}$ je $a_{CN}^{D} = \overline{AD}\omega^2 = a\omega^2 = 1\left[\frac{\mathrm{m}}{s^2}\right]$, a usmereno je od tačke C ka tački D.

Projektovanjem jednačine (d) na *x* i *y* osu dobijaju se skalarne jednačine

$$a_{Cx} = -a_{CT}^{D} \cos 45° - a_{CN}^{D} \cos 45°$$

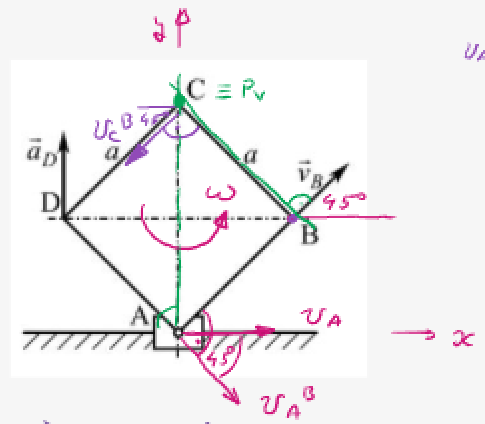
$$a_{Cy} = a_D + a_{CT}^{D} \sin 45° - a_{CN}^{D} \sin 45°$$

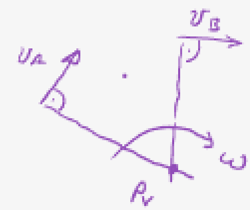
iz kojih sledi

$$a_{Cx} = \frac{\sqrt{2}}{2}\left[\frac{\mathrm{m}}{s^2}\right], \qquad a_{Cy} = -\sqrt{2}\left[\frac{\mathrm{m}}{s^2}\right]$$

Intenzitet ubrzanje tačke C u datom položaju jednak je

$$a_C = \sqrt{a_{Cx}^2 + a_{Cy}^2} = \frac{\sqrt{10}}{2}\left[\frac{\mathrm{m}}{s^2}\right]$$





$\vec{v}_C = \vec{v}_B + \vec{v}_C^{\,B}$

ПРЕКО А $\vec{v}_A = \vec{v}_B + \vec{v}_A^{\,B}$; $\vec{v}_A^{\,B} \perp \overline{AB}$

$v_A^B = \overline{AB}\,\omega = ?$

$x: \quad v_A = v_B \cos 45^\circ + v_A^B \cos 45^\circ \quad (1)$

$y: \quad 0 = v_B \cancel{\sin 45^\circ} - v_A^B \cancel{\sin 45^\circ} \quad (2)$

$\boxed{v_A^B = v_B = 1}$

$v_A^B = \overline{AB}\,\omega \longrightarrow \boxed{\omega = \frac{v_A^B}{\overline{AB}} = \frac{v_A^B}{a} = \frac{1}{1} = 1}\ s^{-1}$

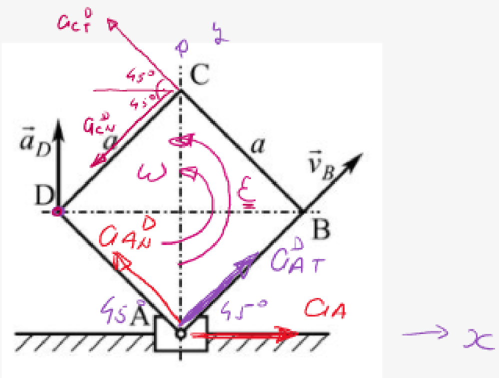
$$\vec{v}_C = \vec{v}_B + \vec{v}_C^{\,B} \quad ; \quad \vec{v}_C^{\,B} \perp \overline{BC}$$

$$v_C^B = \overline{BC}\,\omega = a\omega = 1 \cdot 1 = 1$$

$$x: \quad v_{Cx} = v_B \cos 45^\circ - v_C^B \cos 45^\circ$$

$$y: \quad v_{Cy} = v_B \sin 45^\circ - v_C^B \sin 45^\circ$$

$$\left.\begin{aligned} v_{Cx} &= 1 \cdot \frac{\sqrt{2}}{2} - 1 \cdot \frac{\sqrt{2}}{2} = 0 \\ v_{Cy} &= 1 \cdot \frac{\sqrt{2}}{2} - 1 \cdot \frac{\sqrt{2}}{2} = 0 \end{aligned}\right\} \quad v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = 0$$



ПРЕКО А

$$\vec{a}_A = \vec{a}_B + \vec{a}_A^B$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{AT}^B + \vec{a}_{AN}^B \, ; \qquad \vec{a}_{AN}^D \, , \; A \rightarrow D$$

$$a_{AN}^D = \overline{AD}\,\omega^2 = 1 \cdot 1^2 = 1$$

$$\vec{a}_{AT}^D \perp \overline{AD} \quad (\text{преко } \varepsilon)$$

$$a_{AT}^D = \overline{AD}\,\varepsilon = ?$$

x: (3) $a_A = 0 + a_{AT}^D \cos 45° - a_{AN}^D \cos 45°$

y: (4) $0 = a_D + a_{AT}^D \sin 45° + a_{AN}^D \sin 45°$

(4) $0 = \cancel{\frac{\sqrt{2}}{2}}^{1} + a_{AT}^D \cancel{\frac{\sqrt{2}}{2}} + 1 \cdot \cancel{\frac{\sqrt{2}}{2}}$

$a_{AT}^D = -2$ $\rightarrow$ $\varepsilon = \dfrac{a_{AT}^D}{\overline{AD}} = -2$ s^{-2}

$$\vec{a}_C = \vec{a}_D + \vec{a}_{CT}^{D} + \vec{a}_{CN}^{D};$$

$$a_{CT}^{D} = \overline{CD}\,\varepsilon = 1\cdot(-2) = -2$$

$$a_{CN}^{D} = \overline{CD}\,\omega^2 = 1\cdot 1^2 = 1$$

$$x: \quad a_{Cx} = 0 - a_{CT}^{D}\cos 45^\circ - a_{CN}^{D}\cos 45^\circ$$

$$y: \quad a_{Cy} = a_D + a_{CT}^{D}\sin 45^\circ - a_{CN}^{D}\sin 45^\circ$$

$$a_{Cx} = -(-2)\frac{\sqrt{2}}{2} - 1\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

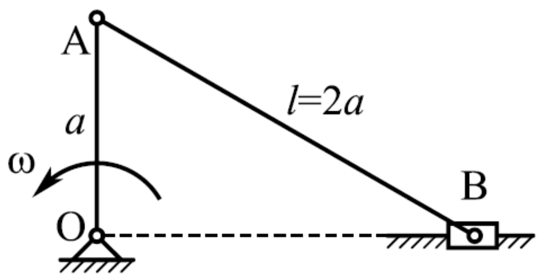
$$a_{Cy} = \cancel{\frac{\sqrt{2}}{2}} + (-2)\frac{\sqrt{2}}{2} \cancel{- 1\cdot\frac{\sqrt{2}}{2}} = -\sqrt{2}$$

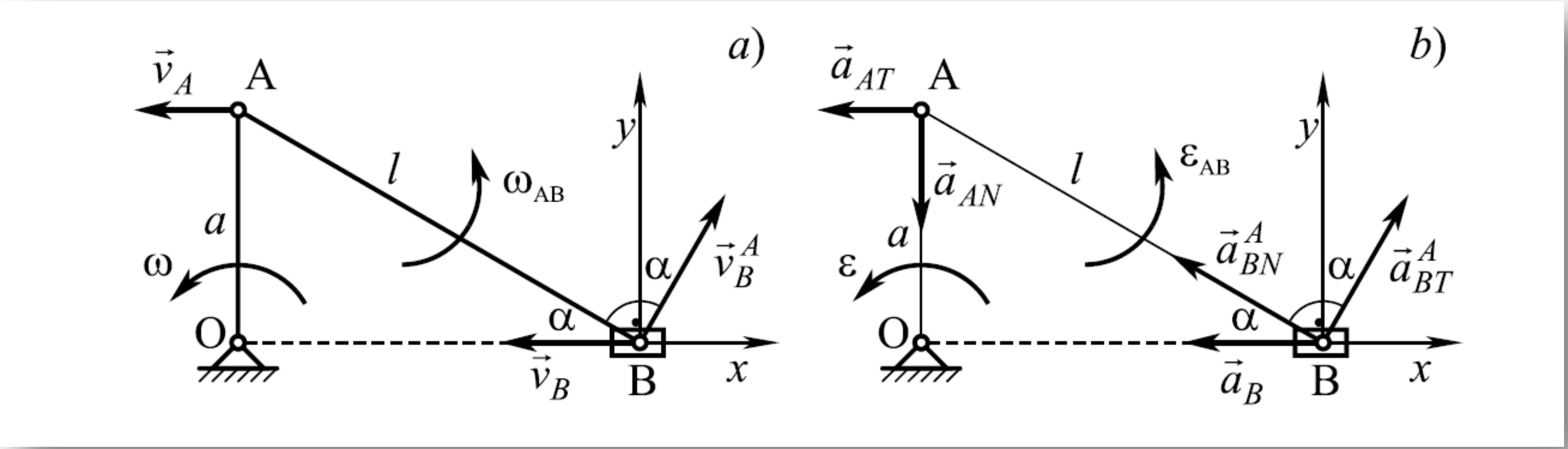
$$a_C = \sqrt{a_{Cx}^2 + a_{Cy}^2}$$

$$a_C = \sqrt{\frac{1}{2} + 2} = \sqrt{\frac{5}{2}}$$

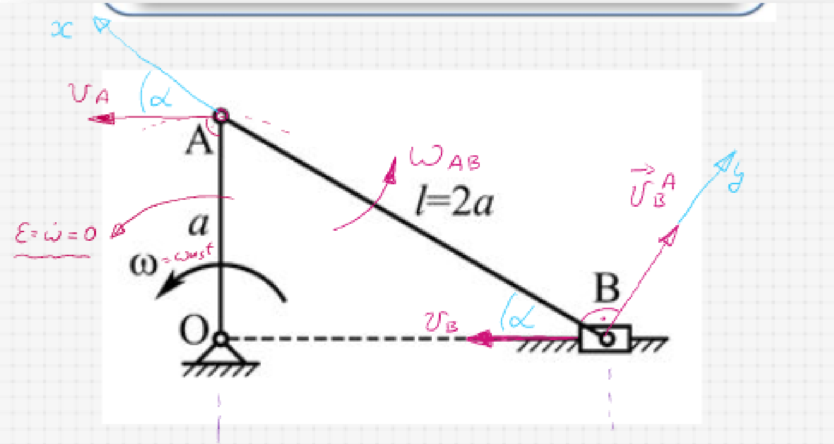
Zadatak 2

Zadatak 2.21 Krivaja OA, klipnog mehanizma, obrće se konstantnom ugaonom brzinom ω. Odrediti brzinu i ubrzanje klipa B i ugaonu brzinu i ugaono ubrzanje klipnjače AB u prikazanom položaju klipnog mehanizma ($\overline{OA} = a$, $\overline{AB} = l = 2a$).





a)
$\vec{v}_A$
A
l
ω_{AB}
y
a
ω
α
$\vec{v}_B^{\,A}$
α
O
$\vec{v}_B$
B
x
b)
$\vec{a}_{AT}$
A
$\vec{a}_{AN}$
l
ε_{AB}
y
a
ε
$\vec{a}_{BN}^{\,A}$
α
$\vec{a}_{BT}^{\,A}$
α
O
$\vec{a}_B$
B
x



A → $\overline{OA}$, $v_A = \overline{OA}\cdot\omega = a\,\omega$

B → $\overline{AB}$ 　　$\vec{v}_B = \vec{v}_A + \vec{v}_B^A$
B

$\vec{v}_B^A \perp \overline{AB}$ (смер ω_{AB})

$v_B^A = \overline{AB}\,\omega_{AB} = ?$

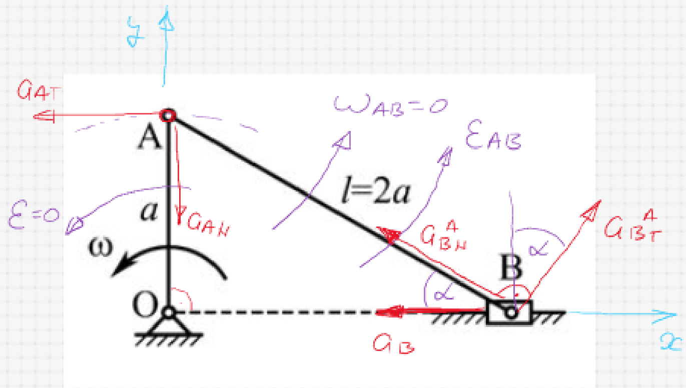
x: $v_B \cancel{\cos}\alpha = v_A \cancel{\cos}\alpha$ (1)

y: $-v_B \sin\alpha = -v_A \sin\alpha + v_B^A$ (2)

(1) $v_B = v_A = a\,\omega$

(2) $v_B^A = v_A \sin\alpha - v_B \sin\alpha = 0$

$$\omega_{AB} = \frac{v_B^A}{\overline{AB}} = 0$$



$\sin\alpha = \frac{a}{2a} = \frac{1}{2}$

$\alpha = 30^\circ$

$\vec{a}_A = \cancel{\vec{a}_{AT}}^{\,0} + \vec{a}_{AN}$; $a_{AT} = \overline{OA}\,\varepsilon = 0$

$a_{AN} = \overline{OA}\cdot\omega^2 = a\omega^2$

$\vec{a}_B = \vec{a}_A + \vec{a}_{BT}^{A} + \vec{a}_{BN}^{A}$

$\vec{a}_B = \vec{a}_{AT} + \vec{a}_{AN} + \vec{a}_{BT}^{A} + \vec{a}_{BN}^{A}$; $\vec{a}_{BN}^{A}$, $B \rightarrow A$

$a_{BN}^{A} = \overline{AB}\,\omega_{AB}^2 = 0$

$\vec{a}_{BT}^{A} \perp \overline{BA}$ (према ε_{AB})

$a_{BT}^{A} = \overline{AB}\,\varepsilon_{AB} = ?$

x: $-a_B = -\cancel{a_{AT}}^{\,0} + a_{BT}^{A}\sin\alpha - \cancel{a_{BN}^{A}}^{\,0}\cos\alpha$ (3)

y: $0 = -a_{AN} + a_{BT}^{A}\cos\alpha + \cancel{a_{BN}^{A}}^{\,0}\sin\alpha$ (4)

(4) $a_{BT}^{A} = \frac{a_{AN}}{\cos\alpha} = \frac{a\omega^2}{\cos\alpha}$

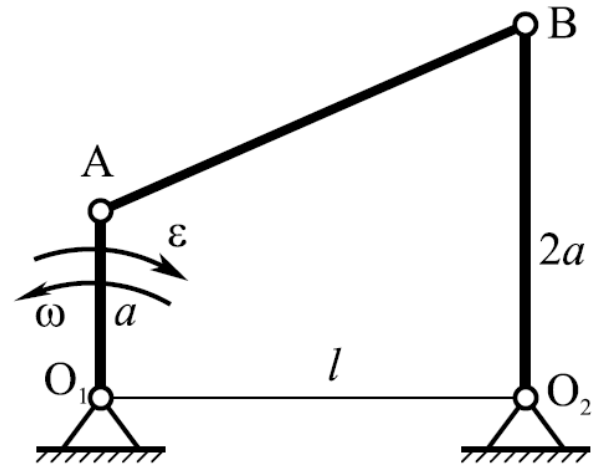
$\varepsilon_{AB} = \frac{a_{BT}^{A}}{\overline{AB}} = \frac{a_{BT}^{A}}{2a} = \frac{\omega^2}{2\cos\alpha}$

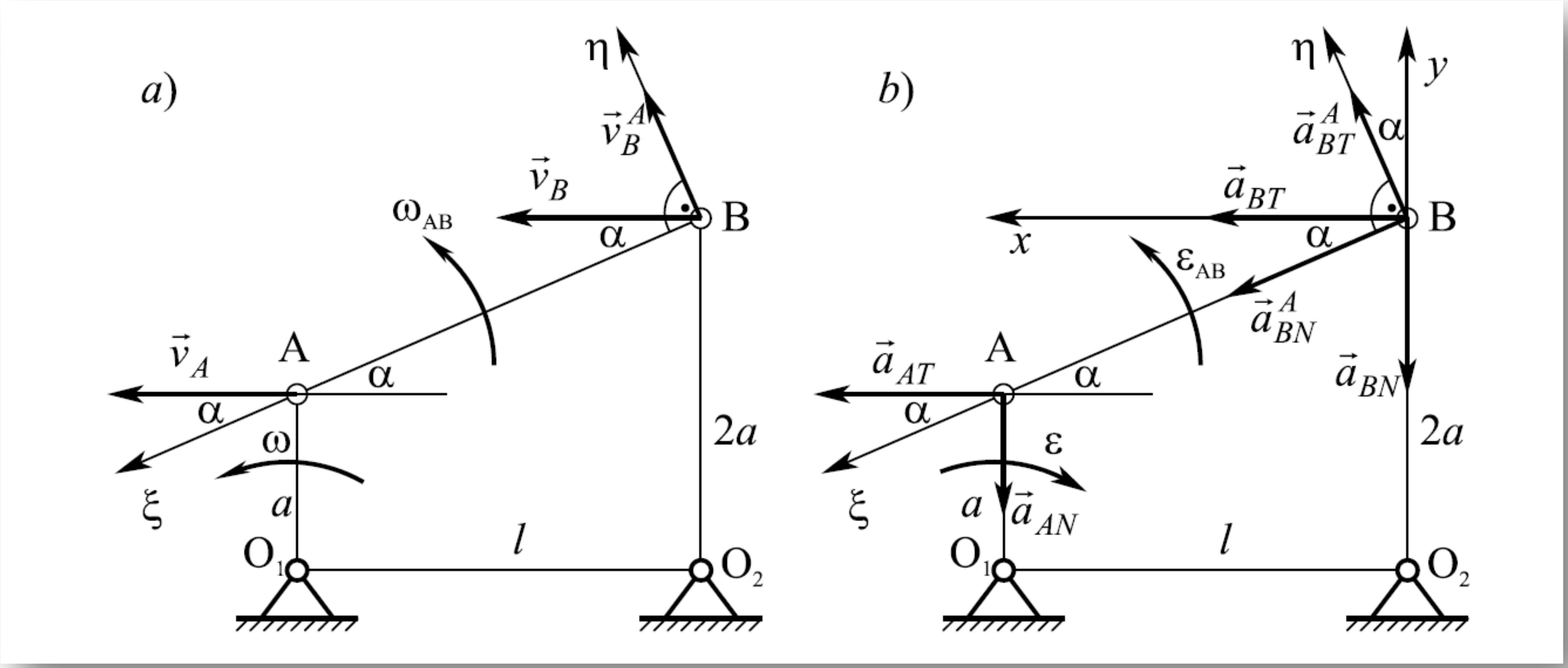
(3) $a_B = -a_{BT}^{A}\sin\alpha = -\frac{a\omega^2}{\cos\alpha}\cdot\sin\alpha$

$a_B = -a\omega^2\,\mathrm{tg}\,\alpha$

Zadatak 3

Zadatak 2.23 U prikazanom položaju zglobnog četvorougla poznati su ugaona brzina i ugaono ubrzanje vodećeg elementa O_1A, ω i ε. Odrediti brzinu i ubrzanje tačke B u datom položaju mehanizma.

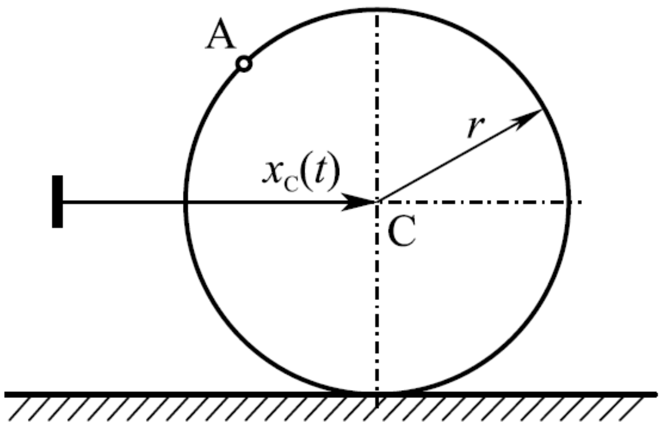




a)
η
$\vec{v}_B^A$
$\vec{v}_B$
B
ω_{AB}
α
$\vec{v}_A$
A
α
α
ω
2a
ξ
a
O1
l
O2
b)
η
y
$\vec{a}_{BT}^A$
α
$\vec{a}_{BT}$
B
x
α
ε_{AB}
$\vec{a}_{BN}^A$
$\vec{a}_{AT}$
A
α
$\vec{a}_{BN}$
α
ε
2a
ξ
a
$\vec{a}_{AN}$
O1
l
O2

Zadatak 4

Zadatak 2.24 Disk, poluprečnika r, kotrlja se bez klizanja po horizontalnom podu. Centar diska se kreće po poznatom zakonu $x_C(t)$. Odrediti brzinu i ubrzanje centra diska, ugaonu brzinu i ugaono ubrzanje diska i trajektoriju tačke na obodu diska. Odrediti iste veličine za slučaj da je $x_C(t) = Vt$, gde je $V = const$. Koliki su brzina i ubrzanje tačke na obodu diska u trenutku dodira sa podom.

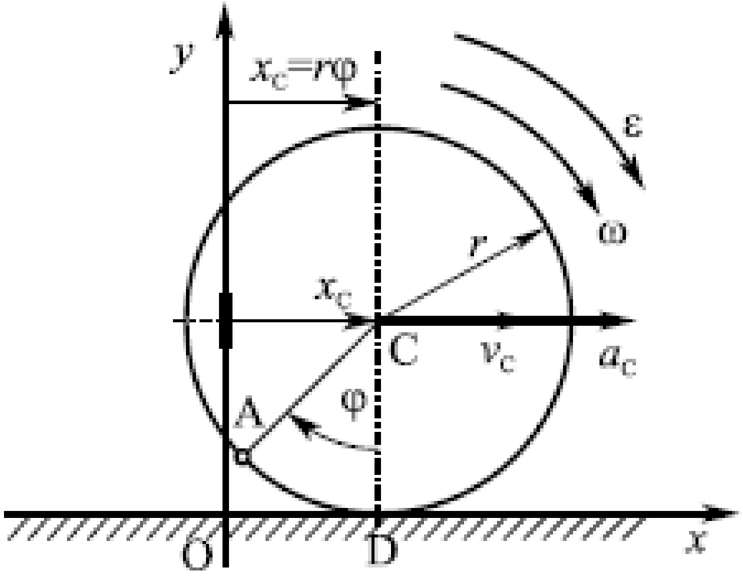


Brzina i ubrzanje centra diska su, Slika 2.31,:

$$\vec{v}_c(t) = v_c(t)\vec{i} = \dot{x}_c(t)\,\vec{i}, \qquad \vec{a}_c(t) = a_c(t)\vec{i} = \ddot{x}_c(t)\vec{i}$$

i u slučaju kada je $x_c(t) = Vt$ iznose

$$\vec{v}_c(t) = V\,\vec{i}, \qquad \vec{a}_c(t) = 0$$



Slika 2.31

Kako se disk kotrlja bez klizanja ($\overline{OD} = \widehat{AD}$ – dužine dodirnih lukova su jednake, Slika 2.31) to postoji veza između koordinate x_C, koja definiše položaj centra diska, i ugla obrtanja: $x_C(t) = r\varphi(t)$, odnosno

$$\varphi(t) = \frac{x_C(t)}{r}$$

Diferenciranjem poslednje jednačine dobijaju se ugaona brzina i ugaono ubrzanje diska:

$$\omega(t) = \dot{\varphi}(t) = \frac{\dot{x}_C(t)}{r} = \frac{v_C(t)}{r}, \qquad \varepsilon(t) = \ddot{\varphi}(t) = \frac{\ddot{x}_C(t)}{r} = \frac{a_C(t)}{r}$$

koji u slučaju kada je $x_C(t) = Vt$ iznose

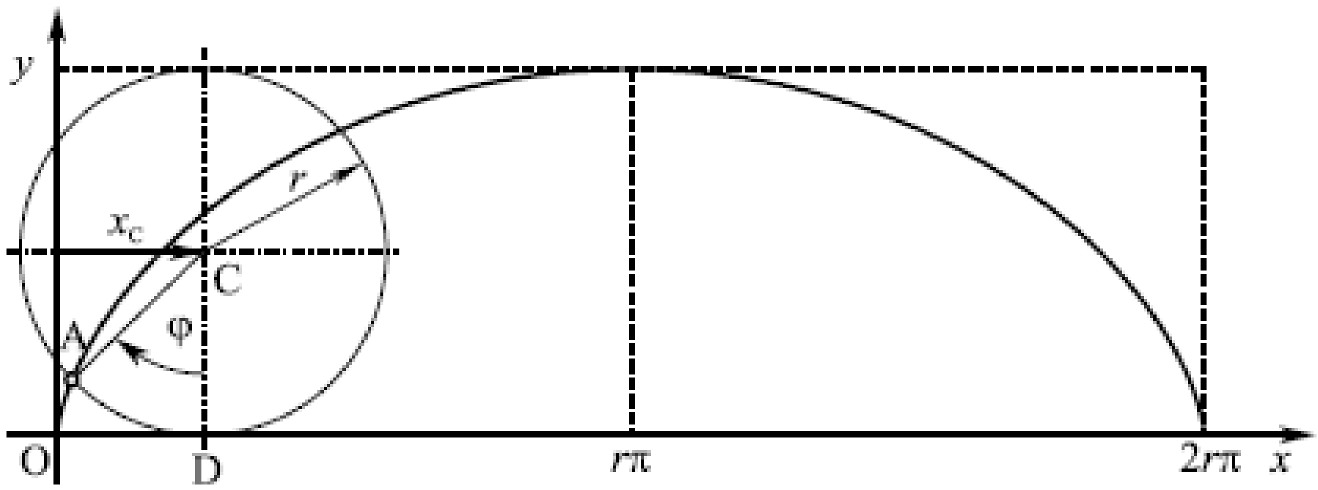
$$\omega(t) = \frac{V}{r} = const, \qquad \varepsilon(t) = 0$$

Koordinate tačke A na obodu diska, izražene preko ugla φ, Slika 2.31, glase:

$$x_A(t) = r\varphi(t) - r\sin\varphi(t)$$

$$y_A(t) = r - r\cos\varphi(t)$$

što su parametarske jednačine cikloide, Slika 2.32.



Slika 2.32

U slučaju kada je $x_C(t) = Vt$, $\varphi(t) = \frac{Vt}{r}$, parametarske jednačine kretanja tačke A su

$$x_A(t) = Vt - r\sin\left(\frac{Vt}{r}\right), y_A(t) = r - r\cos\left(\frac{Vt}{r}\right)$$

Brzina i ubrzanje tačke A određeni su sa

$$\dot{x}_A(t) = V - V\cos\left(\frac{Vt}{r}\right), \dot{y}_A(t) = V\sin\left(\frac{Vt}{r}\right)$$

$$\ddot{x}_A(t) = \frac{V^2}{r}\sin\left(\frac{Vt}{r}\right), \ddot{y}_A(t) = \frac{V^2}{r}\cos\left(\frac{Vt}{r}\right)$$

U trenutku dodira tačke A sa podom , t_d, važi

$$y_A(t_d) = \underbrace{r - r\cos\left(\frac{Vt_d}{r}\right) = 0}$$

odnosno $\cos\left(\frac{Vt_d}{r}\right) = 1$, odakle je $\frac{Vt_d}{r} = n2\pi, n = 0,1,2,\cdots$, $\sin\left(\frac{Vt_d}{r}\right) = 0$.

Prema tome, u trenutku dodira tačke A sa podom važi:

$$x_A(t_d) = n2r\pi, y_A(t_d) = 0$$

$$\dot{x}_A(t_d) = 0, \dot{y}_A(t_d) = 0$$

$$\ddot{x}_A(t_d) = 0, \ddot{y}_A(t_d) = \frac{V^2}{r}$$

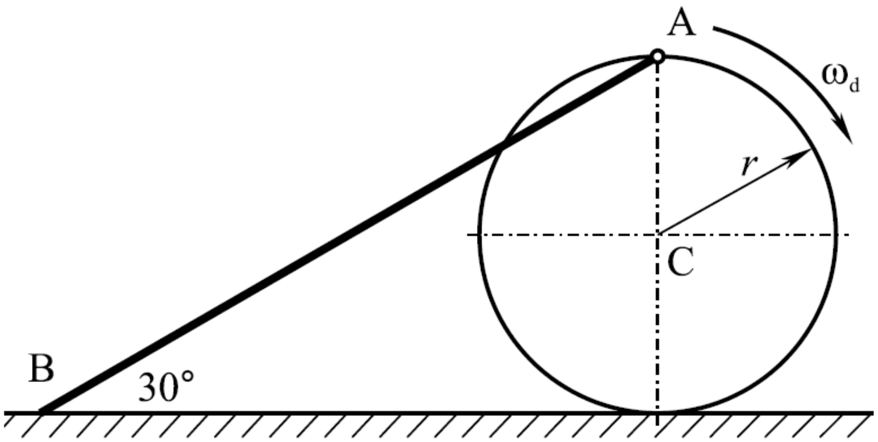
što znači da je u trenutku dodira tačke A sa podom njena brzina jednaka nuli, ali je ubrzanje različito od nule.

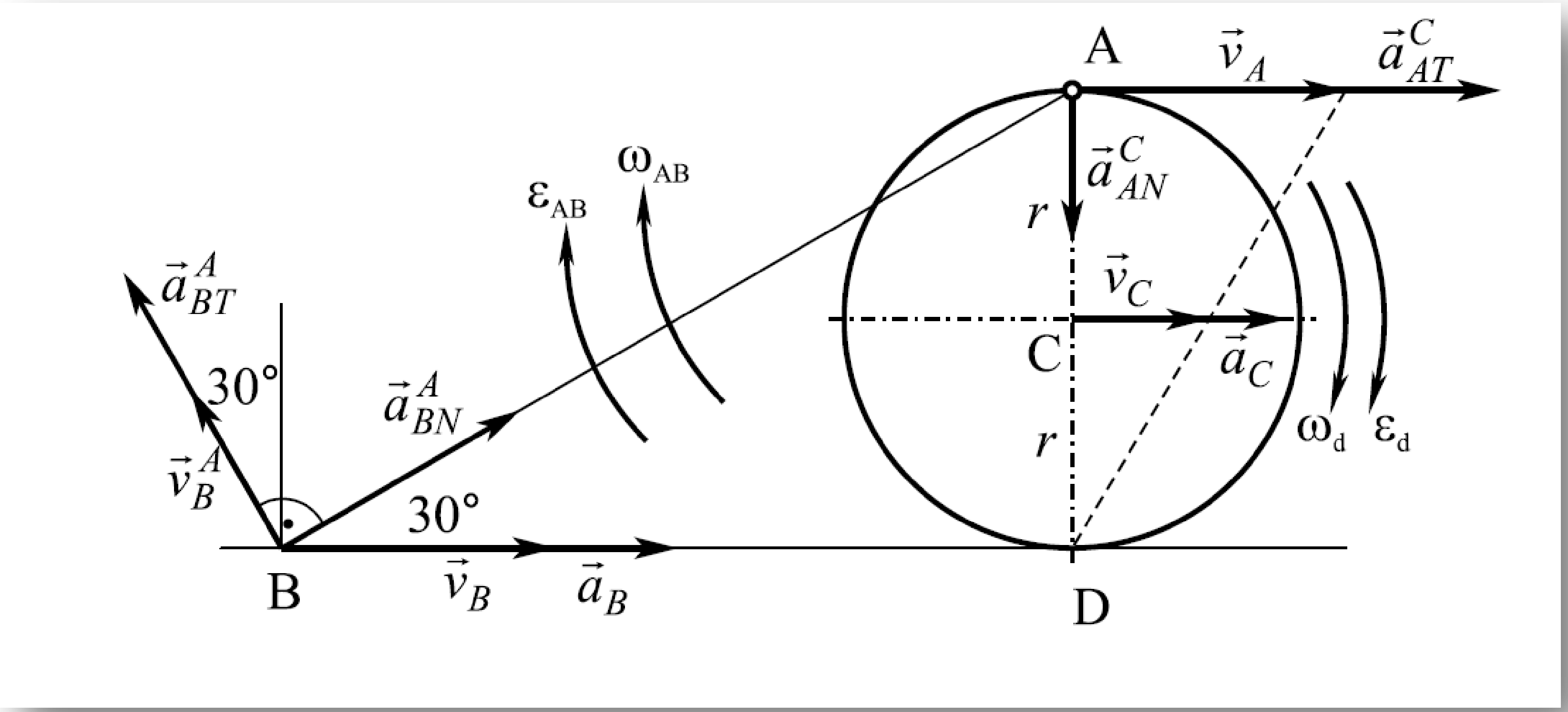
Kod kotrljanja bez klizanja tela po nepokretnoj podlozi brzina dodirne tačke jednaka je nuli.

Ako je u pitanju kotrljanje bez klizanja po pokretnom telu onda su brzine dodirnih tačaka jednake (definicija kotrljanja bez klizanja).

Zadatak 5

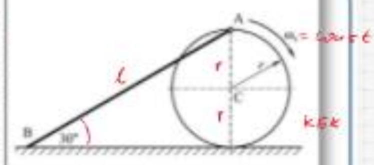
Zadatak 2.25 Disk, radijusa r, kotrlja se bez klizanja po horizontalnom podu. Za tačku A na njegovom obodu vezan je štap AB, dužine l. Drugi kraj štapa B klizi po podu. Ako je ugaona brzina diska ω_d konstantna, odrediti brzinu i ubrzanje tačke B u prikazanom položaju sistema krutih tela.





A
$\vec{v}_A$
$\vec{a}_{AT}^{C}$
$\vec{a}_{AN}^{C}$
r
ω_{AB}
ε_{AB}
$\vec{a}_{BT}^{A}$
30°
$\vec{a}_{BN}^{A}$
$\vec{v}_B^{A}$
30°
$\vec{v}_C$
C
$\vec{a}_C$
ω_d
ε_d
r
B
$\vec{v}_B$
$\vec{a}_B$
D

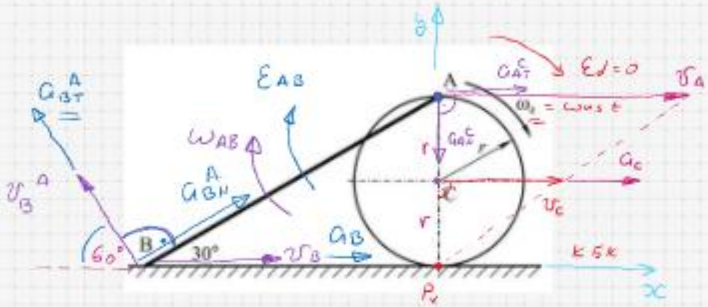
Zadatak 2.25 Disk, radijusa r, kotrlja se bez klizanja po horizontalnom podu. Za tačku A na njegovom obodu vezan je štap AB, dužine l. Drugi kraj štapa B klizi po podu. Ako je ugaona brzina diska ω_d konstantna, odrediti brzinu i ubrzanje tačke B u prikazanom položaju sistema krutih tela.



$\overline{AB} = l = 4r$

$\sin 30° = \frac{2r}{l}$

$l = \frac{2r}{\sin\alpha} = \frac{2r}{\frac{1}{2}} = 4r$



$v_A = \overline{P_vA} \cdot \omega_d = 2r\,\omega_d$

$\vec{v}_A = \vec{v}_C + \vec{v}_A^{\,C}$

$\vec{a}_A = \vec{a}_C + \vec{a}_{AT}^{\,C} + \vec{a}_{AN}^{\,C}$; $\vec{a}_{AN}^{\,C}$, $A \to C$

$a_{AN}^{C} = \overline{AC} \cdot \omega_d^2 = r\,\omega_d^2$

$\vec{a}_{AT}^{\,C} \perp \overline{AC}$ (ε_d)

$a_{AT}^{C} = \overline{AC} \cdot \varepsilon_d = 0$

$x:$ $a_{Ax} = a_C + a_{AT}^{C}$

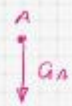
$y:$ $a_{Ay} = -a_{AN}^{C}$

$v_C = \overline{P_vC} \cdot \omega_d = r\,\omega_d \quad / \frac{d}{dt}$

$a_{CT} = a_C = r\,\varepsilon_d = 0$

$a_{Ax} = 0 + 0 = 0$

$a_{Ay} = -r\,\omega_d^2$



$a_{Ay} = -r\omega_d^2$ }

a_A

„AB" $\vec{v}_B = \vec{v}_A + \vec{v}_B^A$; $\vec{v}_B^A \perp \overline{AB}$ (ω_{AB})

$v_B^A = \overline{AB}\,\omega_{AB} = ?$

x: $v_B = v_A - v_B^A \cos 60°$ (1)

y: $0 = 0 + v_B^A \sin 60°$ (2)

(2) $\rightarrow$ $v_B^A = 0$ $\rightarrow$ $\omega_{AB} = \frac{v_B^A}{\overline{AB}} = 0$

(1) $\rightarrow$ $v_B = v_A = 2r\omega_d$

$\vec{a}_B = \vec{a}_A + \vec{a}_{BT}^A + \cancel{\vec{a}_{BN}^A}$ 0 ; $\vec{a}_{BN}^A$, $B \rightarrow A$

$a_{BN}^A = \overline{AB}\cdot\omega_{AB}^2 = 0$

$\vec{a}_{BT}^A \perp \overline{AB}$ (ε_{AB})

$a_{BT}^A = \overline{AB}\,\varepsilon_{AB} = ?$

x: $a_B = a_{Ax} - a_{BT}^A \cos 60°$ (3)

y: $0 = a_{Ay} + a_{BT}^A \sin 60°$ (4)

(4) $a_{BT}^A = -\frac{a_{Ay}}{\sin 60°} = -\frac{(-r\omega_d^2)}{\frac{\sqrt{3}}{2}}$

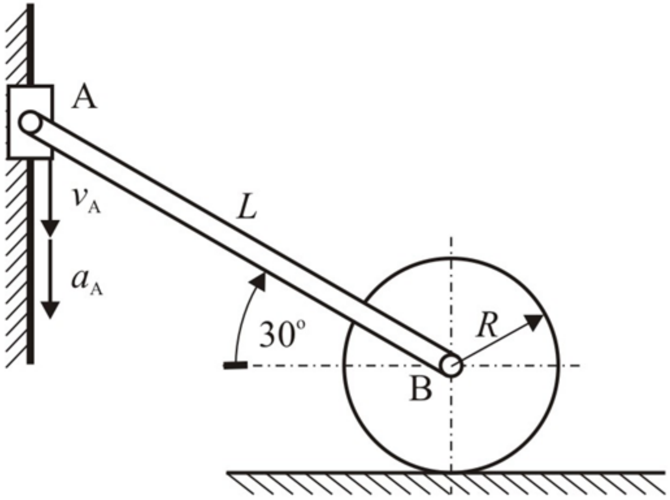
$a_{BT}^A = +\frac{2}{\sqrt{3}} r\omega_d^2 = \frac{2\sqrt{3}}{3} r\omega_d^2$

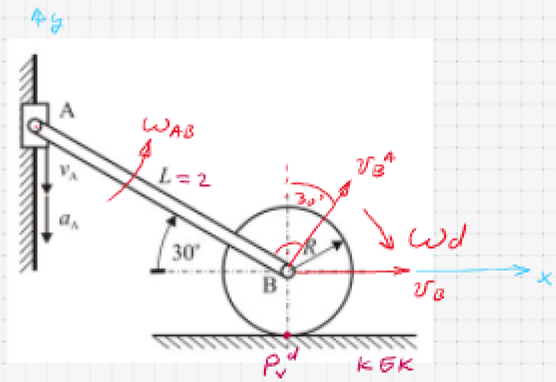
$\varepsilon_{AB} = \frac{a_{BT}^A}{\overline{AB}} = \frac{\frac{2\sqrt{3}}{3}\cancel{r}\,\omega_d^2}{\cancel{4}_2\,\cancel{r}} = \frac{\sqrt{3}}{6}\omega_d^2$

(3) $a_B = 0 - \frac{\cancel{2}\sqrt{3}}{3} r\omega_d^2 \cdot \frac{1}{\cancel{2}} = -\frac{\sqrt{3}}{3} r\omega_d^2$

Zadatak 6

Štap AB, dužine L=2 m, vezan je zglobno svojim krajevima za klizač A, koji je kreće po vertikalnoj vođici, i za centar diska, radijusa R=0.5 m, koji se kotrlja bez klizanja po horizontalnom podu. U položaju sistema u kome štap gradi sa horizontalom ugao 30° intenziteti brzine i ubrzanja klizača A iznose: $v_A = \sqrt{3}$ m/s, a_A=2 m/s², (smerovi su dati na slici). Odrediti brzinu i ubrzanje centra diska, kao i ugaonu brzinu i ugaono ubrzanje diska u datom položaju.





$$\vec{v}_B = \vec{v}_A + \vec{v}_B^{A} \quad ; \quad v_B^{A} = \overline{AB}\,\omega_{AB} = ?$$

$$x: \quad v_B = 0 + v_B^{A} \sin 30^\circ \quad (1)$$

$$y: \quad 0 = -v_A + v_B^{A} \cos 30^\circ \quad (2)$$

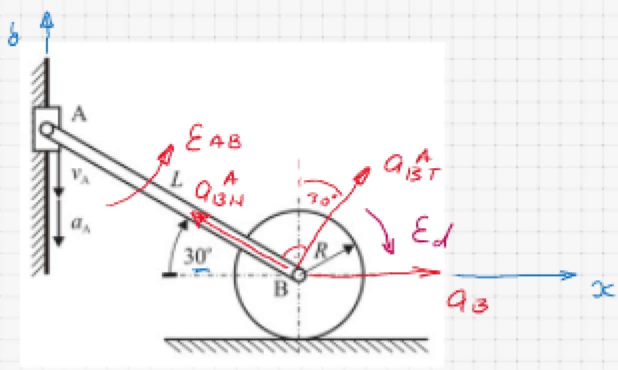
$$(2) \quad v_B^{A} = \frac{v_A}{\cos 30^\circ} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}} = 2 \ \frac{m}{s}$$

$$\omega_{AB} = \frac{v_B^{A}}{\overline{AB}} = \frac{2}{2} = 1 \ s^{-1}$$

$$(1) \quad \boxed{v_B = 2 \cdot \frac{1}{2} = 1 \ \frac{m}{s}}$$

$$v_B = \overline{P_v^{d} B}\,\omega_d = R\,\omega_d$$

$$\boxed{\omega_d = \frac{v_B}{R} = \frac{1}{\frac{1}{2}} = 2 \ s^{-1}}$$



$$\vec{a}_B = \vec{a}_A + \vec{a}_{BT}^A + \vec{a}_{BN}^A \quad ; \quad a_{BN}^A = \overline{AB}\,\omega_{AB}^2 = 2\cdot 1^2 = 2\ \frac{m}{s^2}$$

$$a_{BT}^A = \overline{AB}\cdot\varepsilon_{AB} = ?$$

x: (3) $a_B = 0 + a_{BT}^A \sin 30^\circ - a_{BN}^A \cos 30^\circ$

y: (4) $0 = -a_A + a_{BT}^A \cos 30^\circ + a_{BN}^A \sin 30^\circ$

(4) $0 = -2 + a_{BT}^A \frac{\sqrt{3}}{2} + \cancel{2}\cdot\frac{1}{\cancel{2}} \rightarrow a_{BT}^A = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}\ \frac{m}{s^2}$

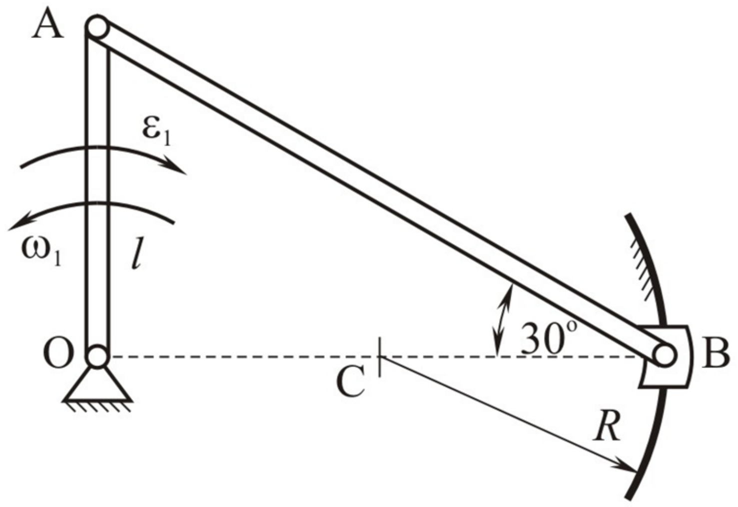
$$\varepsilon_{AB} = \frac{a_{BT}^A}{\overline{AB}} = \frac{\frac{2\sqrt{3}}{3}}{2} = \frac{\sqrt{3}}{3}\ s^{-2}$$

(3) $a_B = \frac{\cancel{2}\sqrt{3}}{3}\,\frac{1}{\cancel{2}} - \cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}} = -\frac{2\sqrt{3}}{3}$

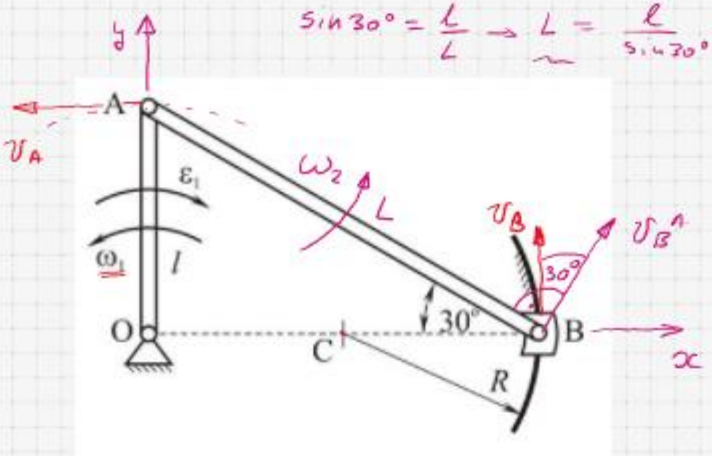
$$\varepsilon_d = \frac{a_{BT}}{R} = \frac{a_B}{R} = \frac{-\frac{2\sqrt{3}}{3}}{\frac{1}{2}} = -\frac{4\sqrt{3}}{3}\ s^{-2}$$

Zadatak 7

Štap OA dužine l=1 m zglobno je vezan u tački O za podlogu, dok je drugim krajem zglobno vezan u tački A za štap AB. Štap AB je zglobno vezan za klizač B koji se kreće po kružnoj vođici radijusa $R=\frac{\sqrt{3}}{2}\,\mathrm{m}$. U prikazanom položaju sistema štap OA ima ugaonu brzinu $\omega_1 = 1\,\mathrm{s}^{-1}$ i ugaono ubrzanje $\varepsilon_1 = \sqrt{3}\,\mathrm{s}^{-2}$ (smerovi su dati na slici). Odrediti brzinu i ubrzanje tačaka A i B, kao i ugaonu brzinu i ugaono ubrzanje štapa AB.



$$\sin 30^\circ = \frac{l}{L} \rightarrow L = \frac{l}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2 = \overline{AB}$$



OA – обртање око осе, $TP_A = K\{O, l\}$

AB – раванско кр.

B – $TP_B = K\{C, R\}$

$$v_A = l\omega_1 = 1 \cdot 1 = 1 \ \frac{m}{s}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_B^{\,A} \quad ; \quad v_B^A = \overline{AB}\,\omega_2 = ?$$

$$x: \quad 0 = -v_A + v_B^A \sin 30^\circ \qquad (1)$$

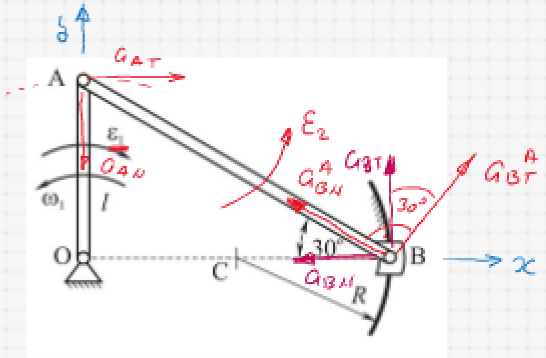
$$y: \quad v_B = 0 + v_B^A \cos 30^\circ \qquad (2)$$

$$(1) \rightarrow \quad v_B^A = \frac{v_A}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2\ \frac{m}{s}$$

$$(2) \rightarrow \quad v_B = \cancel{2} \cdot \frac{\sqrt{3}}{\cancel{2}} = \sqrt{3}\ \frac{m}{s}$$

$$\omega_2 = \frac{v_B^A}{\overline{AB}} = \frac{\sqrt{3}}{2}\ s^{-1}$$

$= 2 = \overline{AB}$



$\vec{a}_A = \vec{a}_{AT} + \vec{a}_{AN}$; $a_{AN} = \overline{OA}\cdot\omega_1^2 = 1\cdot 1^2 = 1 \ \frac{m}{s^2}$

$a_{AT} = \overline{OA}\,\varepsilon_1 = 1\cdot\sqrt{3} = \sqrt{3} \ \frac{m}{s^2}$

$a_A = \sqrt{a_{AT}^2 + a_{AN}^2} = \dots = 2 \ \frac{m}{s^2}$

$\vec{a}_B = \vec{a}_A + \vec{a}_B^{A}$

$\vec{a}_{BT} + \vec{a}_{BN} = \vec{a}_{AT} + \vec{a}_{AN} + \vec{a}_{BT}^{A} + \vec{a}_{BN}^{A}$; $a_{BN}^{A} = \overline{AB}\,\omega_2^2 = 2\cdot\left(\frac{\sqrt{3}}{2}\right)^2 = 2\cdot\frac{3}{4} = \frac{3}{2} \ \frac{m}{s^2}$

x: (3) $-a_{BN} = a_{AT} + a_{BT}^{A}\sin 30° - a_{BN}^{A}\cos 30°$

$a_{BT}^{A} = \overline{AB}\cdot\varepsilon_2 = ?$

y: (4) $a_{BT} = -a_{AN} + a_{BT}^{A}\cos 30° + a_{BN}^{A}\sin 30°$

$a_{BN} = \frac{v_B^2}{R} = \frac{(\sqrt{3})^2}{\frac{\sqrt{3}}{2}} = 2\sqrt{3} \ \frac{m}{s^2}$

(3) $-2\sqrt{3} = \sqrt{3} + a_{BT}^{A}\,\frac{1}{2} - \cancel{2}\sqrt{3}\cdot\frac{\sqrt{3}}{\cancel{2}}$

$$\frac{1}{2}\,a_{BT}^{A} = -2\sqrt{3} - \sqrt{3} + 3$$

$$a_{BT}^{A} = 2\,(-3\sqrt{3} + 3)$$

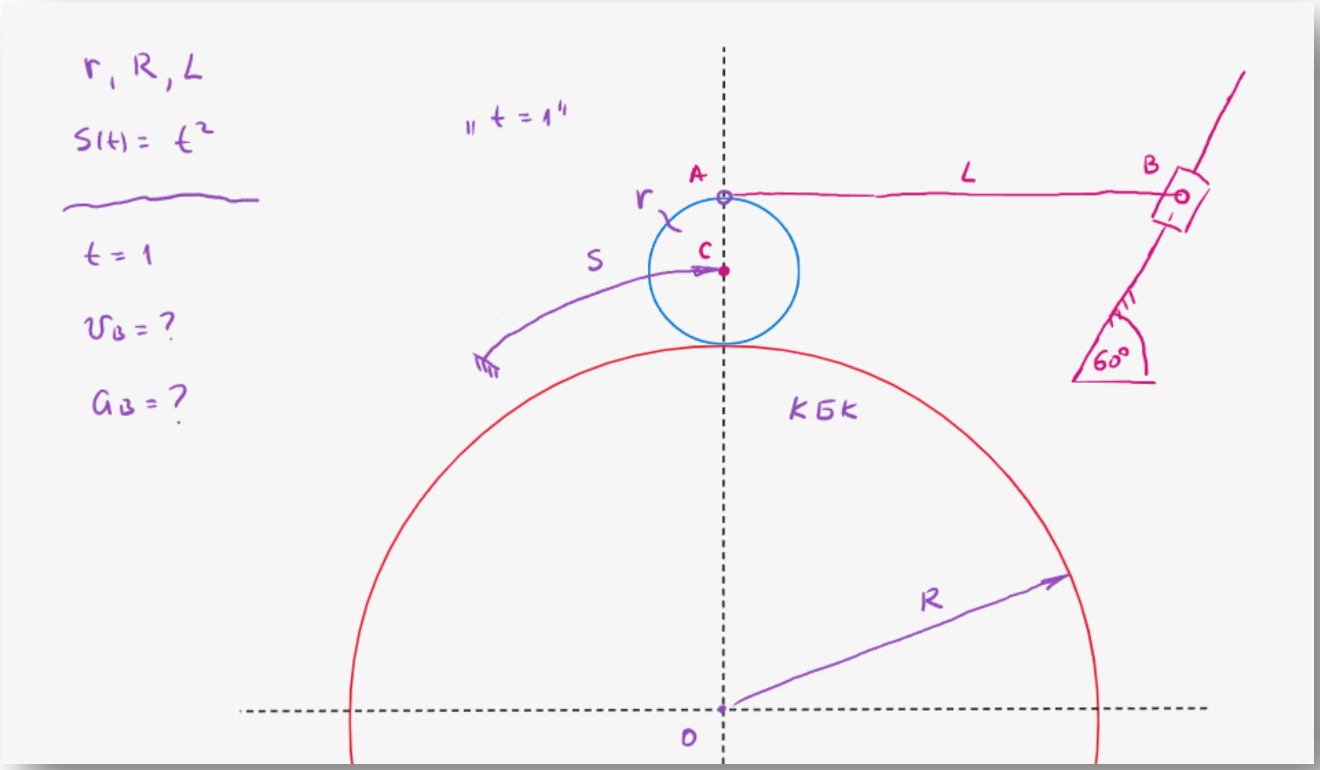
$$a_{BT}^{A} = -2\,(3\sqrt{3} - 3) = -6\,(\sqrt{3} - 1)\ \frac{m}{s^2}$$

$$\varepsilon_z = \frac{a_{BT}^{A}}{\overline{AB}} = \frac{-6\,(\sqrt{3}-1)}{2} = -3\,(\sqrt{3}-1)\ s^{-2}$$

(4) $a_{BT} = -1 - 6\,(\sqrt{3}-1)\,\frac{\sqrt{3}}{2} + 2\sqrt{3}\cdot\frac{1}{2} = \dots$

$$a_B = \sqrt{a_{BT}^2 + a_{BN}^2} = \dots$$

Zadatak 8



Kinematika – vežbe 4

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.