Mehanika 2 (Kinematika)
Predavanja 4

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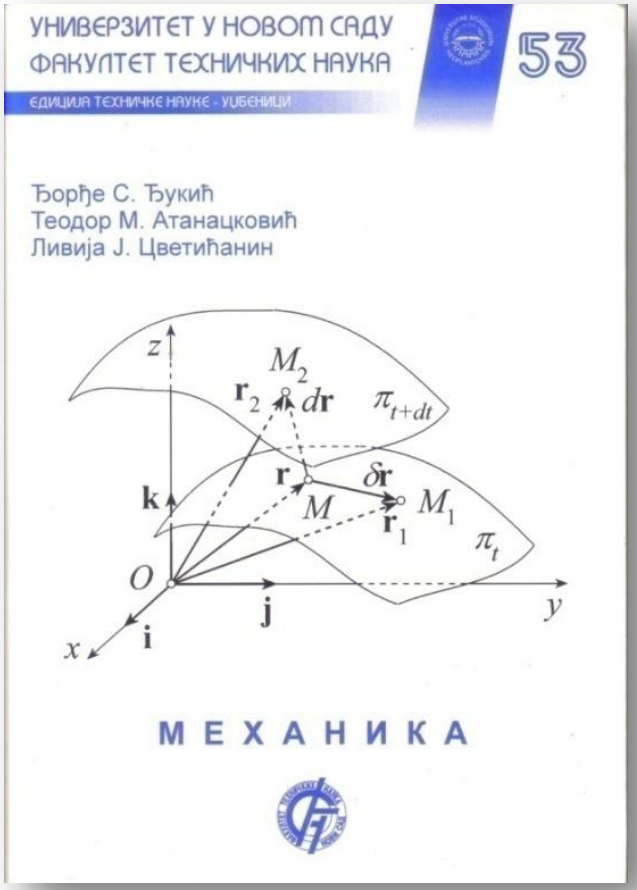
Novi Sad, 2023.

Literatura

- Đorđe S. Đukić, Teodor M. Atanacković, Livija J. Cvetićanin: Mehanika, Fakultet tehničkih nauka u Novom Sadu, Novi Sad, 2003.



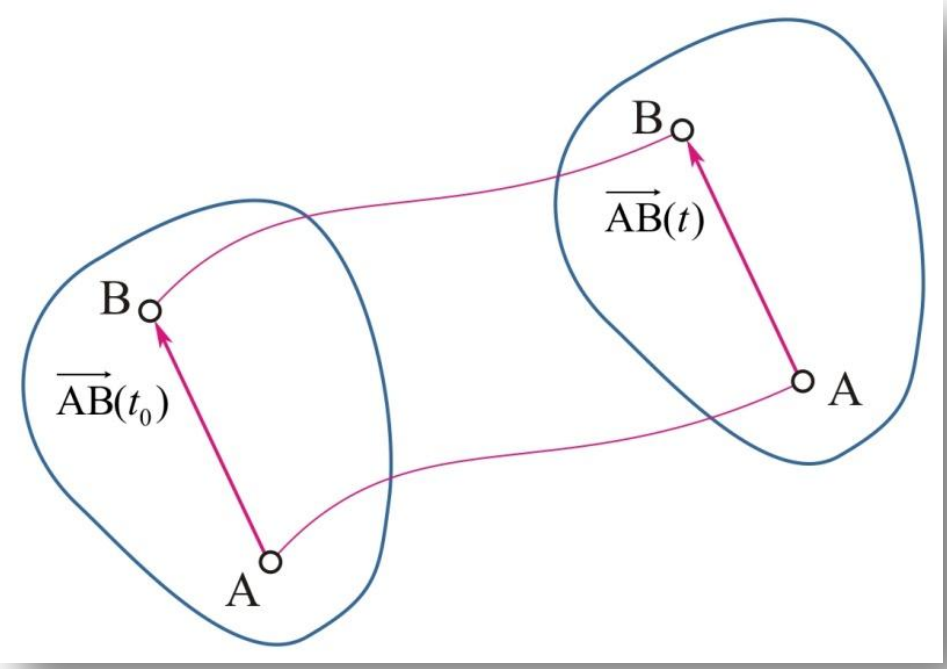
Šta ćemo naučiti?

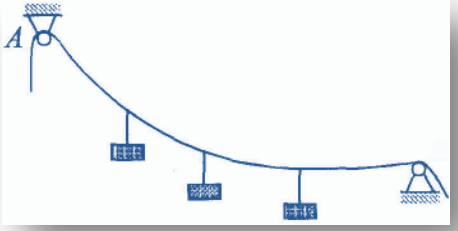
9. **Kinematika translatornog kretanja krutog tela**
10. **Kinematika obrtanja krutog tela oko nepokretne ose (zakon kretanja, ugaona brzina i ugaono ubrzanje)**
11. **Kinematika obrtanja krutog tela oko nepokretne ose (brzina i ubrzanje tačaka tela)**
12. **Ravnomerno i ravnomerno promenljivo obrtanje krutog tela oko nepokretne ose**

9. Kinematika translatornog kretanja krutog tela

Translatorno kretanja krutog tela

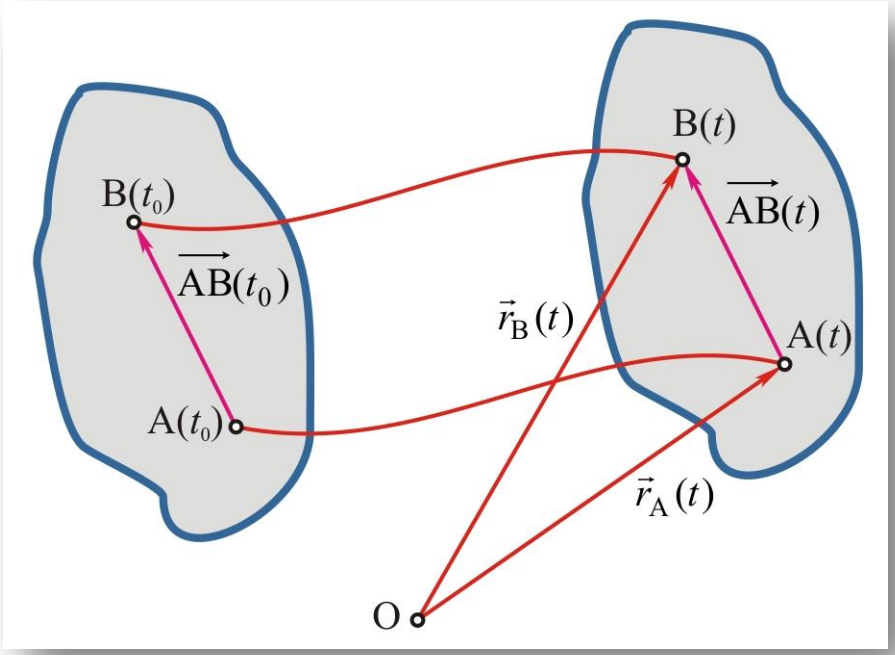
- Kretanje tela je translatorno, ako svaki zamišljeni pravac u telu za vreme kretanja ostaje sam sebi paralelan. Kretanje ima (najviše) tri stepena slobode.





Translatorno kretanja krutog tela

- Pri translatornom kretanju tela putanje, brzine i ubrzanja svih tačaka tela su u bilo kom trenutku vremena iste.



$$\overrightarrow{AB}(t) = \overrightarrow{const} = \overrightarrow{AB}(t_0)$$

$$\dot{\overrightarrow{AB}}(t) = 0$$

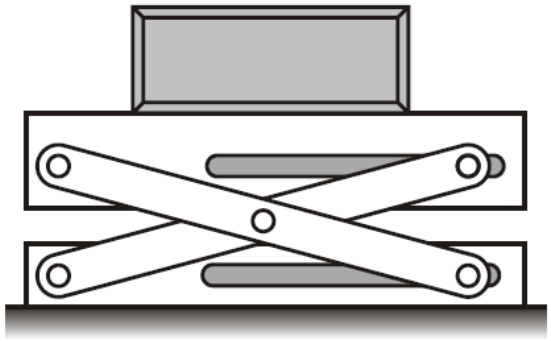
$$\vec{r}_B(t) = \vec{r}_A(t) + \overrightarrow{AB}(t)$$

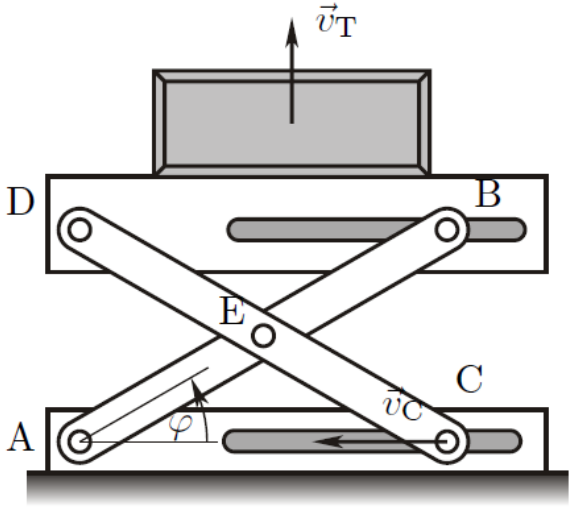
$$\dot{\vec{r}}_B(t) = \dot{\vec{r}}_A(t) \rightarrow \vec{v}_B(t) = \vec{v}_A(t)$$

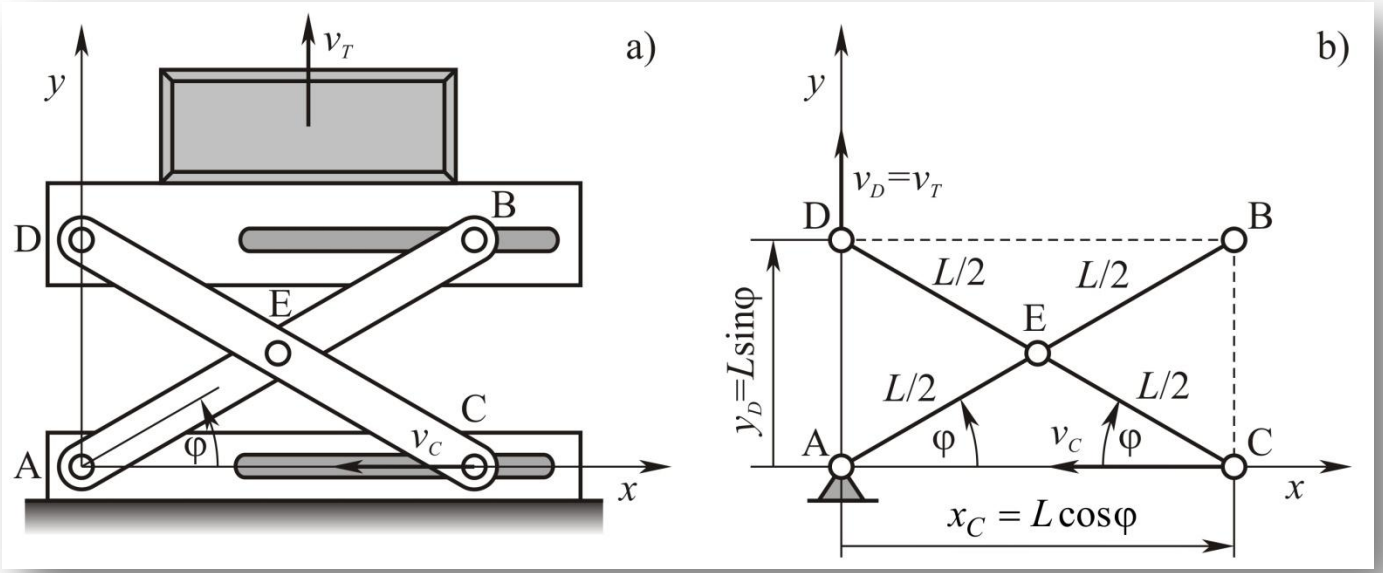
$$\ddot{\vec{r}}_B(t) = \ddot{\vec{r}}_A(t) \rightarrow \vec{a}_B(t) = \vec{a}_A(t)$$

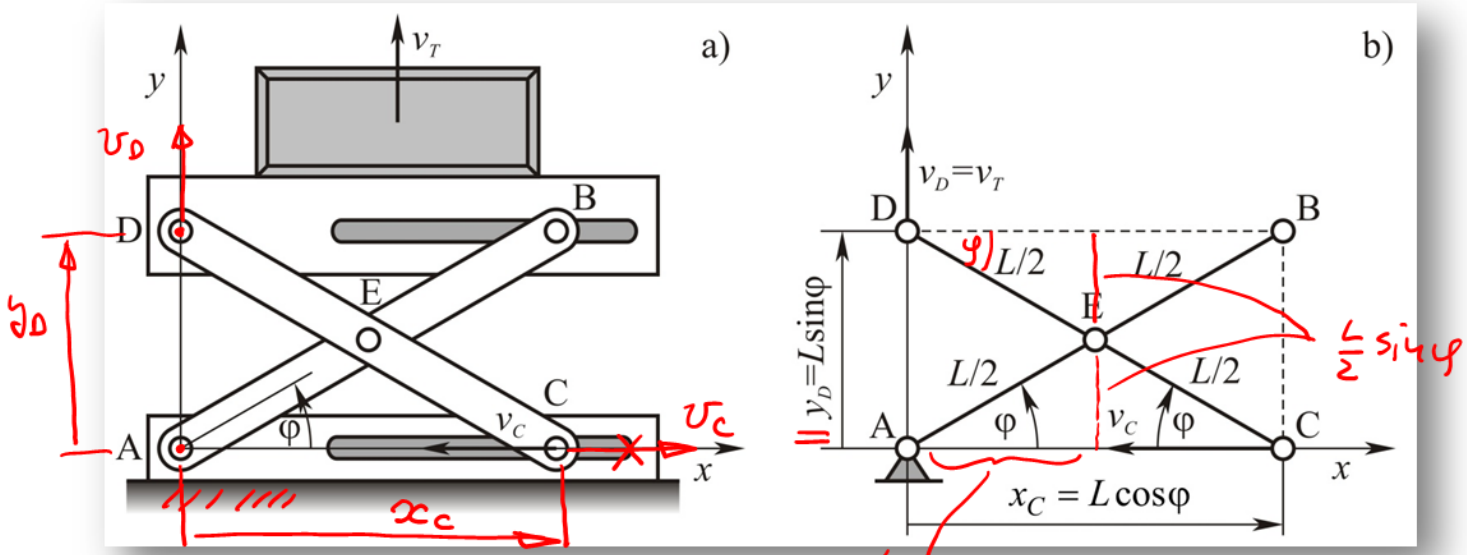
Primer

Одредити брзину и убрзање терета, ако угао који полуга AB, механизма за подизање, гради са хоризонталом мења по закону $\varphi = \omega t$ ($\omega = \text{const.}$). Која је веза између брзине терета и брзине тачке C? Дужине полуга AB и CD механизма су једнаке ($\overline{\text{AB}} = \overline{\text{CD}} = L$), а зглоб E се налази на њиховој средини. Како би требало да се мења угао φ да би се терет подизао константном брзином?






a)
b)
v_T
y
x
D
B
E
C
A
φ
v_C
$v_D=v_T$
L/2
L/2
L/2
L/2
$y_D=L\sin\varphi$
$x_C = L\cos\varphi$



$\varphi = \omega t$

$y_D = L\sin\varphi = L\sin(\omega t)$

$v_D = \dot{y}_D = L\omega\cos(\omega t) = v_T$

$a_D = \ddot{y}_D = -L\omega^2\sin(\omega t) = a_T$

$x_C = L\cos\varphi = L\cos(\omega t)$

$v_C = \dot{x}_C = -L\omega\sin(\omega t)$

$a_C = \ddot{x} = -L\omega^2\cos(\omega t)$

$\frac{v_D}{v_C} = \frac{L\omega\cos(\omega t)}{-L\omega\sin(\omega t)} = -\operatorname{ctg}(\omega t)$

$\varphi = ?$ $\rightarrow$ $v_T = \text{const}$

$y_D = L \sin\varphi$

$v_D = \boxed{v_T = L\,\dot{\varphi}\cos\varphi}$

$\dot{\varphi} = \frac{v_T}{L}\,\frac{1}{\cos\varphi}$

$\frac{d\varphi}{dt} = \frac{v_T}{L}\cdot\frac{1}{\cos\varphi}$

$\int_{\varphi_0}^{\varphi}\cos\varphi\,d\varphi = \frac{v_T}{L}\int_0^t dt$

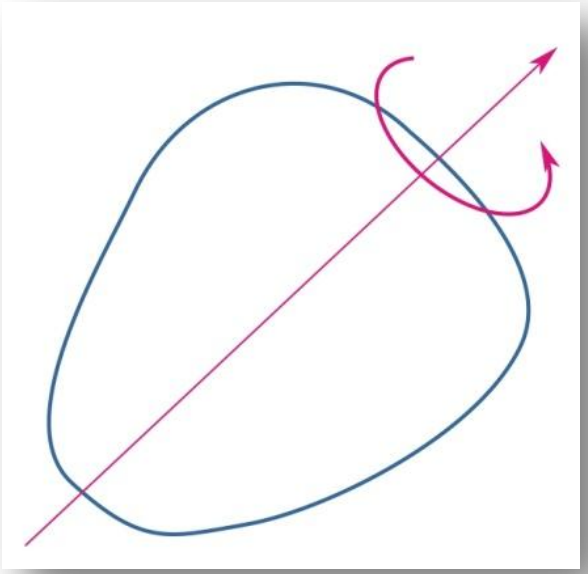
$\sin\varphi\Big|_{\varphi_0}^{\varphi} = \frac{v_T}{L}\,t\Big|_0^t$

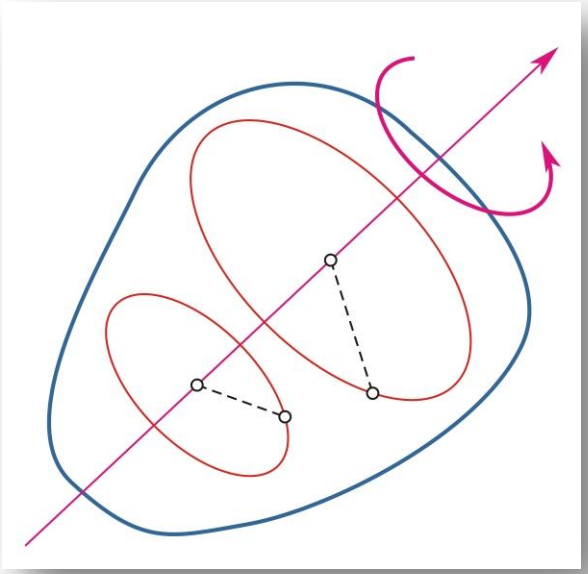
$\sin\varphi - \sin\varphi_0 = \frac{v_T}{L}\,t$

$\sin\varphi = \sin\varphi_0 + \frac{v_T}{L}\,t$

10. Kinematika obrtanja krutog tela oko nepokretne ose (zakon kretanja, ugaona brzina i ugaono ubrzanje)

Obrtanje krutog tela oko nepokretne ose





Obrtanje krutog tela oko nepokretne ose je takvo kretanje tela kod kog se njegove tačke kreću po kružnicama čiji je centar na nepokretnoj osi i koje su u ravnima upravnim na tu osu (osa obrtanja).

Obrtanja oko nepokretne ose – zakon kretanja

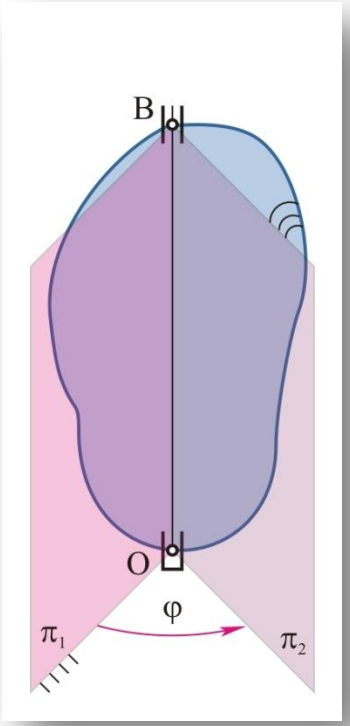
1 stepen slobode kretanja

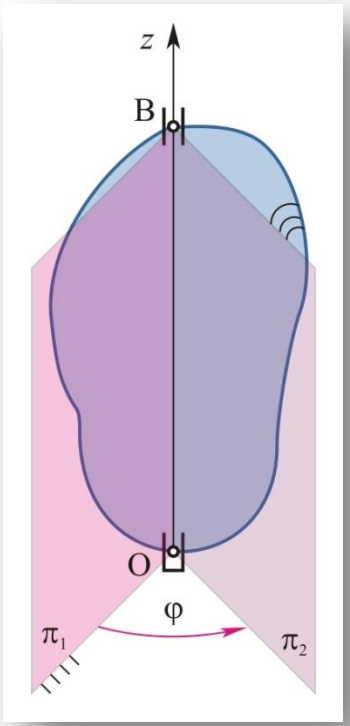
Ugao obrtanja

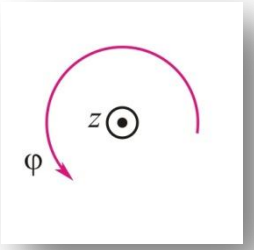
φ

Zakon kretanja – parametarska jednačina kretanja

$\varphi = \varphi(t)$







Obrtanja oko nepokretne ose – ugaona brzina

Srednja ugaona brzina

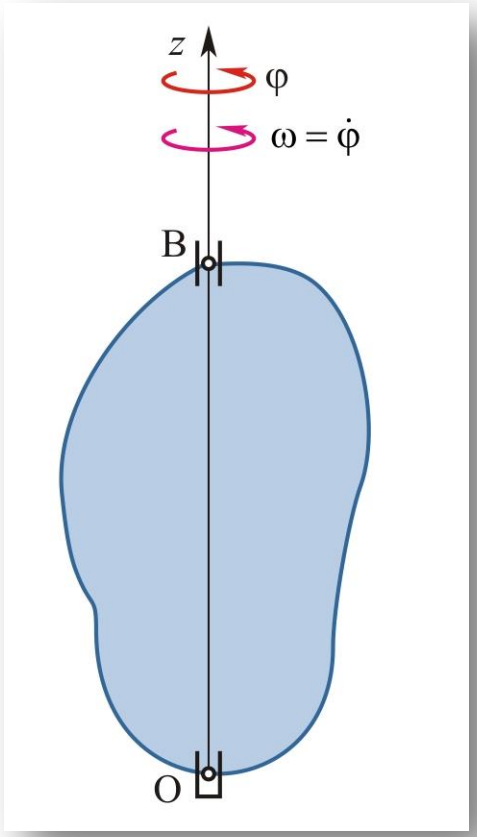
$$\omega_{sr} = \frac{\Delta\varphi}{\Delta t}$$

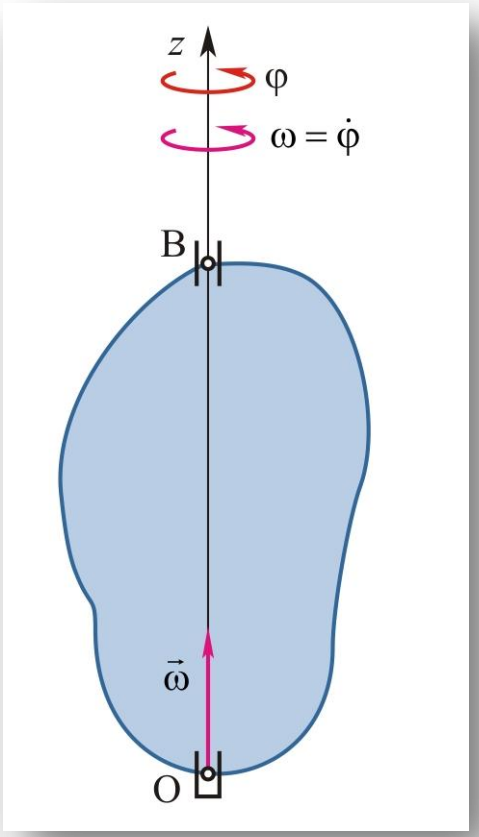
Ugaona brzina

$$\omega = \lim_{\Delta t \to 0} \omega_{sr} = \lim_{\Delta t \to 0} \frac{\Delta\varphi}{\Delta t} = \frac{d\varphi}{dt} = \dot{\varphi}$$

Vektor ugaone brzine

$$\vec{\omega}(t) = \omega(t)\vec{k}$$





Obrtanja oko nepokretne ose – ugaono ubrzanje

Srednje ugaono ubrzanje

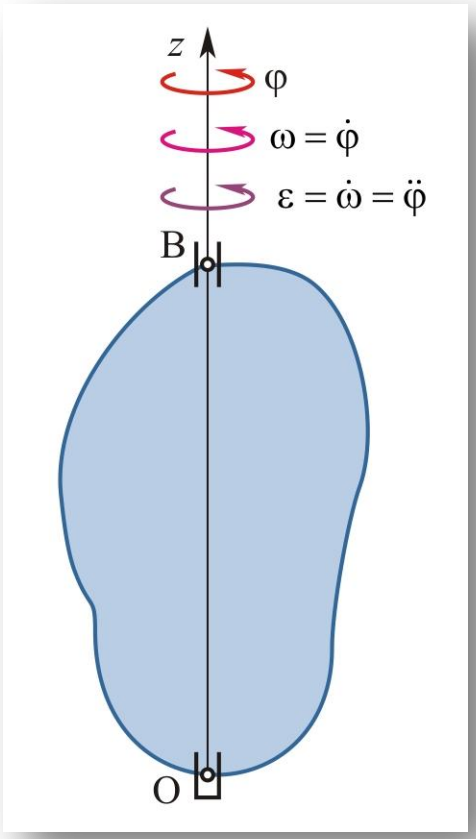
$$\varepsilon_{sr} = \frac{\Delta\omega}{\Delta t}$$

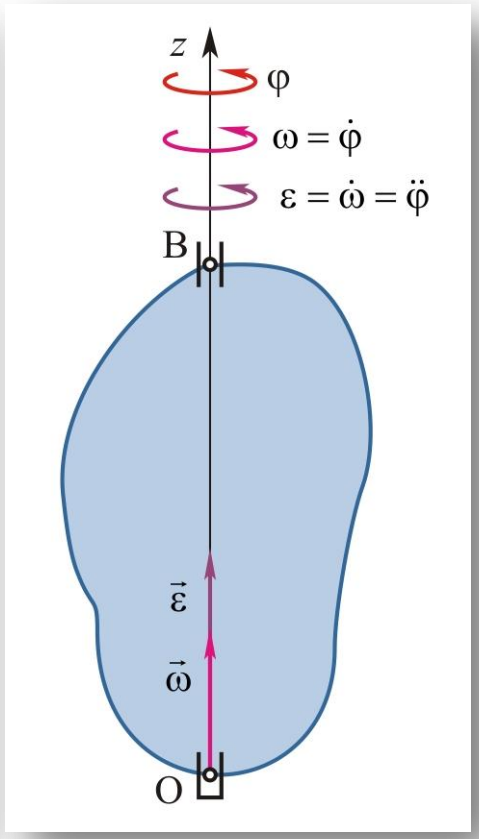
Ugaono ubrzanje

$$\varepsilon = \lim_{\Delta t \to 0} \varepsilon_{sr} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \dot{\omega} = \ddot{\varphi}$$

Vektor ugaonog ubrzanja

$$\vec{\varepsilon}(t) = \varepsilon(t)\vec{k}$$





11. Kinematika obrtanja krutog tela oko nepokretne ose (brzina i ubrzanje tačaka tela)

Obrtanja oko nepokretne ose – brzine tačaka tela

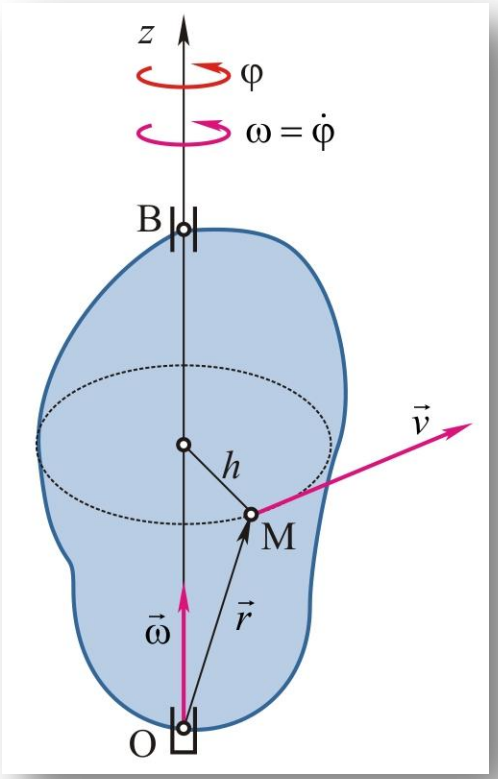
$$\vec{v} = v_T \vec{T} = \dot{s}\vec{T}$$

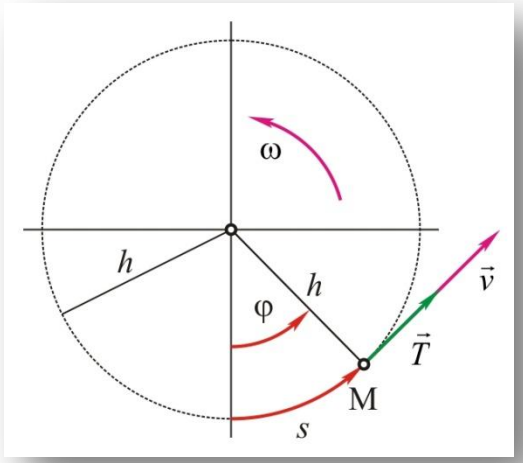
$$s = h\varphi$$

$$\vec{v} = h\dot{\varphi}\vec{T} = h\omega\vec{T}$$

$$v = h\omega$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$





Obrtanja oko nepokretne ose – ubrzanja tačaka tela

$$\vec{a} = a_T \vec{T} + a_N \vec{N}$$

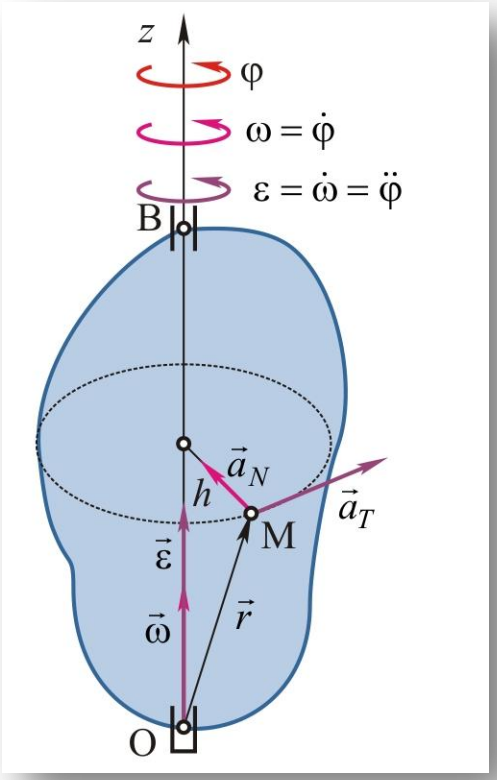
$$\vec{a} = \ddot{s}\vec{T} + \frac{\dot{s}^2}{h}\vec{N}$$

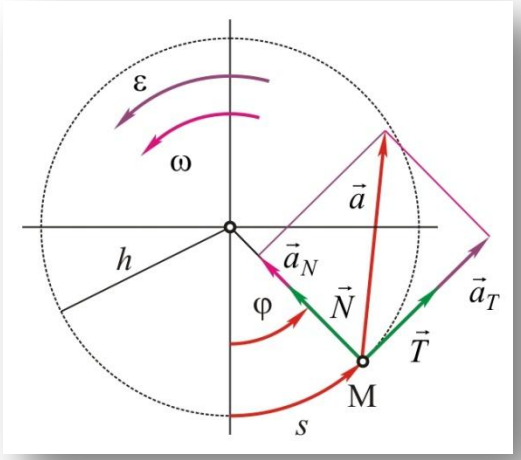
$$\vec{a} = h\ddot{\varphi}\vec{T} + h\dot{\varphi}^2\vec{N}$$

$$\vec{a} = h\varepsilon\vec{T} + h\omega^2\vec{N}$$

$$a_T = h\varepsilon$$

$$a_N = h\omega^2$$





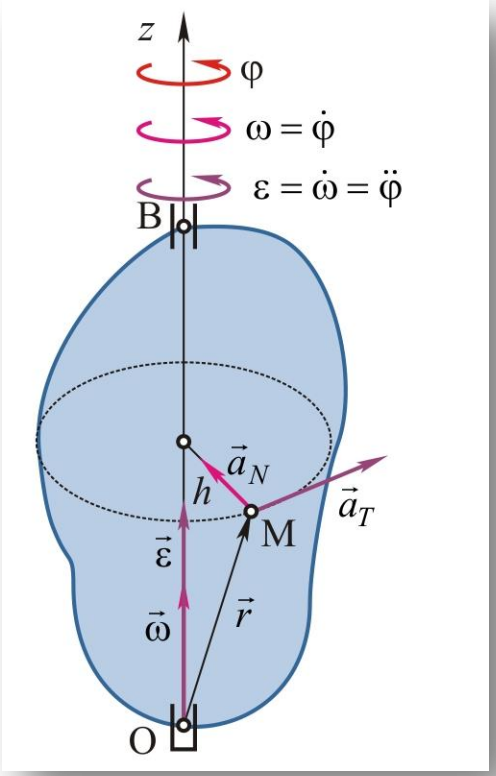
Obrtanja oko nepokretne ose – ubrzanja tačaka tela

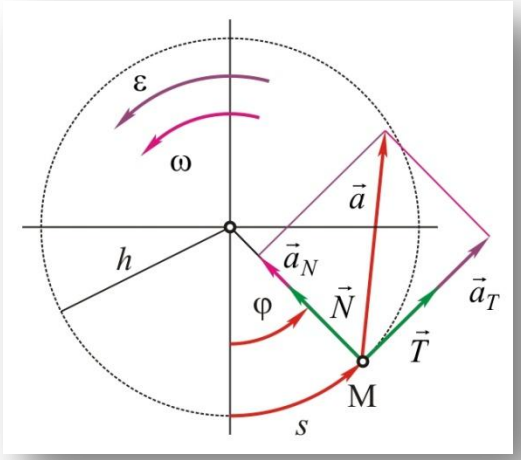
$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \dot{\vec{v}} = \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}}$$

$$\vec{a} = \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times \vec{v}$$

$$\vec{a}_T = \vec{\varepsilon} \times \vec{r}$$
$$\vec{a}_N = \vec{\omega} \times \vec{v}$$





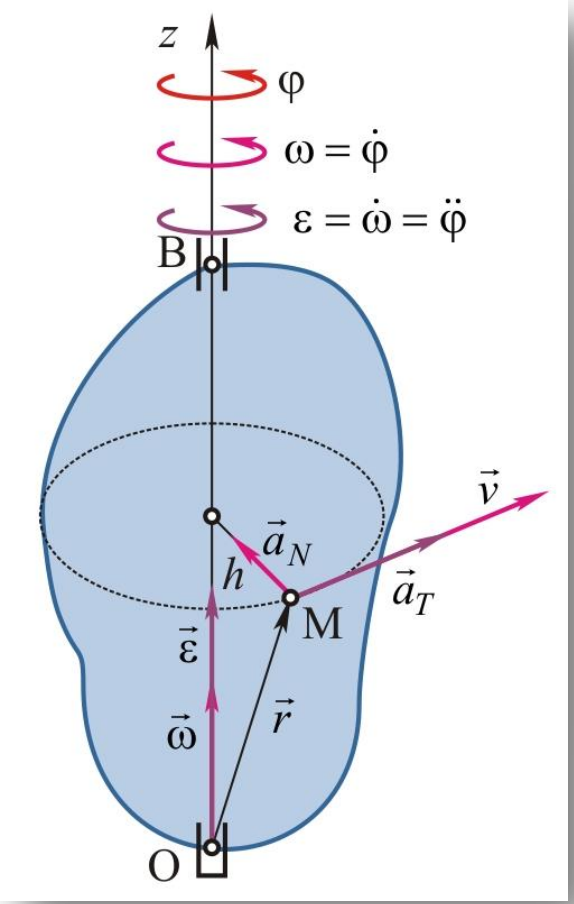
Obrtanja oko nepokretne ose – brzine i ubrzanja tačaka tela

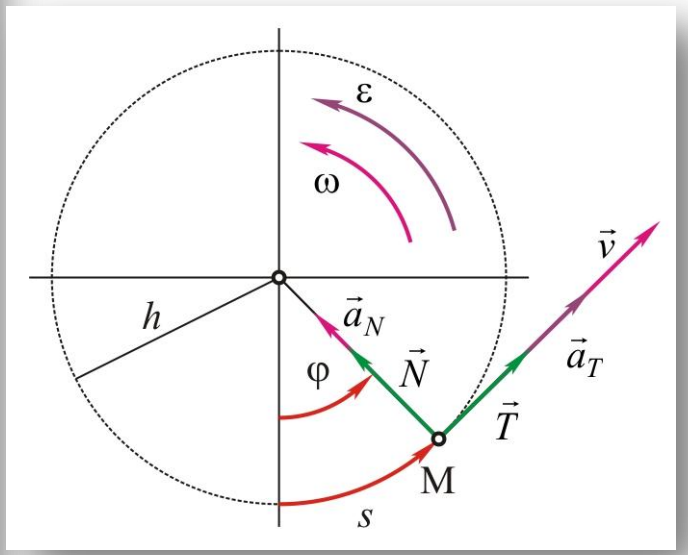
$$\vec{v} = h\omega\vec{T}$$

$$\vec{a} = h\varepsilon\vec{T} + h\omega^2\vec{N}$$

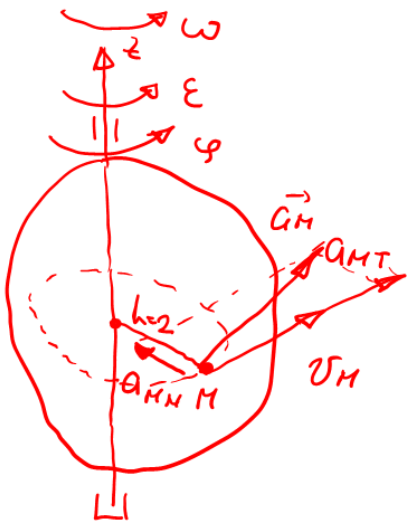
$$\vec{v} = \vec{\omega}\times\vec{r}$$

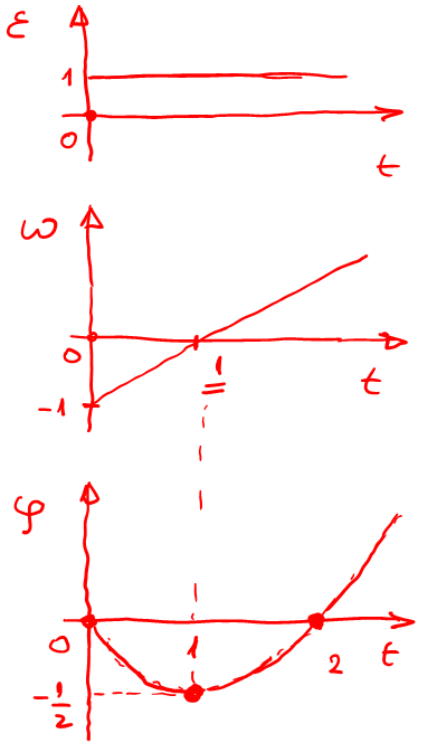
$$\vec{a} = \vec{\varepsilon}\times\vec{r} + \vec{\omega}\times\vec{v}$$





Primer





$\varphi(t) = \frac{t^2}{2} - t = t\left(\frac{t}{2} - 1\right)$

$\omega(t) = \dot{\varphi}(t) = t - 1$

$\varepsilon(t) = \dot{\omega}(t) = \ddot{\varphi}(t) = 1 = \text{const}$

$\varphi(1) = 1 \cdot \left(\frac{1}{2} - 1\right) = -\frac{1}{2}$

$t = 2$

$\varphi(2) = 0$

$\omega(2) = 2 - 1 = 1$

$\varepsilon(2) = 1$

$v_M(2) = h\,\omega(2) = 2 \cdot 1 = 2$

$a_{MT}(2) = h\,\varepsilon(2) = 2 \cdot 1 = 2$

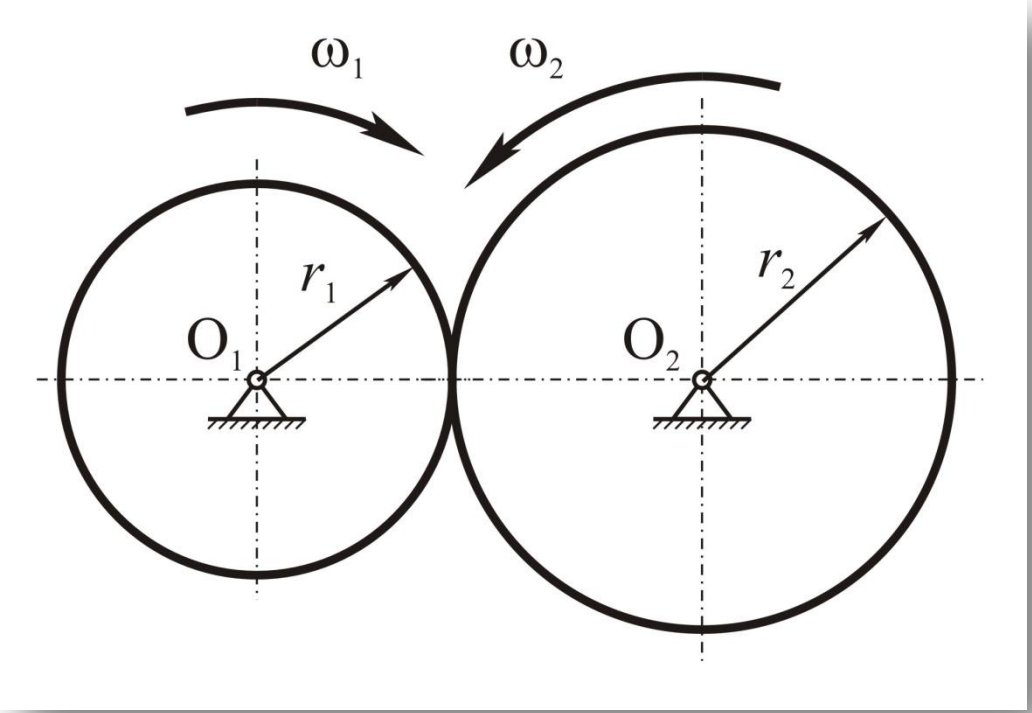
$a_{MN}(2) = h\,\omega^2(2) = 2 \cdot 1^2 = 2$

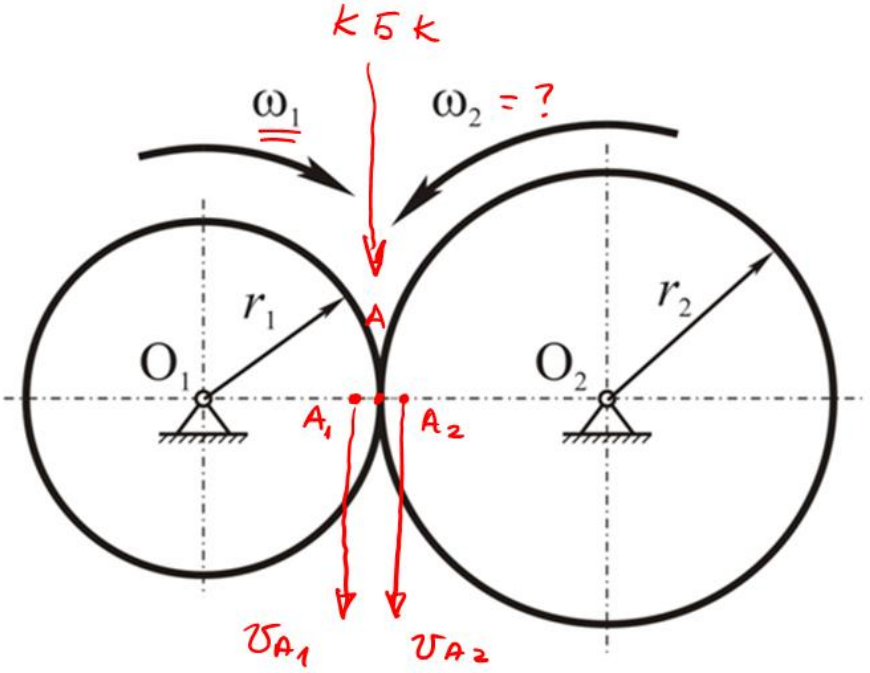
$a_M(2) = \sqrt{a_{MT}^2(2) + a_{MN}^2(2)}$

$= \sqrt{2^2 + 2^2} = 2\sqrt{2}$

Primer

Ugaona brzina pogonskog zupčanika, poluprečnika r_1, iznosi ω_1. Odrediti ugaonu brzinu ω_2 gonjenog zupčanika, poluprečnika r_2.





КБК → КОТРЉАЊЕ БЕЗ КЛИЗАЊА

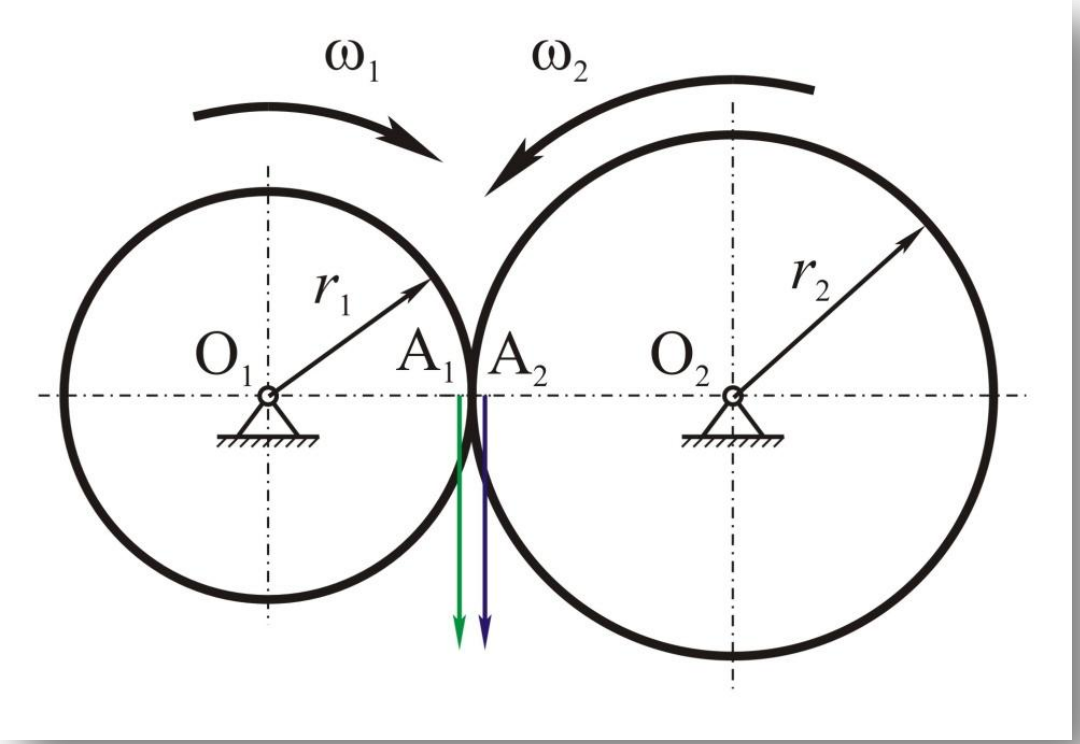
$v_{A1} = r_1 \omega_1$

$v_{A2} = r_2 \omega_2$

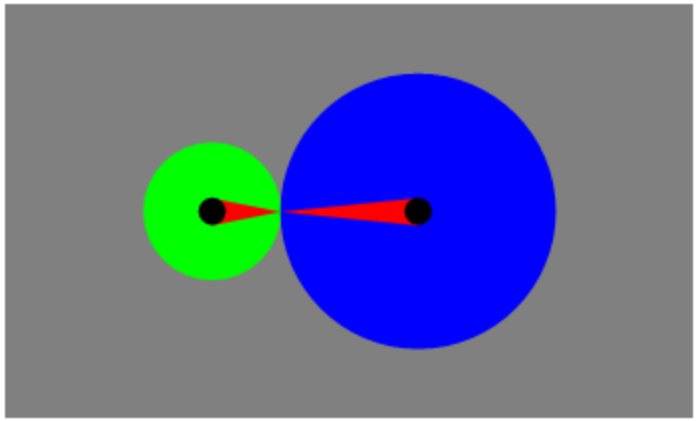
КБК → $v_{A1} = v_{A2} \rightarrow r_1 \omega_1 = r_2 \omega_2$

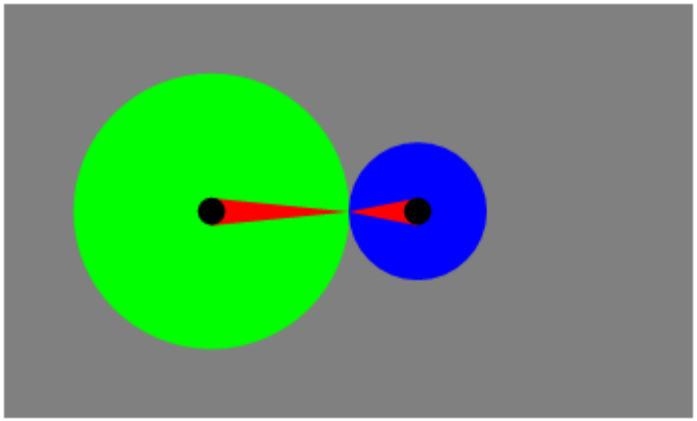
$* \Big| \quad \omega_2 = \frac{r_1}{r_2} \omega_1 = \frac{\omega_1}{\frac{r_2}{r_1}} = \frac{\omega_1}{i} \quad \Big| \qquad i = \frac{r_2}{r_1}$

$* / \frac{d}{dt} \rightarrow \varepsilon_2 = \frac{r_1}{r_2} \varepsilon_1 = \frac{\varepsilon_1}{i}$

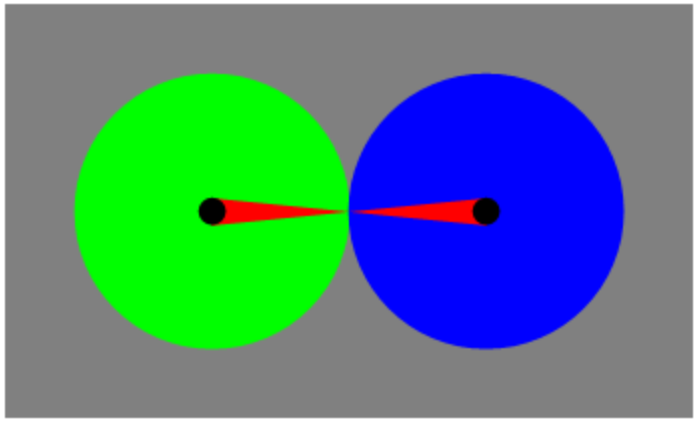


$$v_{A_1} = v_{A_2} \rightarrow r_1\omega_1 = r_2\omega_2 \rightarrow \omega_2 = \frac{r_1}{r_2}\omega_1$$





$$r_2 = 2r_1$$
$$\omega_2 = \frac{1}{2}\omega_1$$

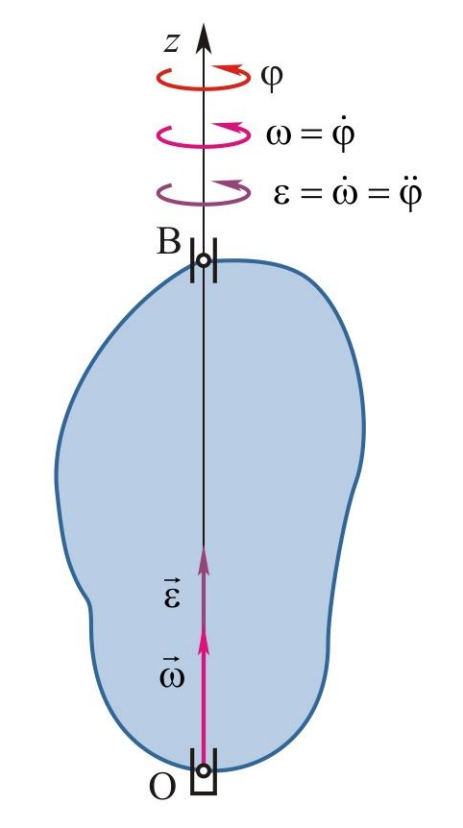


$$r_2 = \frac{r_1}{2}$$
$$\omega_2 = 2\omega_1$$

$$r_2 = r_1$$
$$\omega_2 = \omega_1$$

12. Ravnomerno i ravnomerno promenljivo obrtanje krutog tela oko nepokretne ose

Ravnomerno obrtanje krutog tela oko nepokretne ose

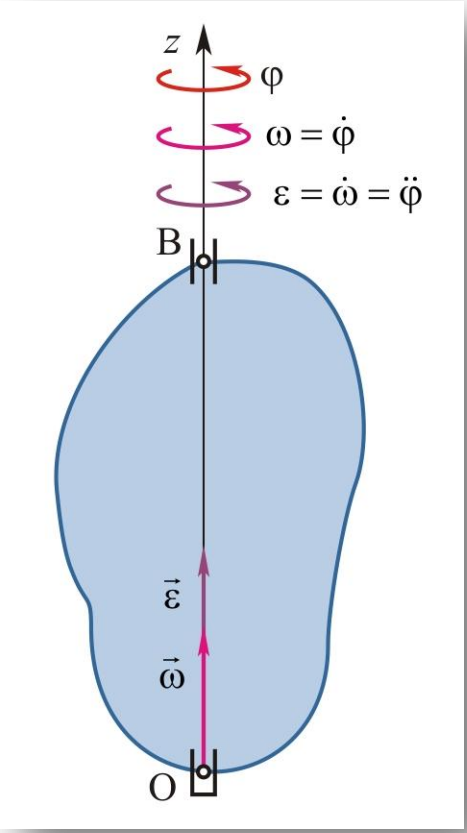


$$\omega = \dot{\varphi} = \omega_0 = const \quad \Rightarrow \quad \varepsilon = \dot{\omega} = \ddot{\varphi} = 0$$

$$\frac{d\varphi}{dt} = \omega_0 \quad \rightarrow \quad d\varphi = \omega_0 dt \quad \rightarrow \quad \int_{\varphi_0}^{\varphi} d\varphi = \int_{t_0=0}^{t} \omega_0 dt$$

$$\varphi = \omega_0 t + \varphi_0$$

Ravnomerno promenljivo obrtanje krutog tela oko nepokretne ose



$$\varepsilon = \dot{\omega} = \ddot{\varphi} = \varepsilon_0 = const$$

$$\frac{d\dot{\varphi}}{dt} = \varepsilon_0 \quad \rightarrow \quad \int_{\omega_0}^{\dot{\varphi}} d\dot{\varphi} = \varepsilon_0 \int_{t_0=0}^{t} dt \quad \rightarrow \quad \dot{\varphi} = \varepsilon_0 t + \omega_0$$

$$\frac{d\varphi}{dt} = \varepsilon_0 t + \omega_0 \quad \rightarrow \quad \int_{\varphi_0}^{\varphi} d\varphi = \int_{t_0=0}^{t} (\varepsilon_0 t + \omega_0) dt \quad \rightarrow \quad \varphi = \varepsilon_0 \frac{t^2}{2} + \omega_0 t + \varphi_0$$

Šta smo naučili?

9. **Kinematika translatornog kretanja krutog tela**
10. **Kinematika obrtanja krutog tela oko nepokretne ose (zakon kretanja, ugaona brzina i ugaono ubrzanje)**
11. **Kinematika obrtanja krutog tela oko nepokretne ose (brzina i ubrzanje tačaka tela)**
12. **Ravnomerno i ravnomerno promenljivo obrtanje krutog tela oko nepokretne ose**

Mehanika 2 (Kinematika)
Predavanja 4

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