Dinamika
Dinamika materijalne tačke – Dinamika relativnog kretanja materijalne tačke,…

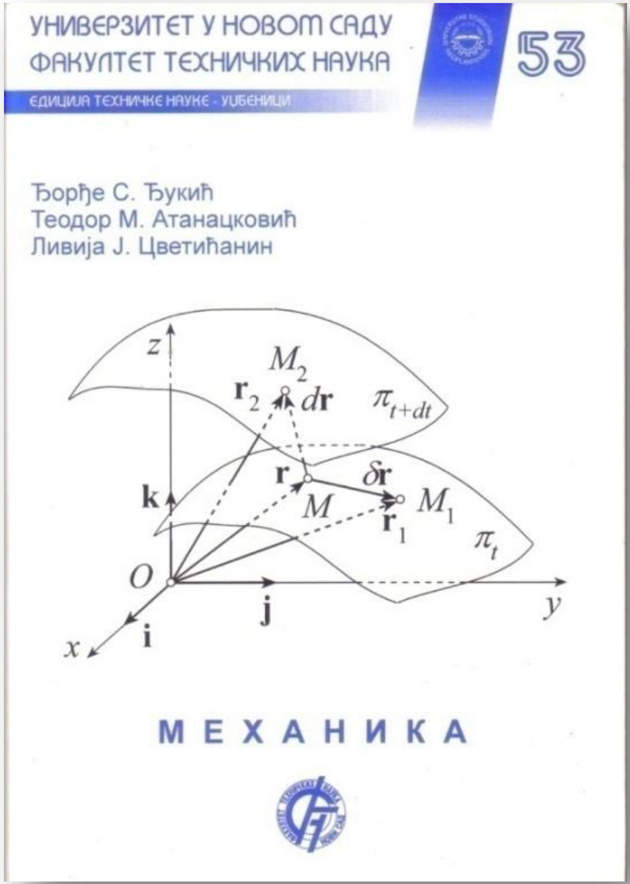
Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.

Literatura

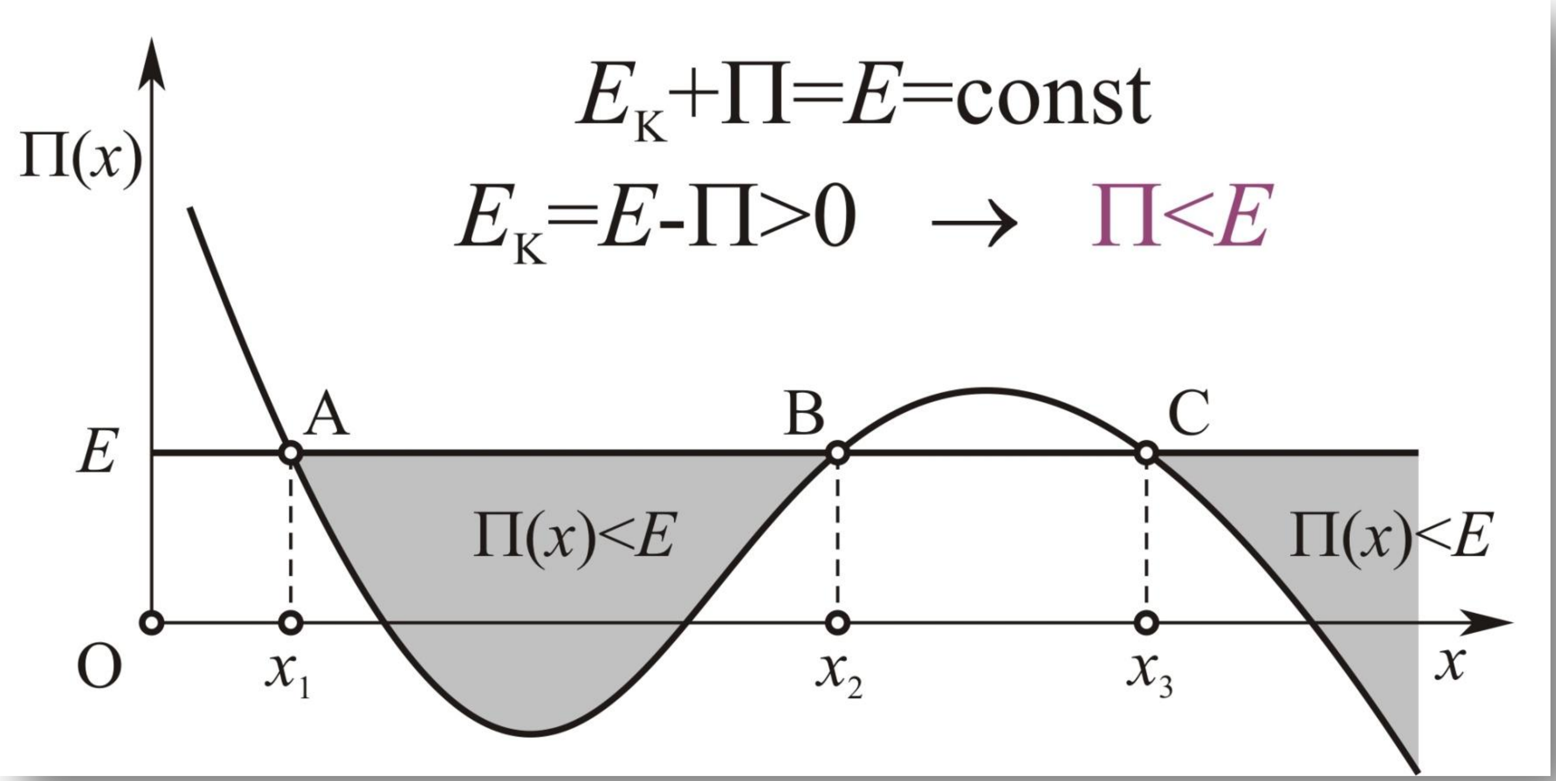
- Đorđe S. Đukić, Teodor M. Atanacković, Livija J. Cvetićanin: Mehanika, Fakultet tehničkih nauka u Novom Sadu, Novi Sad, 2003.



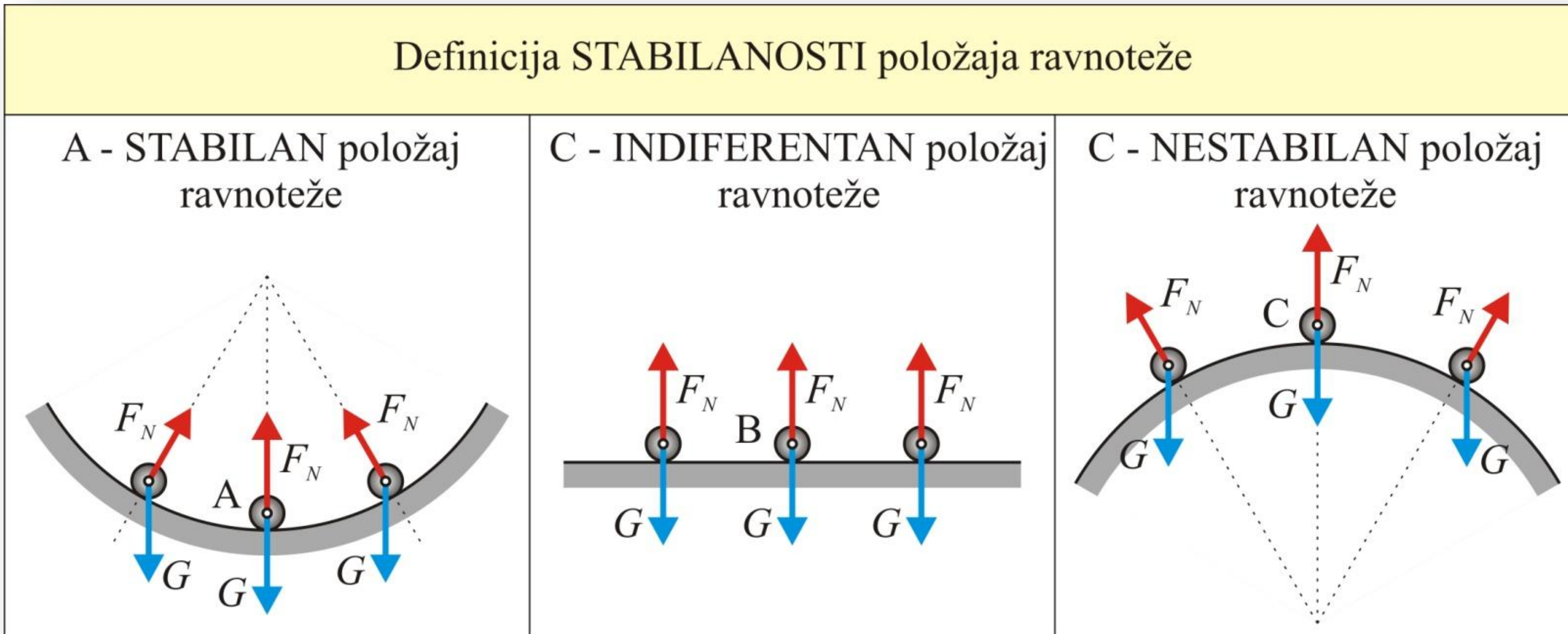
Šta ćemo naučiti?

27. **Stabilnost ravnotežnog položaja. Teorema o stabilnosti i nestabilnosti**
28. **Kretanje po liniji. Matematičko klatno**
29. **Dinamika relativnog kretanje materijalne tačke**

Dijagram potencijalne energije tačke



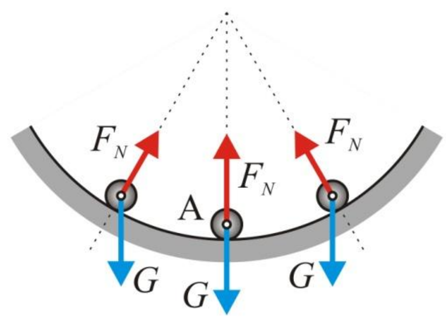
27. Stabilnost ravnotežnog položaja…



Stabilnost ravnotežnog položaja…

Dirihleova teorema o STABILANOSTI položaja ravnoteže

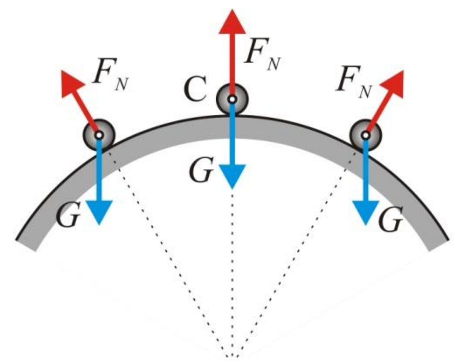
A - STABILAN položaj ravnoteže



U položaju **stabilne ravnoteže** (x_r) potencijana energija ima **minimum**

$$\left(\frac{d^2\Pi}{dx^2}\right)_{x_r} > 0$$

C - NESTABILAN položaj ravnoteže



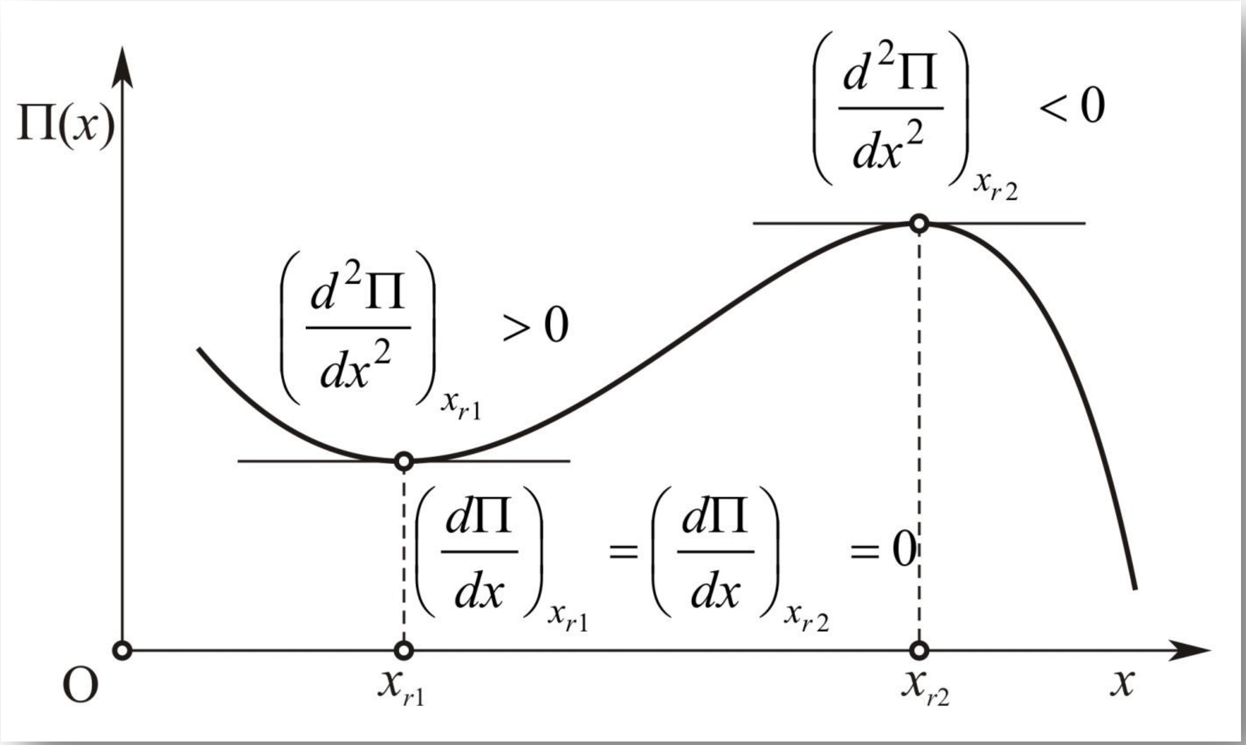
U položaju **nestabilne ravnoteže** (x_r) potencijana energija ima **maksimum**

$$\left(\frac{d^2\Pi}{dx^2}\right)_{x_r} < 0$$

U položaju **ravnoteže** (x_r) potencijana energija ima ekstremnu vrednost

$$\left(\frac{d\Pi}{dx}\right)_{x_r} = 0$$

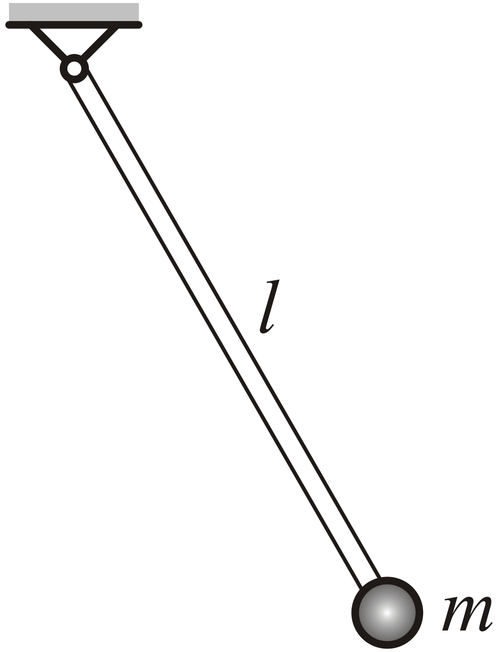
Stabilnost ravnotežnog položaja…



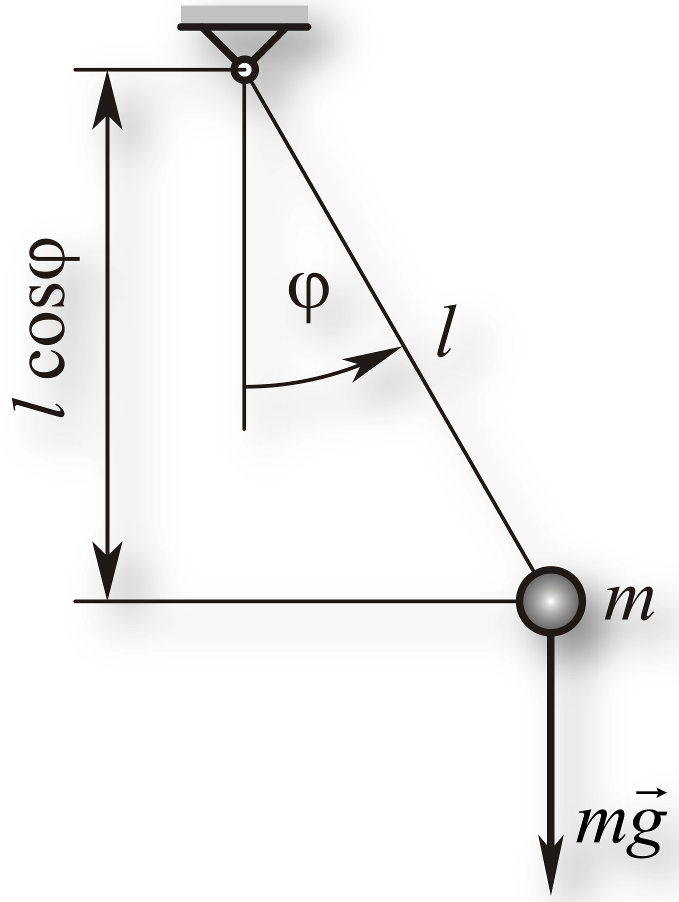
Stabilnost ravnotežnog položaja…

- Primer

  Odrediti položaje stabilnosti matematičkog klatna i ispitati njihovu stabilnost.



Primer…



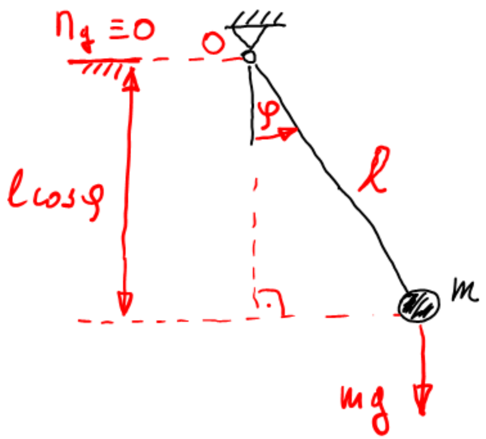
$$\Pi = \Pi(\varphi) = -mgl\cos\varphi$$

$$\frac{d\Pi}{d\varphi} = mg\sin\varphi = 0 \rightarrow \begin{cases} \varphi = 0 \\ \varphi = \pi \end{cases}$$

$$\frac{d^2\Pi}{d\varphi^2} = mg\cos\varphi$$

$$\left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=0} = mg > 0$$

$$\left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=\pi} = -mg < 0$$



$$\Pi = \Pi(\varphi) = -mg \cdot l \cos\varphi$$

$$\frac{d\Pi}{d\varphi} = +mgl\sin\varphi \; ; \quad \frac{d^2\Pi}{d\varphi^2} = mgl\cos\varphi$$

ПОЛ. РАВ. $\left.\frac{d\Pi}{d\varphi}\right|_{\varphi=\varphi_r} = 0$

$$\left.\frac{d\Pi}{d\varphi}\right|_{\varphi=\varphi_r} = mgl\sin\varphi_r = 0 \rightarrow \sin\varphi_r = 0$$

1° $\varphi_{r_1} = 0$

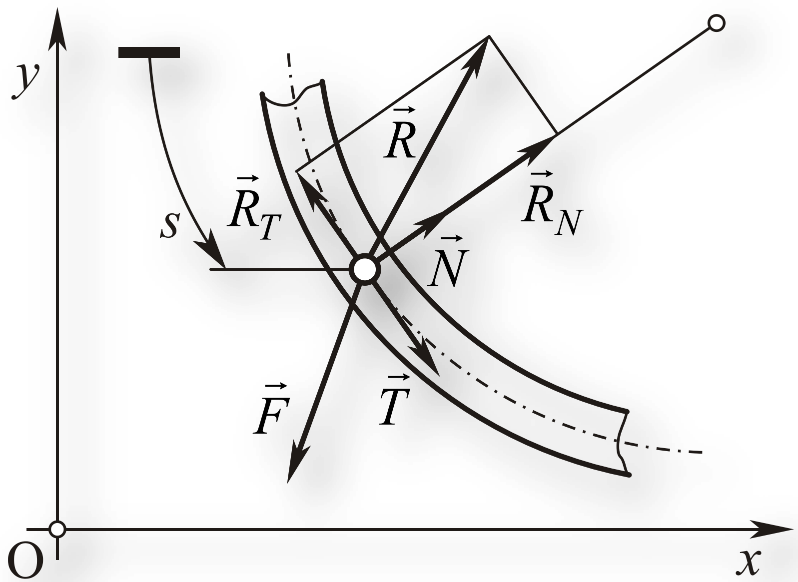
2° $\varphi_{r_2} = \pi$

СТАБИЛНОСТ

$$\left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=\varphi_{r_1}=0} = mgl\underbrace{\cos 0}_{1} = mgl > 0 \rightarrow \Pi_{min} \rightarrow$$ СТАБИЛНА РАВН.

$$\left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=\varphi_{r_2}=\pi} = mgl\underbrace{\cos\pi}_{-1} = -mgl < 0 \rightarrow \Pi_{max} \rightarrow$$ НЕСТАБ. РАВН.

28. Kretanje tačke po liniji...



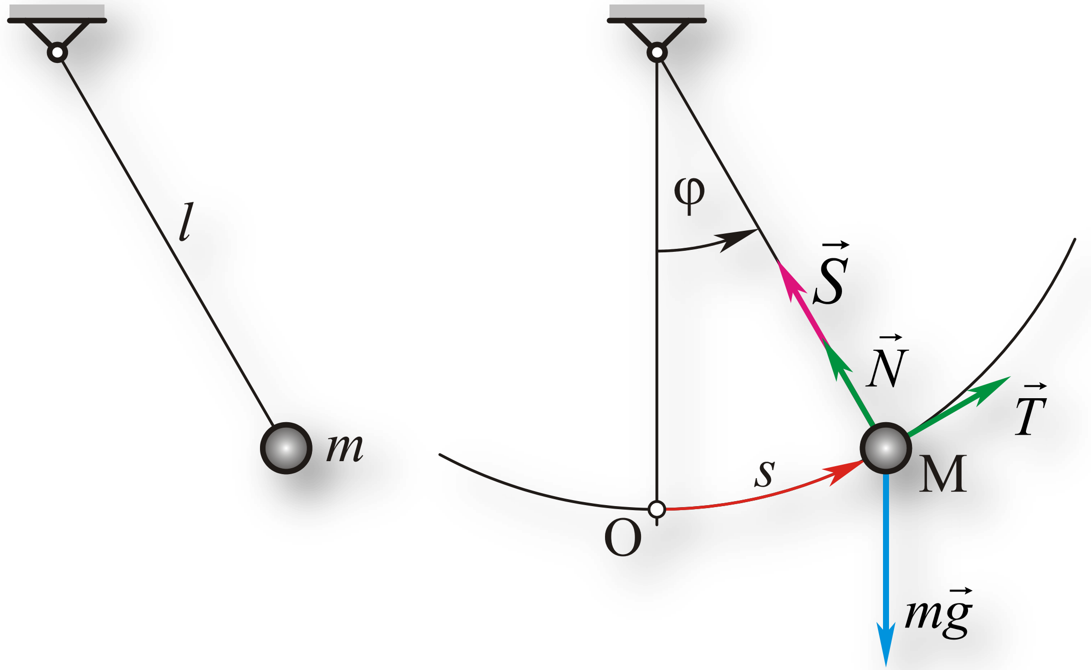
$$m\ddot{s} = F_T + R_T$$

$$m\frac{\dot{s}^2}{R_k} = F_N + R_N$$

$$R_T = 0$$

$$m\ddot{s} = F_T$$

$$m\frac{\dot{s}^2}{R_k} = F_N + R_N$$



Jednačine kretenja:

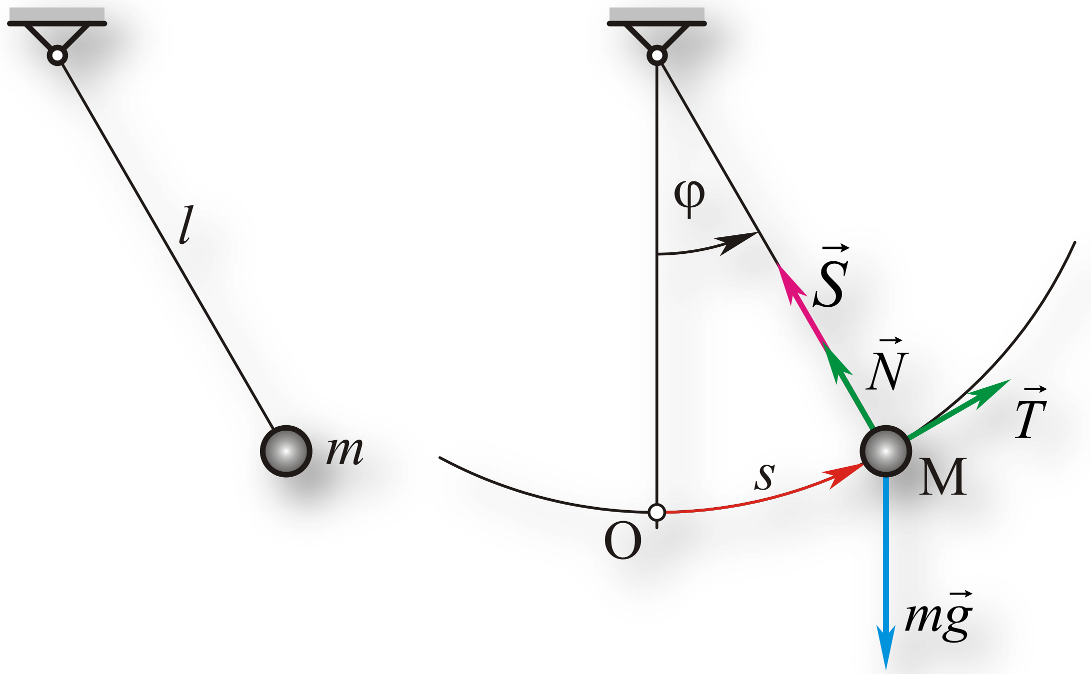
$$m\ddot{s} = -mg\sin\varphi$$

$$m\frac{\dot{s}^2}{l} = -mg\cos\varphi + S$$

$$s = l\varphi$$

$$ml\ddot{\varphi} = -mg\sin\varphi$$

$$\ddot{\varphi} + \frac{g}{l}\sin\varphi = 0$$

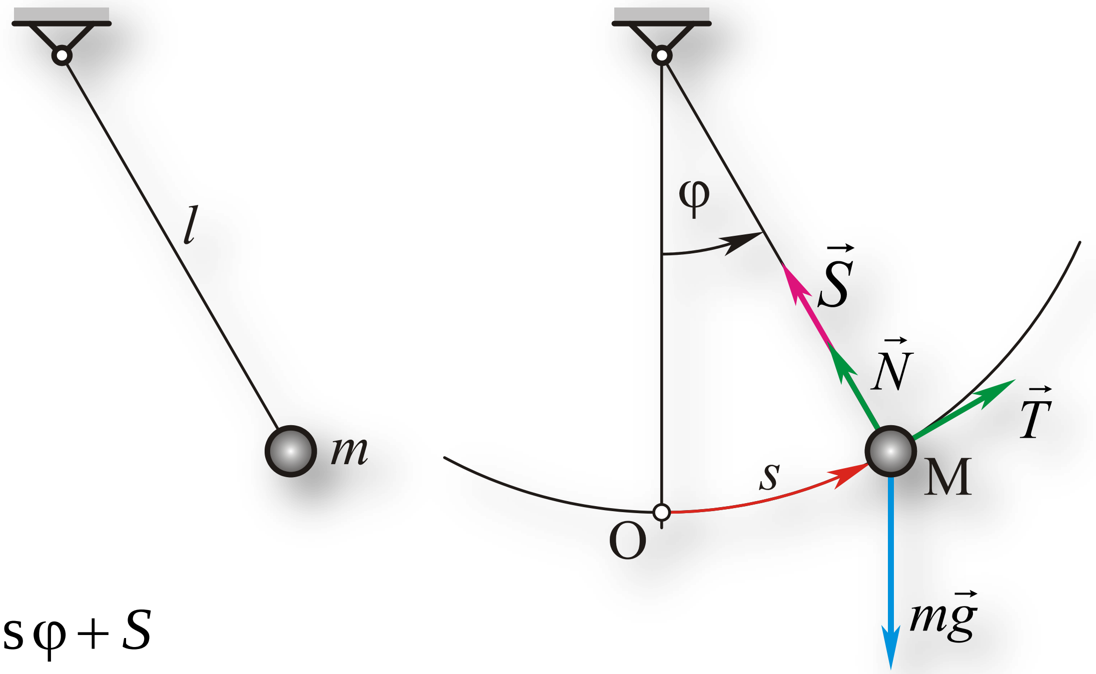


Zakon održanja mehaničke energije:

$$E_K + \Pi = E_{K0} + \Pi_0$$

$$\frac{1}{2}mv^2 - mgl\cos\varphi = \frac{1}{2}mv_0^2 - mgl\cos\varphi_0$$

$$v^2 = v_0^2 - mgl\left(\cos\varphi_0 - \cos\varphi\right)$$

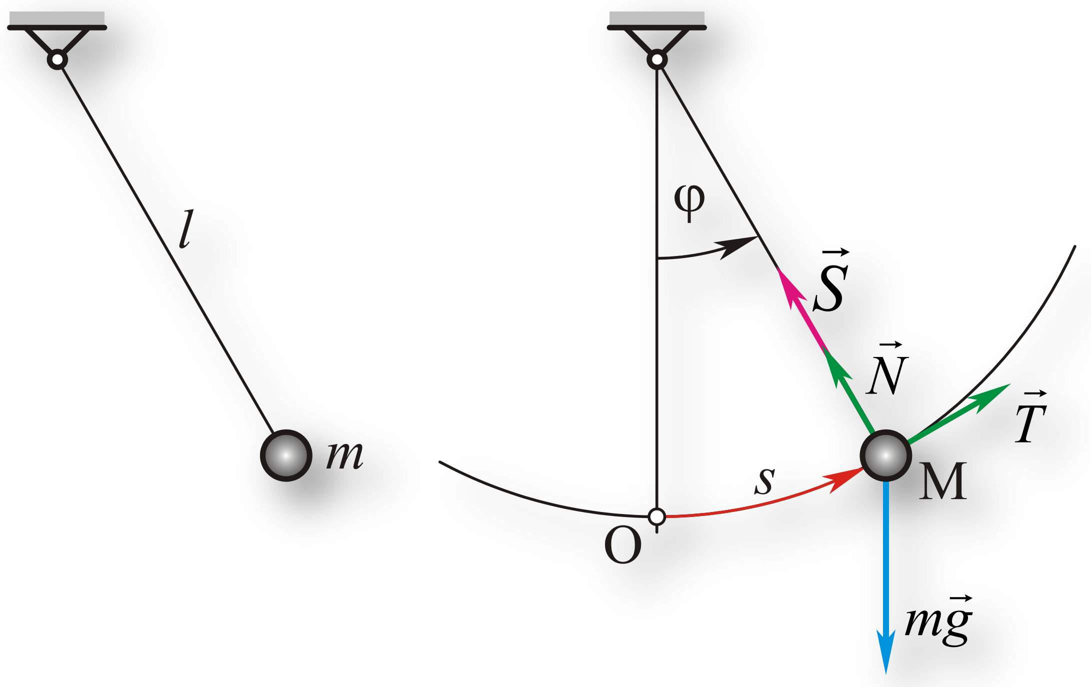


$$m\frac{\dot{s}^2}{l} = -mg\cos\varphi + S$$

Reakcija veze:

$$S = mg\cos\varphi + m\frac{\dot{s}^2}{l} = mg\cos\varphi + m\frac{v^2}{l}$$

$$v^2 = v_0^2 - mgl(\cos\varphi_0 - \cos\varphi)$$



Male (linearne) oscilacije:

$$\varphi << 1 \rightarrow \sin\varphi \approx \varphi$$

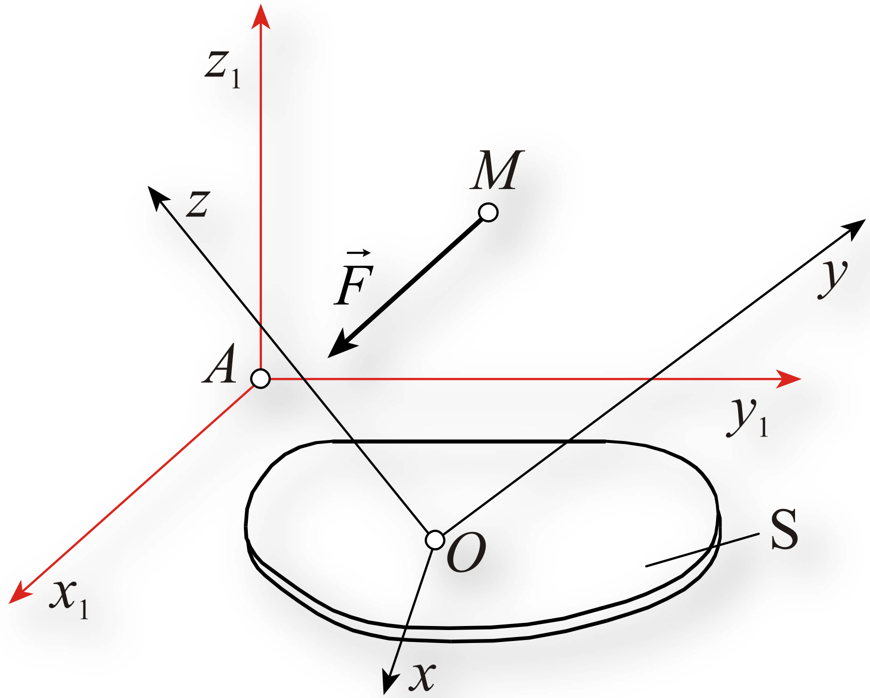
$$\ddot{\varphi} + \omega^2 \varphi = 0$$

$$\omega^2 = \frac{g}{l}$$

$$\varphi(t) = A\cos(\omega t + \alpha)$$

29. Dinamika relativnog kretanje materijalne tačke

$$m\vec{a} = \vec{F}$$



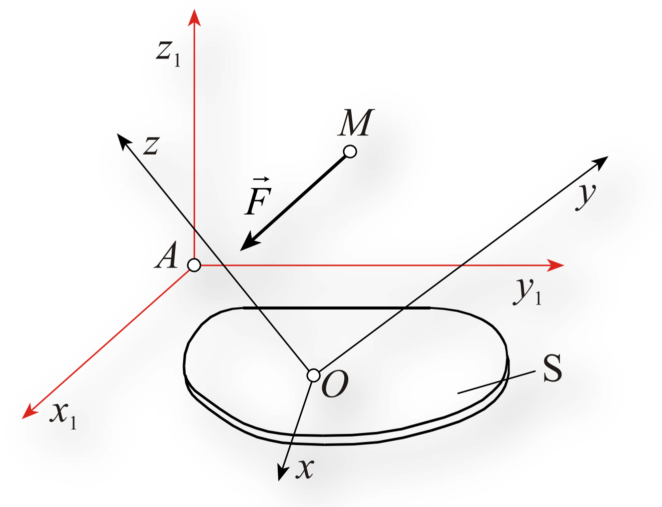
$$m\vec{a} = \vec{F}$$

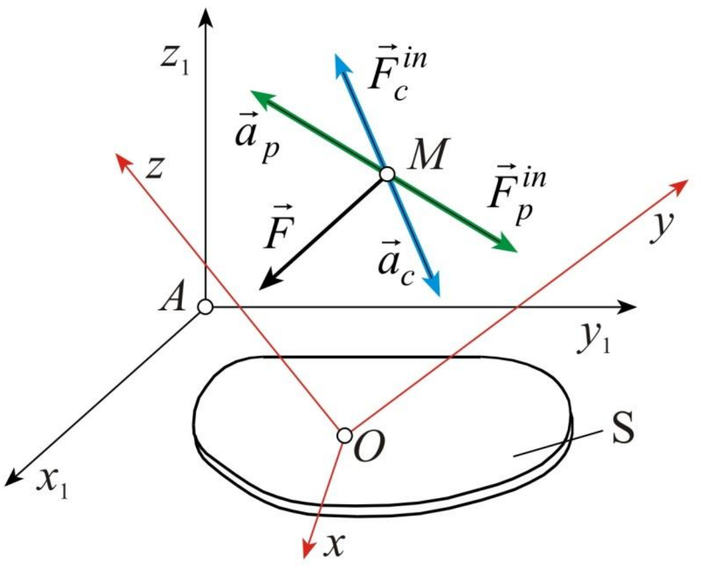
$$m(\vec{a}_p + \vec{a}_r + \vec{a}_c) = \vec{F}$$

$$m\vec{a}_r = \vec{F} + (-m\vec{a}_p) + (-m\vec{a}_c)$$

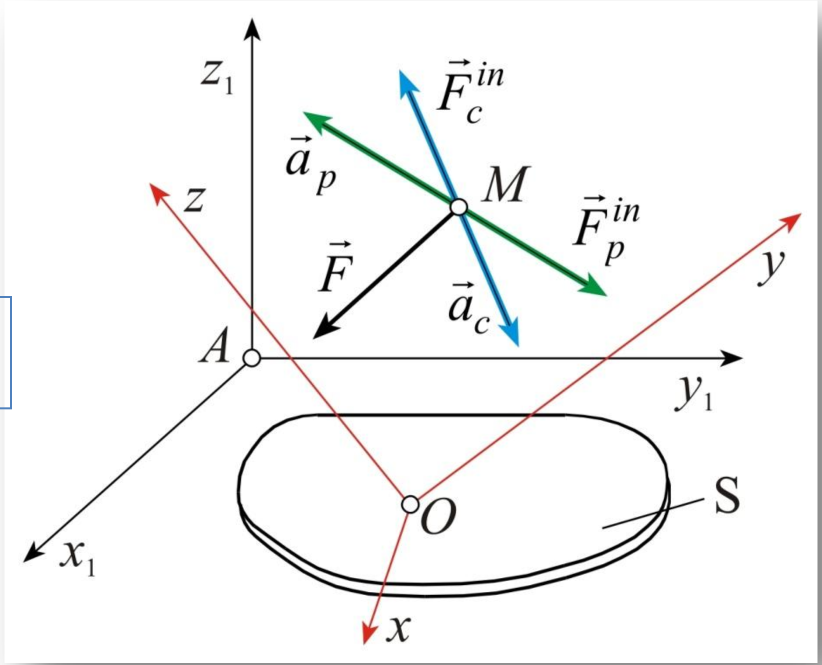
$$m\vec{a}_r = \vec{F} + \vec{F}^{in}_p + \vec{F}^{in}_c$$

$$\vec{F}^{in}_p = -m\vec{a}_p, \quad \vec{F}^{in}_c = -m\vec{a}_c$$





Specijalni slučajevi



1. Pokretni koordinatni sistem Oxyz
se kreće translatorno

$$\vec{\omega}_p = 0 \rightarrow \vec{F}_c^{in} = -m\vec{a}_c = -m\left(2\left(\vec{\omega}_p \times \vec{v}_r\right)\right) = 0$$

$$m\vec{a}_r = \vec{F} + \vec{F}_p^{in}$$

2. Pokretni koordinatni sistem Oxyz se
kreće translatorno pravolinijski ravnomero

$$\vec{a}_p = 0 \rightarrow \vec{F}_p^{in} = -m\vec{a}_p = 0$$

$$m\vec{a}_r = \vec{F}$$

3. Relativna ravnoteža

$$\vec{v}_r = \vec{a}_r = 0, \left(\vec{a}_c = 0\right)$$

$$\vec{F} + \vec{F}_p^{in} = 0$$

Zakon o promeni kinetičke energije pri relativnom kretanju tačke

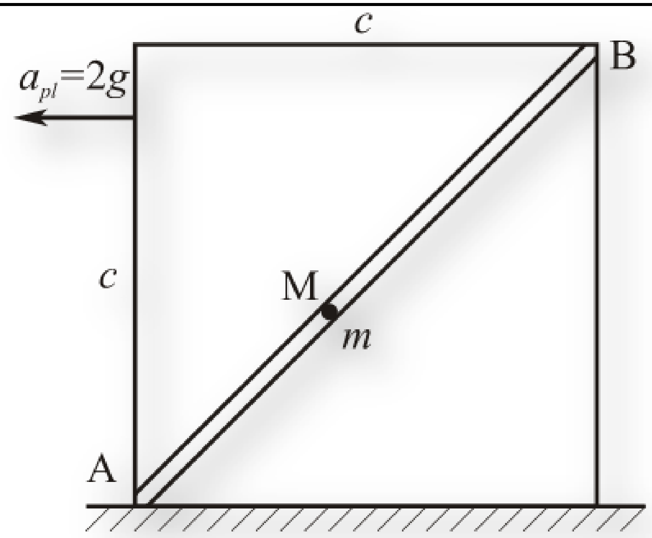
$$dE_{Krel} = dA_{rel}^{\vec{F}} + dA_{rel}^{\vec{F}_p^{in}}$$

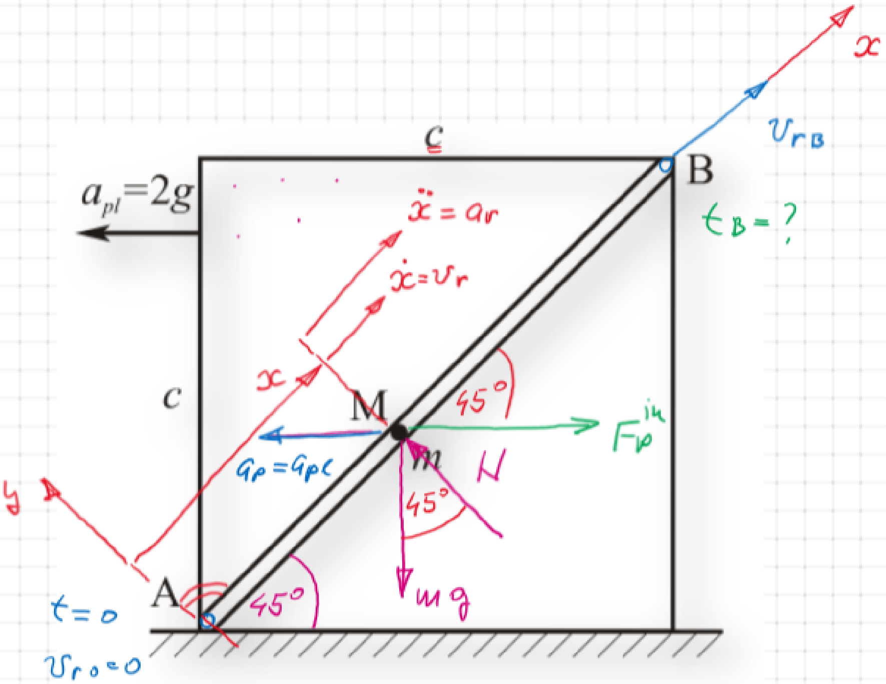
$$E_{Krel1} - E_{Krel0} = A_{rel01}^{\vec{F}} + A_{rel01}^{\vec{F}_p^{in}}$$

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = A_{rel01}^{\vec{F}} + A_{rel01}^{\vec{F}_p^{in}}$$

Primer

Duž žleba u pravcu dijagonale kvadratne ploče, koja se kreće translatorno pravolinijski konstantnim ubrzanjem $a_{pl} = 2g = const$, kreće se materijalna tačka M, mase m. Odrediti relativnu brzinu tačke, u odnosu na ploču, u položaju B. Tačka započinje kretanje bez relativne brzine iz položaja A. Sve otpore kretanju tačke zanemariti.





$$m\vec{a}_r = \vec{F} + \vec{F}_p^{in} + \cancel{\vec{F}_c^{in}}^{\,0}$$

ПРЕН. КР. – ПЛОЧА – ТРАНСЛ. → $\vec{\omega}_p = 0$ →

→ $\vec{a}_c = 2\vec{\omega}_p \times \vec{v}_r = 0$

→ $\vec{F}_c^{in} = -m\vec{a}_c = 0$

РЕЛ. КР. → ПРАВОЛ.

$$m\vec{a}_r = m\vec{g} + \vec{N} + \vec{F}_p^{in} \quad /\cdot\vec{i} \quad /\cdot\vec{j}$$

$$\vec{F}_p^{in} = -m\vec{a}_p = -m\vec{a}_{pl}$$

$$F_p^{in} = m a_p = m a_{pl} = m \cdot 2g$$

$$(1) \quad m\ddot{x} = -mg\sin 45^\circ + F_p^{in}\cdot\cos 45^\circ$$

$$(2) \quad 0 = -mg\cos 45^\circ + N - F_p^{in}\sin 45^\circ$$

$$(2) \rightarrow N = mg\cdot\frac{\sqrt{2}}{2} + F_p^{in}\frac{\sqrt{2}}{2} = mg\frac{\sqrt{2}}{2} + 2mg\cdot\frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}mg$$

$$(1) \quad m\ddot{x} = -mg\frac{\sqrt{2}}{2} + 2mg\cdot\frac{\sqrt{2}}{2} \rightarrow \ddot{x} = g\frac{\sqrt{2}}{2} = const$$

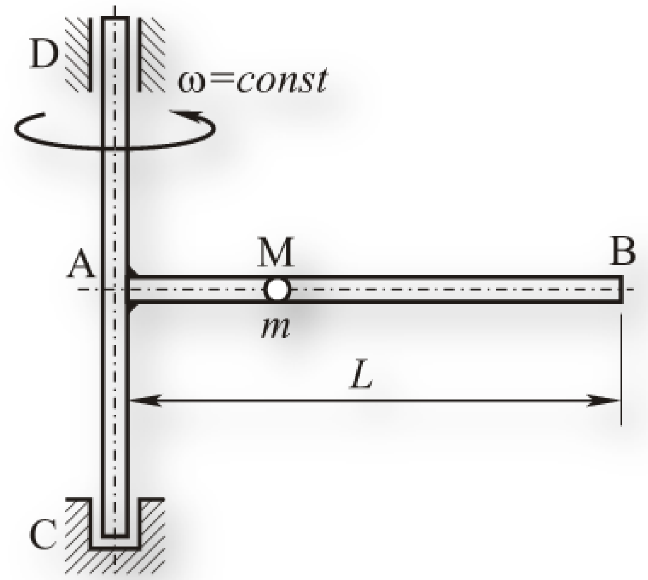
$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{dx}{dt}\frac{d\dot{x}}{dx} = \dot{x}\frac{d\dot{x}}{dx} \qquad \dot{x}\frac{d\dot{x}}{dx} = g\frac{\sqrt{2}}{2} \rightarrow$$

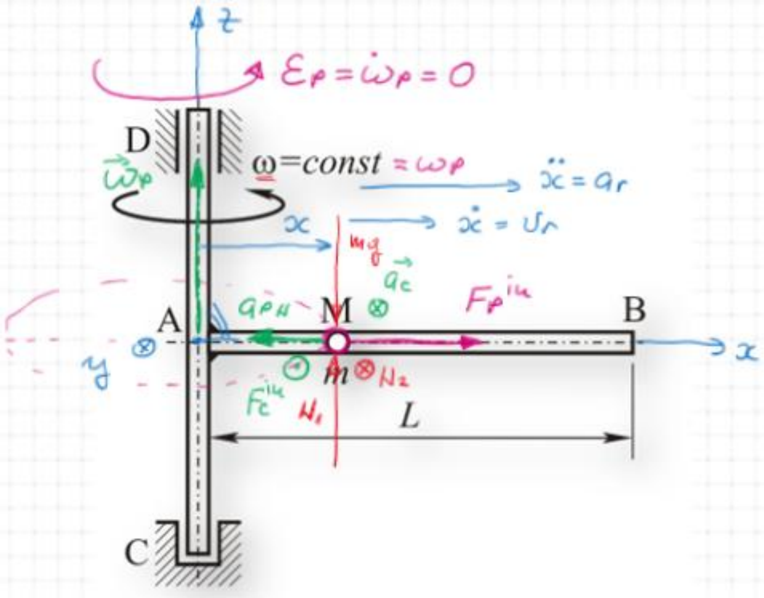
$$\int_{\dot{x}_A = v_{r0} = 0}^{\dot{x}_B = v_{rB}} \dot{x}\,d\dot{x} = g\frac{\sqrt{2}}{2}\int_{x_A = 0}^{x_B = r\sqrt{2}} dx \rightarrow \frac{\dot{x}^2}{2}\Bigg|_0^{v_{rB}} = g\frac{\sqrt{2}}{2}x\Bigg|_0^{r\sqrt{2}} \rightarrow v_{rB}^2 = g\sqrt{2}\cdot r\sqrt{2} = 2gr$$

$$v_{rB} = \sqrt{2gr}$$

Primer

Kuglica, mase m, može da se kreće u horizontalnoj cevi, dužine L, koja se obrće oko vertikalne ose konstantnom ugaonom brzinom $\omega = const$. Kuglica kretanje započinje sa sredine cevi iz stanja mirovanja u odnosu na cev. Kolika je relativna brzina kuglice na izlasku iz cevi.





$$m\vec{a}_r = m\vec{g} + \vec{N}_1 + \vec{N}_2 + \vec{F}_p^{in} + \vec{F}_c^{in}$$

ПРЕН. КР. → ЦЕВ → ОБРТАЊЕ ОКО НЕП. ОСЕ (z) →

→ $TР_{пр} = К\{A, \overline{AM} = x\}$

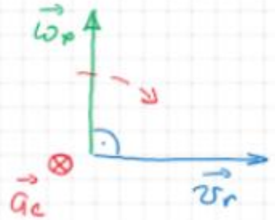
РЕЛ. КР. → ПРАВОЛ.

$$\vec{F}_p^{in} = -m\,\vec{a}_p = -m\,(\vec{a}_{pT} + \vec{a}_{pN}) = -m\,\vec{a}_{pN}$$

$$F_p^{in} = m\,a_p = m\,a_{pN} = m\cdot\overline{AM}\,\omega_p^2 = m\,x\,\omega^2$$

$$a_{pT} = \overline{AM}\,\varepsilon_p = 0$$

$$a_{pN} = \overline{AM}\,\omega_p^2$$



$$\vec{F}_c^{in} = -m\,\vec{a}_c =$$

$$F_c^{in} = m\,a_c = m\cdot 2\omega\dot{x}$$

$$\vec{a}_c = 2\,\vec{\omega}_p \times \vec{v}_r$$

$$a_c = 2\,\omega_p v_r \sin\angle(\vec{\omega}_p, \vec{v}_r) = 2\,\omega\,\dot{x}\,\sin 90^\circ$$

$$\boxed{m\vec{a}_r = m\vec{g} + \vec{N}_1 + \vec{N}_2 + \vec{F}_p^{in} + \vec{F}_c^{in}} \quad / \cdot \vec{i} \; / \cdot \vec{j} \; / \cdot \vec{k}$$

$$(1) \quad \boxed{m\ddot{x} = F_p^{in}}$$

$$(2) \quad 0 = N_2 - F_c^{in} \;\rightarrow\; N_2 = F_c^{in} = m\,2\omega\dot{x}$$

$$(3) \quad 0 = -mg + N_1 \;\rightarrow\; N_1 = mg$$

$$\vec{N} = \vec{N}_1 + \vec{N}_2$$

$$(1) \quad m\ddot{x} = m x\omega^2 \;\rightarrow\; \boxed{\ddot{x} = \omega^2 x}$$

$$\ddot{x} = \dot{x}\frac{d\dot{x}}{dx}$$

$$\boxed{\dot{x}\frac{d\dot{x}}{dx} = \omega^2 x}$$

$$\int_{\dot{x}(0)=v_{r0}=0}^{\dot{x}_B = v_{rB}} \dot{x}\,d\dot{x} = \omega^2 \int_{x(0)=\frac{L}{2}}^{x_B = L} x\,dx \;\rightarrow\; \frac{\dot{x}^2}{2}\Bigg|_0^{v_{rB}} = \omega^2 \frac{x^2}{2}\Bigg|_{\frac{L}{2}}^{L}$$

$$\boxed{v_{rB}^2 = \omega^2\left(L^2 - \left(\frac{L}{2}\right)^2\right)} \; \ldots \quad v_{rB} = \sqrt{}$$

Primer

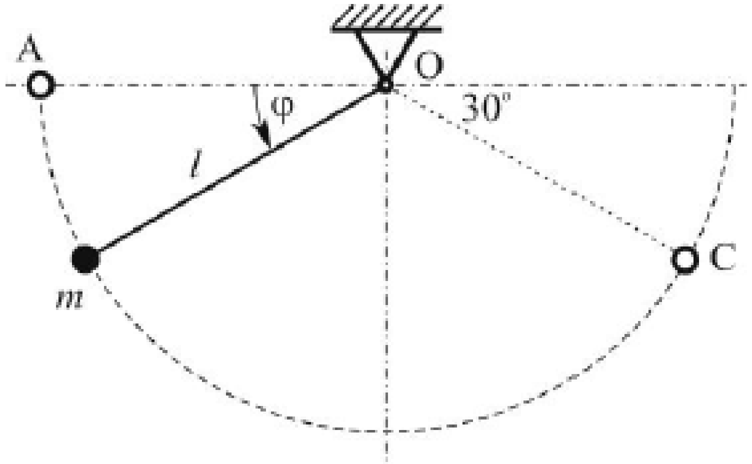
Matematičko klatno, materijalna tačka mase m obešena o nepokretnu tačku neistegljivim užetom dužine l, započinje kretanje iz položaja A bez početne brzine.

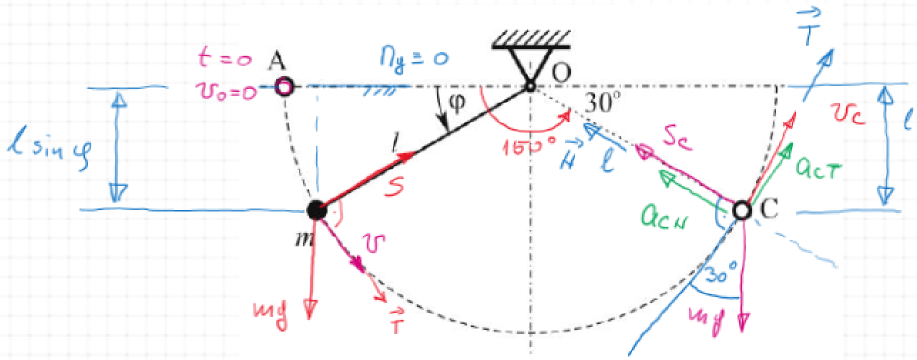
Odrediti:

a) brzinu materijalne tačke u funkciji ugla φ,

b) brzinu tačke u položaju C,

c) silu zatezanja užeta u položaju C.





$$E_K + \Pi = E_{K0} + \Pi_0 \quad (E_{K0} = 0,\ \Pi_0 = 0)$$

$$\frac{1}{2} m v^2 - m g l \sin\varphi = 0$$

$$v^2 = 2 g l \sin\varphi$$

$$C \rightarrow \varphi = 150^\circ = 180^\circ - 30^\circ$$

$$v_C^2 = 2 g l \sin 150^\circ \quad (\sin 150^\circ = \sin 30^\circ = \tfrac{1}{2})$$

$$v_C^2 = g l \rightarrow v_C = \sqrt{g l}$$

$$E_{KC} + \Pi_C = E_{K0} + \Pi_0 \quad (E_{K0} = 0,\ \Pi_0 = 0)$$

$$\frac{1}{2} m v_C^2 - m g l \sin 30^\circ = 0$$

$$v_C^2 = 2 g l \sin 30^\circ \quad (\sin 30^\circ = 1/2)$$

$$v_C^2 = g l$$

$$S_C = ?$$

$$\text{II} \quad m\vec{a} = m\vec{g} + \vec{S}$$

$$\boxed{m\vec{a}_C = m\vec{g} + \vec{S}_C}$$

$$m(\vec{a}_{CT} + \vec{a}_{CN}) = m\vec{g} + \vec{S}_C \quad / \cdot \vec{N}$$

$$m a_{CN} = -m g \sin 30^\circ + S_C$$

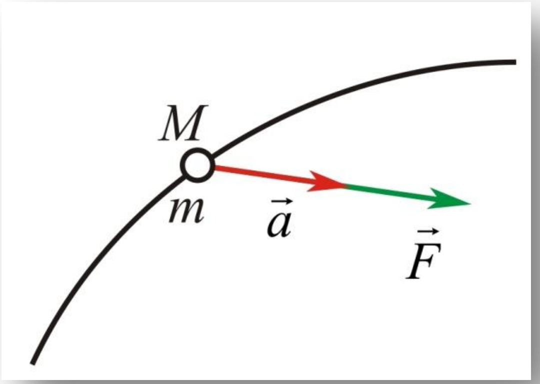
$$S_C = m g \sin 30^\circ + m a_{CN} \quad (\sin 30^\circ = \tfrac{1}{2})$$

$$S_C = \frac{mg}{2} + m \cdot \frac{v_C^2}{l} = \frac{mg}{2} + m \cdot \frac{g l}{l} = \frac{3}{2} m g$$

Dalamberov princip

- Jednačine dinamike se pišu u obliku jednačina statike

$$m\vec{a}=\vec{F}\rightarrow\vec{F}+(-m\vec{a})=0$$

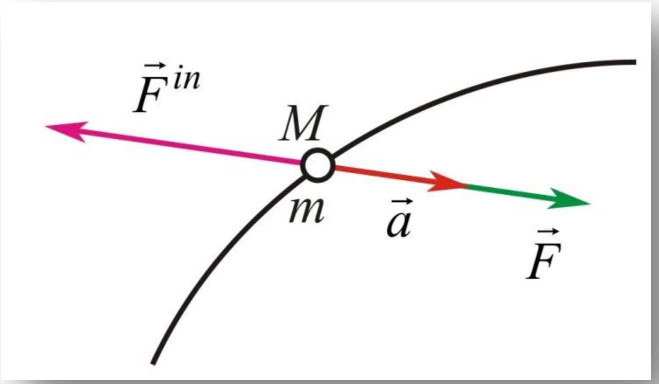


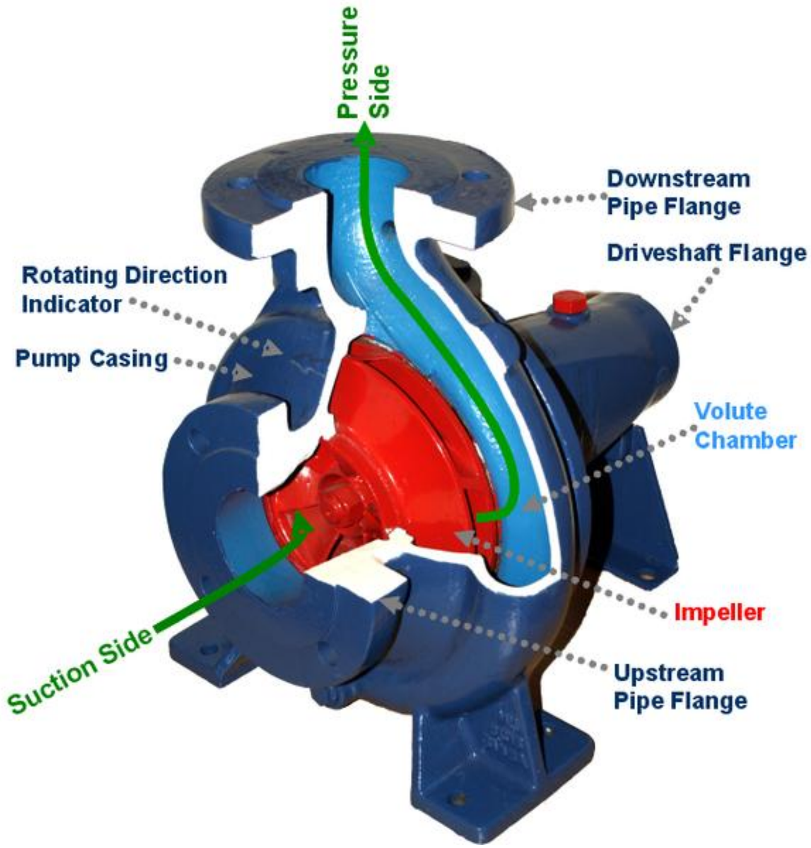
- Inercijalna sila

$$\vec{F}^{in}=-m\vec{a}$$

- Dalamberov princip

$$\vec{F}+\vec{F}^{in}=0$$




Pressure Side
Downstream Pipe Flange
Driveshaft Flange
Rotating Direction Indicator
Pump Casing
Volute Chamber
Impeller
Suction Side
Upstream Pipe Flange


Logar

Šta smo naučili?

27. **Stabilnost ravnotežnog položaja. Teorema o stabilnosti i nestabilnosti**
28. **Kretanje po liniji. Matematičko klatno**
29. **Dinamika relativnog kretanje materijalne tačke**

Dinamika
Dinamika materijalne tačke – Dinamika relativnog kretanja materijalne tačke,…

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