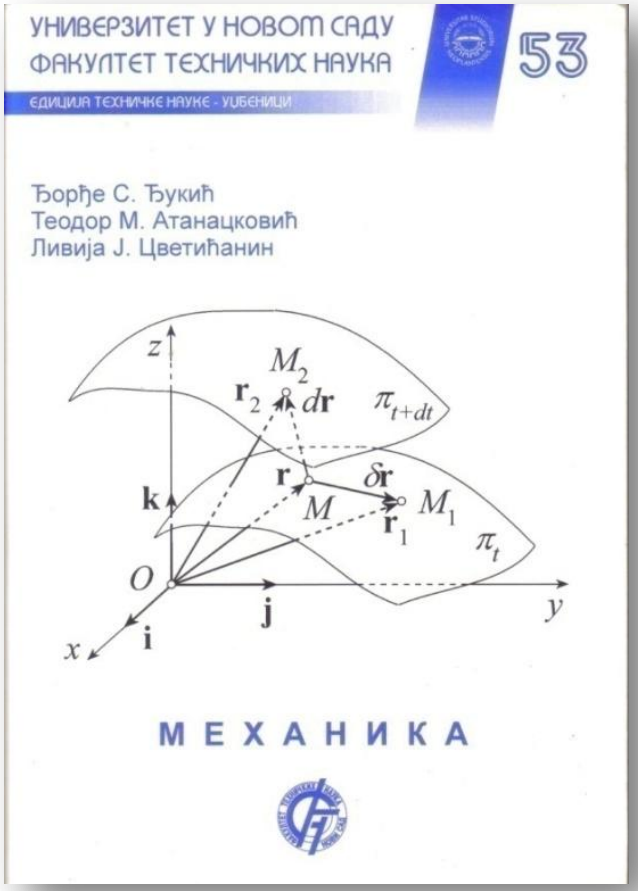
Kinematika – vežbe 3

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.

Literatura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ФАКУЛТЕТ ТЕХНИЧКИХ НАУКА
ЕДИЦИЈА ТЕХНИЧКЕ НАУКЕ - УЏБЕНИЦИ
53
Ђорђе С. Ђукић
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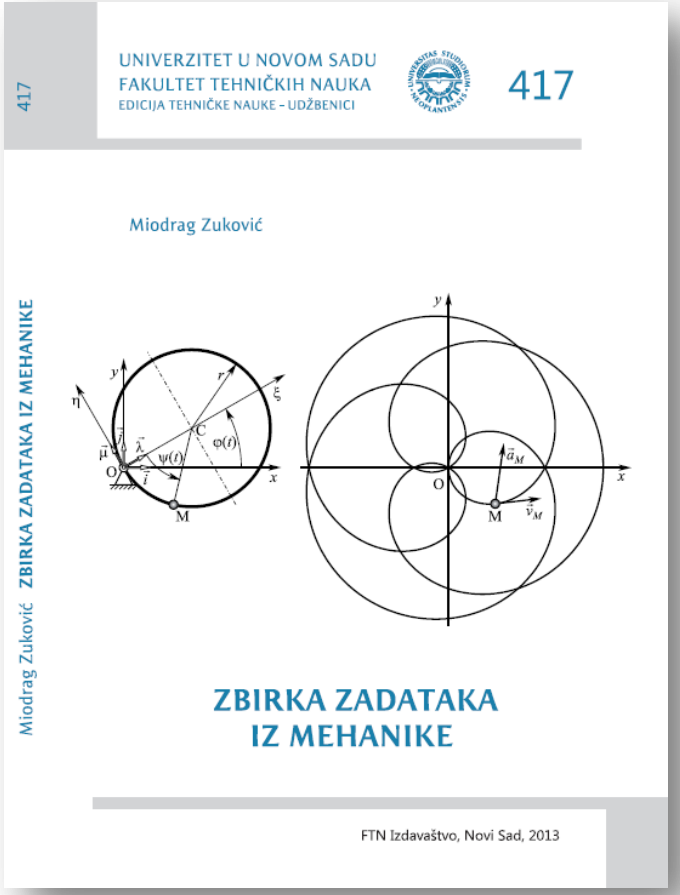
МЕХАНИКА


417
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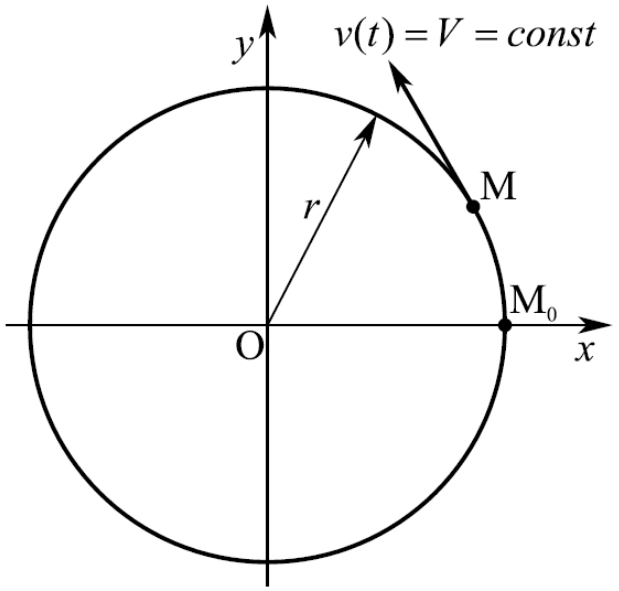
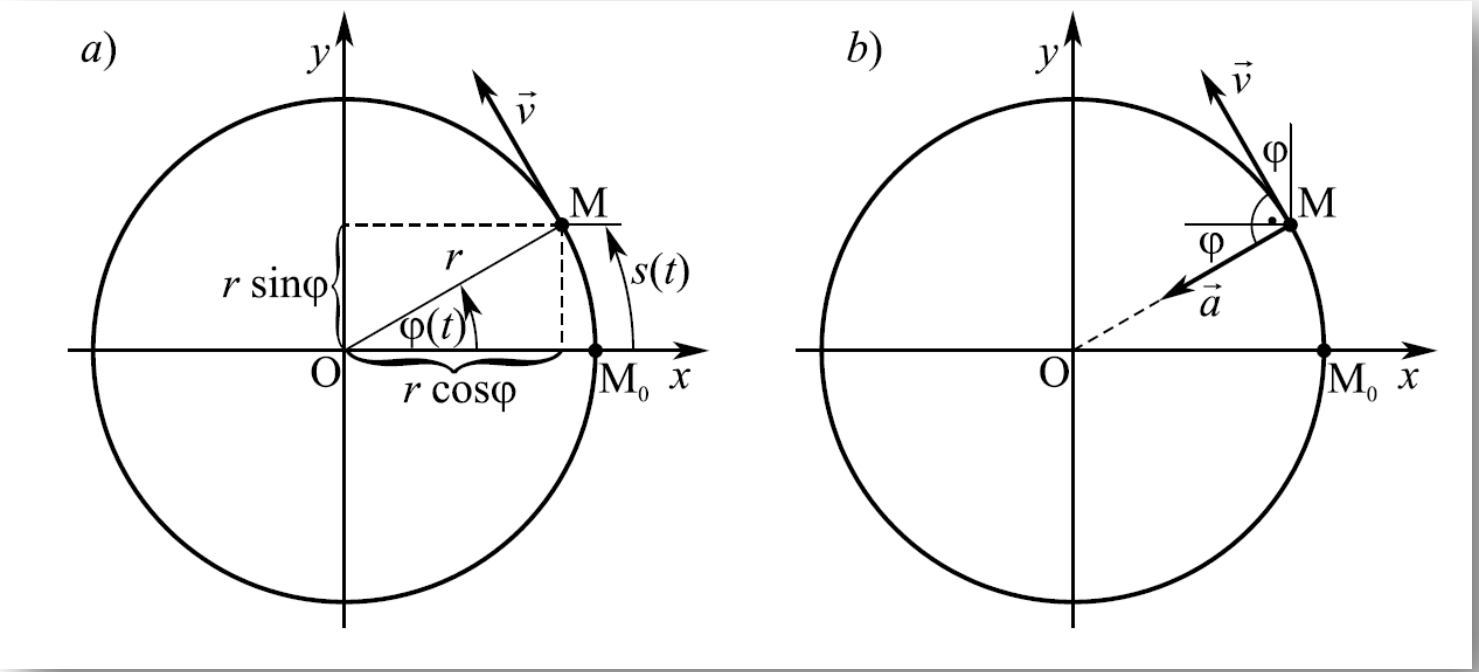
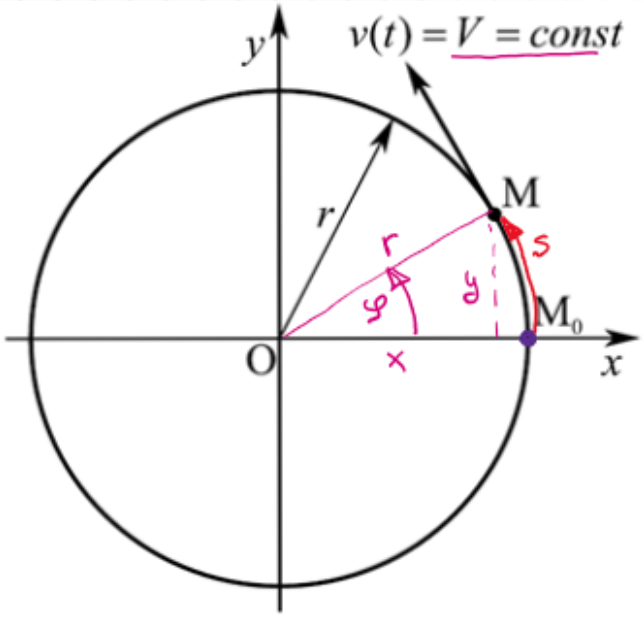
417
Miodrag Zuković
Miodrag Zuković ZBIRKA ZADATAKA IZ MEHANIKE
ZBIRKA ZADATAKA
IZ MEHANIKE
FTN Izdavaštvo, Novi Sad, 2013

Zadatak 1

Zadatak 2.6 Tačka se kreće po kružnici, poluprečnika r, brzinom konstantnog intenziteta $v(t) = V = const$. Odrediti konačne jednačine kretanja tačke $x(t)$ i $y(t)$, ako je koordinatni početak Dekartovog koordinatnog sistema u centru kružnice, a tačka započinje kretanje iz položaja: $x(0) = r, y(0) = 0$. Odrediti i ubrzanje tačke pri ovom kretanju.




a)
y
$\vec{v}$
M
r
r sinφ
φ(t)
s(t)
O
r cosφ
M0
x
b)
y
$\vec{v}$
φ
M
φ
$\vec{a}$
O
M0
x



$x(t) = ?\quad y(t) = ?$

$x = r\cos\varphi$

$y = \sin\varphi$

$s = r\varphi \rightarrow \varphi = \frac{s}{r}$

$\dot{s} = v = V = const$

$\frac{ds}{dt} = V \rightarrow \int_{s(0)=0}^{s} ds = V\int_{0}^{t} dt$

$s\Big|_0^s = V\,t\Big|_0^t \rightarrow s - 0 = V(t-0)$

$s = Vt \rightarrow \varphi = \frac{V}{r}t$

$x(t) = r\cos\left(\frac{V}{r}t\right)$

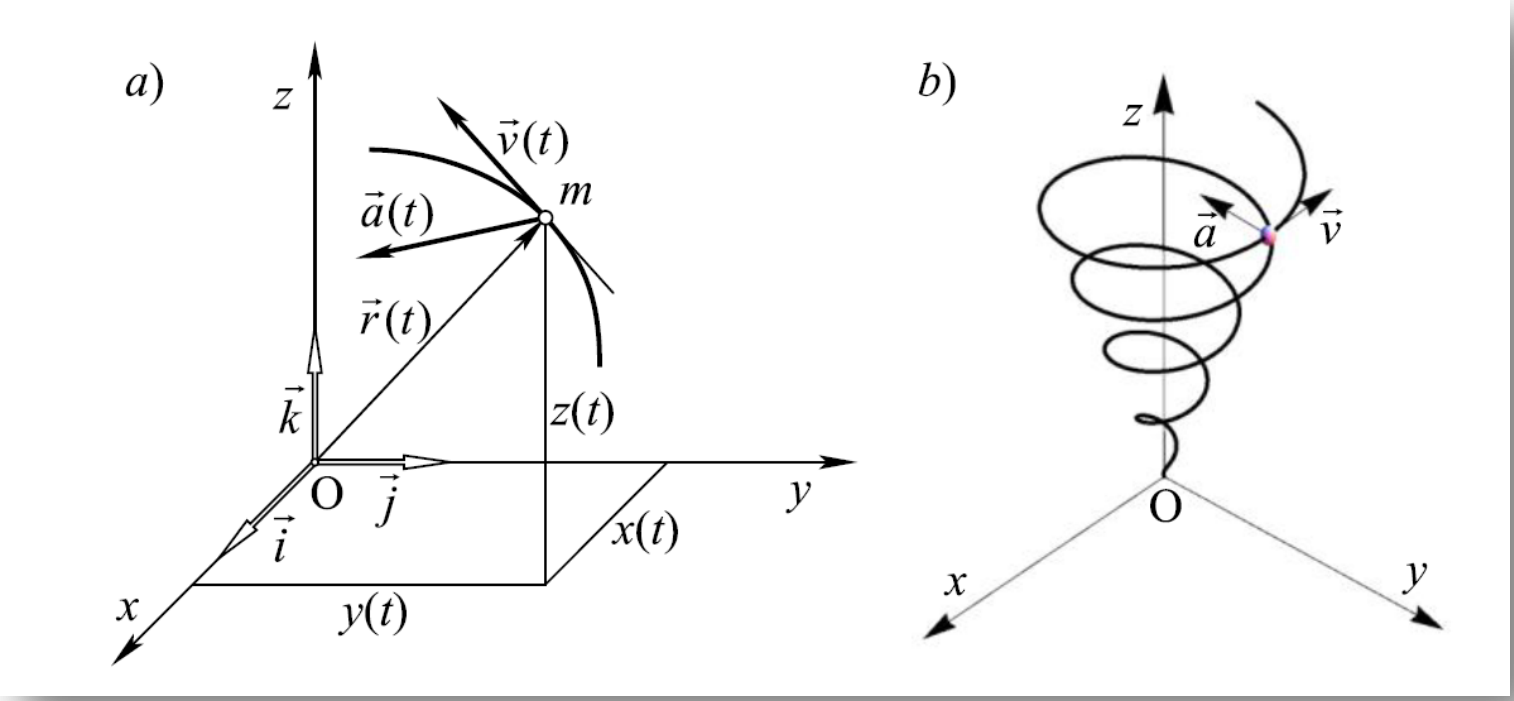
$y(t) = r\sin\left(\frac{V}{r}t\right)$

Zadatak 2

Zadatak 2.8 Kretanje tačke je opisano parametarskim jednačinama

$$x(t) = kt\cos t\,, y(t) = kt\sin t\,, z(t) = pt$$

Odrediti brzinu i ubrzanje tačke u proizvoljnom trenutku vremena t.


a)
z
$\vec{v}(t)$
m
$\vec{a}(t)$
$\vec{r}(t)$
$\vec{k}$
z(t)
O
$\vec{j}$
y
$\vec{i}$
x(t)
x
y(t)
b)
z
$\vec{a}$
$\vec{v}$
O
x
y

$$\dot{x}(t) = k\cos t - kt\sin t \qquad \ddot{x}(t) = -k\sin t - k\sin t - kt\cos t$$

$$\dot{y}(t) = k\sin t + kt\cos t \qquad \ddot{y}(t) = k\cos t + k\cos t - kt\sin t$$

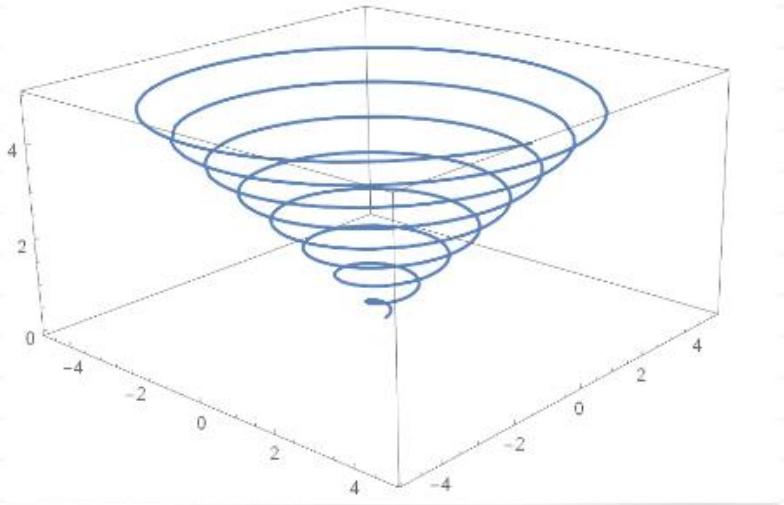
$$\dot{z}(t) = p \qquad \ddot{z}(t) = 0$$

$$x = kt\cos t \;/^2$$

$$y = kt\sin t \;/^2$$

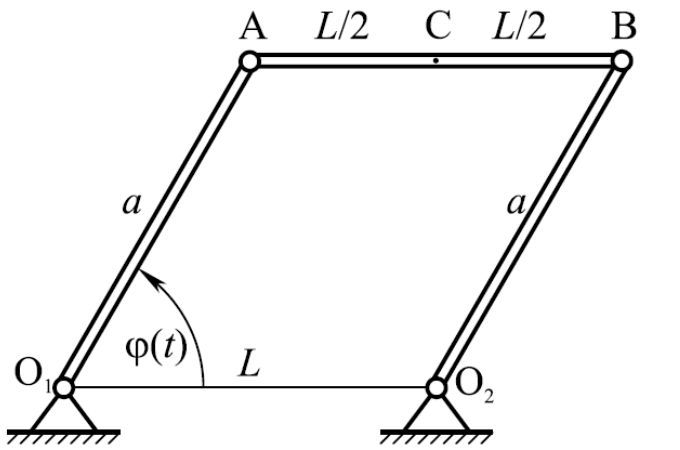
$$+ \qquad x^2 + y^2 = k^2t^2$$

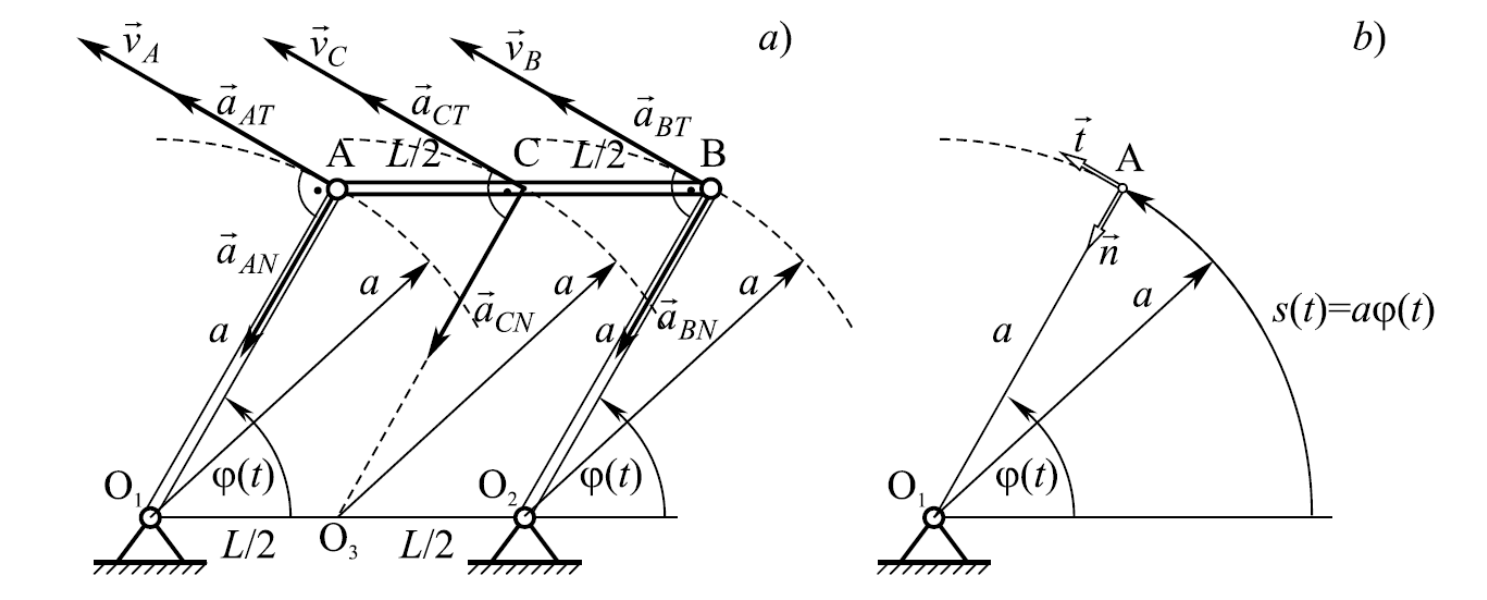
$$\sqrt{x^2+y^2} = kt$$



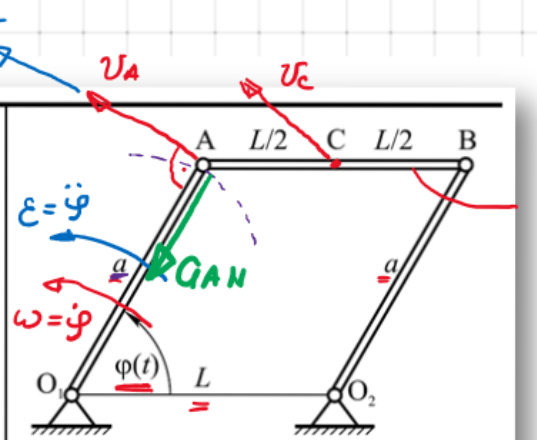
Zadatak 3

Zadatak 2.12 Štapovi O_1A i O_2B, jednakih dužina a, vezani su za podlogu cilindričnim zglobovima. U A i B za njih je zglobno vezan štap AB, dužine L. Rastojanje između zglobova O_1 i O_2 je L. Smatrajući poznatom funkcijom $\varphi(t)$, zakon promene ugla obrtanja štapa O_1A, odrediti brzinu i ubrzanje centra C štapa AB.




a)
b)
$\vec{v}_A$
$\vec{v}_C$
$\vec{v}_B$
$\vec{a}_{AT}$
$\vec{a}_{CT}$
$\vec{a}_{BT}$
A
C
B
L/2
L/2
$\vec{a}_{AN}$
$\vec{a}_{CN}$
$\vec{a}_{BN}$
a
a
a
a
a
O_1
O_2
O_3
φ(t)
φ(t)
L/2
L/2
$\vec{t}$
$\vec{n}$
A
a
a
s(t)=aφ(t)
O_1
φ(t)

Zadatak 2.12 Štapovi O_1A i O_2B, jednakih dužina a, vezani su za podlogu cilindričnim zglobovima. U A i B za njih je zglobno vezan štap AB, dužine L. Rastojanje između zglobova O_1 i O_2 je L. Smatrajući poznatom funkcijom $\varphi(t)$, zakon promene ugla obrtanja štapa O_1A, odrediti brzinu i ubrzanje centra C štapa AB.



ТРАЖИ СЛ. КР.

$\vec{v}_C = ?$, $\vec{a}_C = ?$

$\vec{v}_C = \vec{v}_B = \vec{v}_A$

$\vec{a}_C = \vec{a}_B = \vec{a}_A$

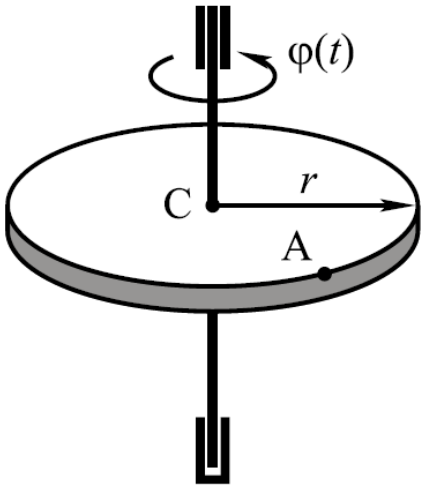
$v_A = \overline{O_1A} \cdot \omega = a\,\dot{\varphi}$

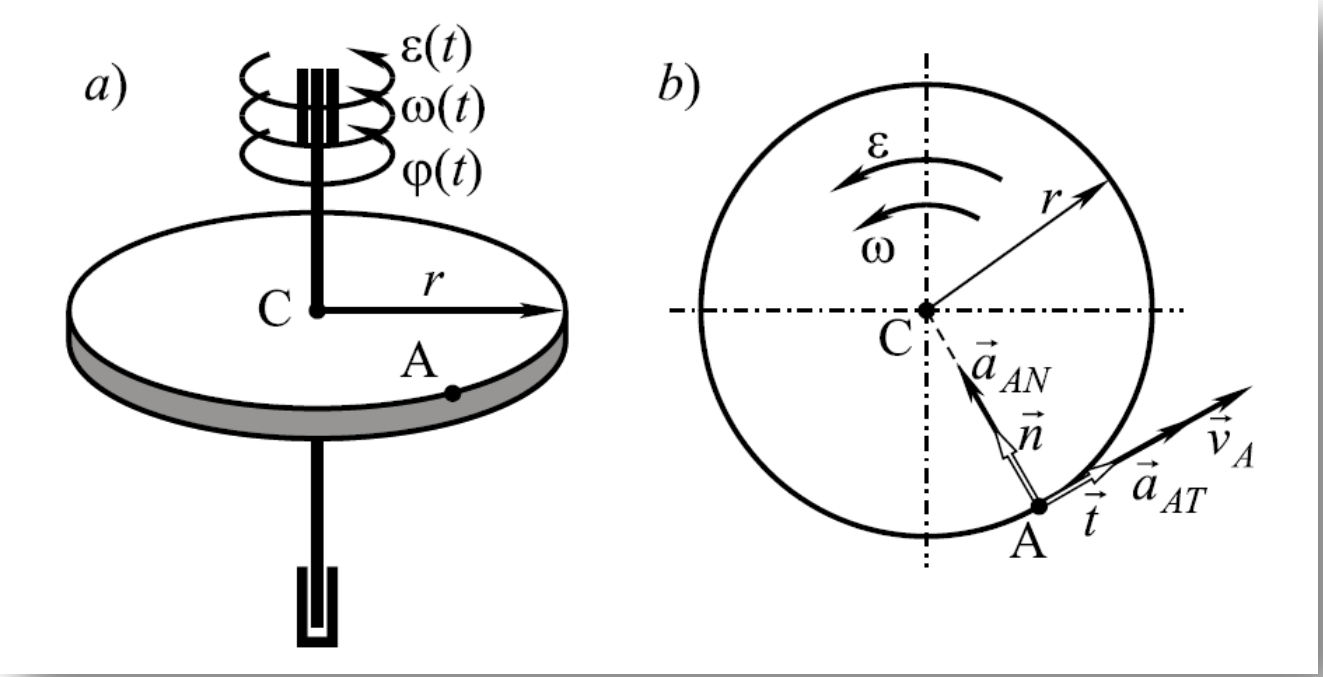
$a_{AT} = \overline{O_1A} \cdot \varepsilon = a \cdot \ddot{\varphi}$

$a_{AN} = \frac{v_A^2}{a} = \frac{a^2\omega^2}{a} = a \cdot \omega^2 = a\,\dot{\varphi}^2$

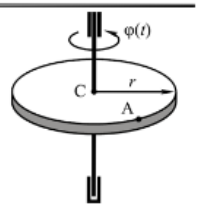
Zadatak 4

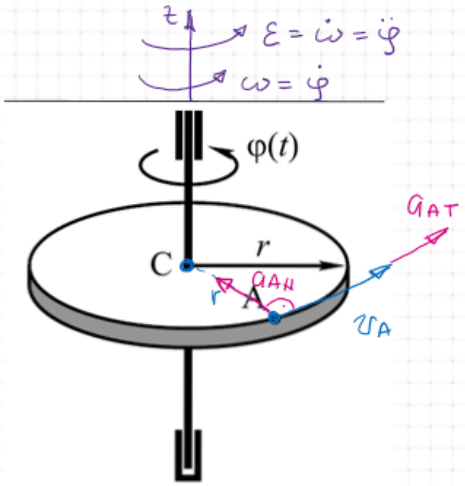
Zadatak 2.13 Disk, poluprečnika $r = 2\text{m}$, obrće se oko nepokretne ose po zakonu $\varphi(t) = t - \frac{t^2}{2}[rad]$. Odrediti ugaonu brzinu i ugaono ubrzanje diska u trenucima $t_0 = 0, t_1 = 1\text{s}$ i $t_2 = 2\text{s}$. Odrediti brzinu i ubrzanje tačke A na obodu diska u ovim trenucima.




a)
ε(t)
ω(t)
φ(t)
C
r
A
b)
ε
ω
r
C
$\vec{a}_{AN}$
$\vec{n}$
$\vec{v}_A$
$\vec{a}_{AT}$
$\vec{t}$
A

Zadatak 2.13 Disk, poluprečnika $r = 2m$, obrće se oko nepokretne ose po zakonu $\varphi(t) = t - \frac{t^2}{2}[rad]$. Odrediti ugaonu brzinu i ugaono ubrzanje diska u trenucima $t_0 = 0, t_1 = 1s$ i $t_2 = 2s$. Odrediti brzinu i ubrzanje tačke A na obodu diska u ovim trenucima.





$\varphi(t) = t - \frac{t^2}{2}$

$\omega(t) = \dot{\varphi}(t) = 1 - t$

$\varepsilon(t) = \ddot{\varphi} = -1 = const$

$t_0 = 0$

$\omega(0) = 1$

$\varepsilon(0) = -1$

$t_1 = 1$

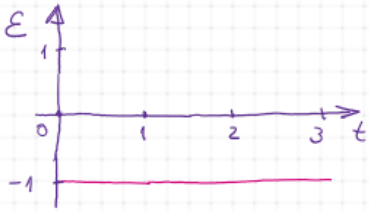
$\omega(1) = 0$

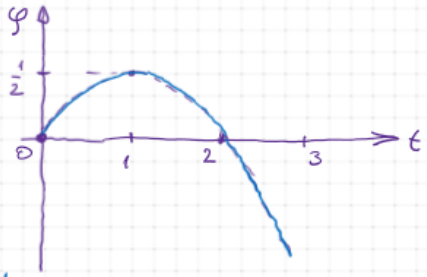
$\varepsilon(1) = -1$

$t_2 = 2$

$\omega(2) = -1$

$\varepsilon(2) = -1$





$v_A = r\omega = 2(1-t)$

$a_{AT} = r\varepsilon = 2 \cdot (-1) = -2 = const$

$a_{AN} = r\omega^2 = 2(1-t)^2 = \frac{v_A^2}{r}$

$t_0 = 0$

$v_A(0) = 2$

$a_{AT}(0) = -2$

$a_{AN}(0) = 2$

$a_{A(0)} = \sqrt{a_{AT}^2(0) + a_{AN}^2(0)} = 2\sqrt{2}$

$t_1 = 1$

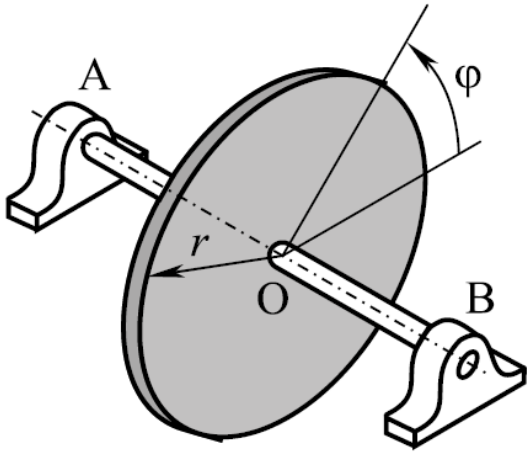
$v_A(1) = 0$

$a_{AT}(1) = -2$

$a_{AN}(1) = 0$

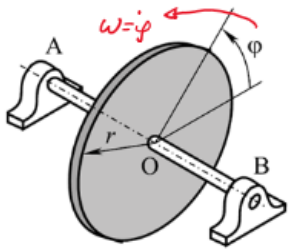
Zadatak 5

Zadatak 2.14 Zamajac započinje kretanje iz stanja mirovanja i kreće se jednoliko ubrzano. Nakon 10 minuta ima ugaonu brzinu koja odgovara $120\frac{\text{obrt}}{\text{min}}$. Koliko obrtaja je načinio zamajac u ovih 10 minuta. Usvojiti da je $\varphi(0) = 0$..



$\varepsilon = \ddot{\varphi}$

Zadatak 2.14 Zamajac započinje kretanje iz stanja mirovanja i kreće se jednoliko ubrzano. Nakon 10 minuta ima ugaonu brzinu koja odgovara $120\frac{\text{obrt}}{\text{min}}$. Koliko obrtaja je načinio zamajac u ovih 10 minuta. Usvojiti da je $\varphi(0) = 0$..



$\omega(0) = 0$; $\varepsilon = const = \varepsilon_0$

posle 10 min $\rightarrow$ $n = 120\ \frac{\text{obrt}}{\text{min}}$

$\varphi(0) = 0$

$$\omega = \frac{n \cdot 2\pi}{60} \quad \left[\frac{rad}{s}\right]$$

$$\omega = \frac{n\pi}{30} \quad \left[\frac{rad}{s}\right]$$

$n\left[\frac{o}{min}\right]$

$\varepsilon = \boxed{\varepsilon_0 = \ddot{\varphi}}$

$\ddot{\varphi} = \varepsilon_0$

$$\frac{d\dot{\varphi}}{dt} = \varepsilon_0 \rightarrow \int d\dot{\varphi} = \varepsilon_0 \int dt$$

(a) $\dot{\varphi} = \varepsilon_0 t + C_1$ $\rightarrow$ $\frac{d\varphi}{dt} = \varepsilon_0 t + C_1 \quad / \cdot dt$

$$\int d\varphi = \varepsilon_0 \int t\,dt + C_1 \int dt \rightarrow (6)\quad \varphi = \varepsilon_0 \cdot \frac{t^2}{2} + C_1 t + C_2$$

$t = 0 \quad \varphi(0) = 0 \quad \omega(0) = 0$

$t = 10\ min = 600\ s \qquad n = 120\ \frac{obrt}{min}$

$\omega(600) = \frac{120 \cdot \pi}{30} = 4\pi$

$\varphi(0) \overset{b}{=} \boxed{C_2 = 0}$

$\omega(0) \overset{a}{=} \boxed{C_1 = 0}$

$\omega(600) = \varepsilon_0 \cdot 600 + C_1 = 4\pi$

$\varepsilon_0 = \frac{4\pi}{600} = \frac{\pi}{150}$

$\varepsilon_{(t)} = \frac{\pi}{150} = const$

$\omega(t) = \dot{\varphi}(t) \overset{a}{=} \frac{\pi}{150} t$

$\varphi(t) \overset{b}{=} \frac{\pi}{300} t^2$

$\varphi(600) = \frac{\pi}{300} 600^2 \quad (10\ min)$

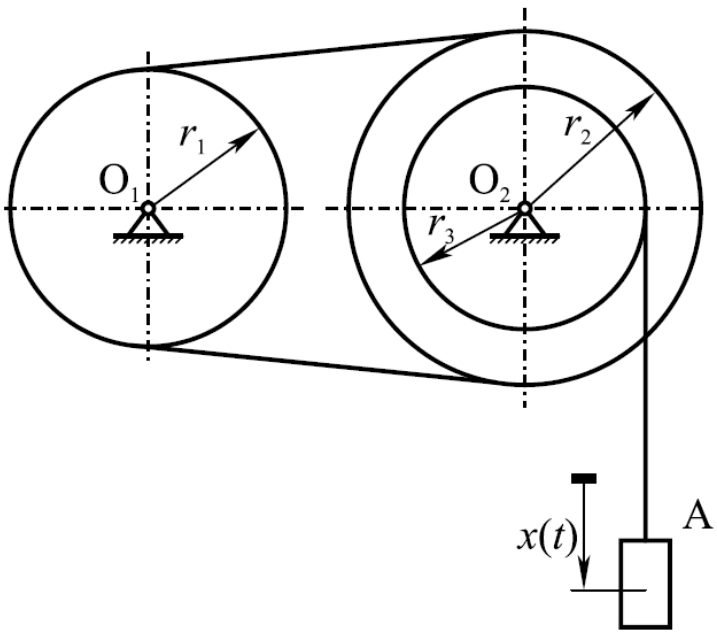
$= \frac{\pi}{300} 3600 \quad (1200)$

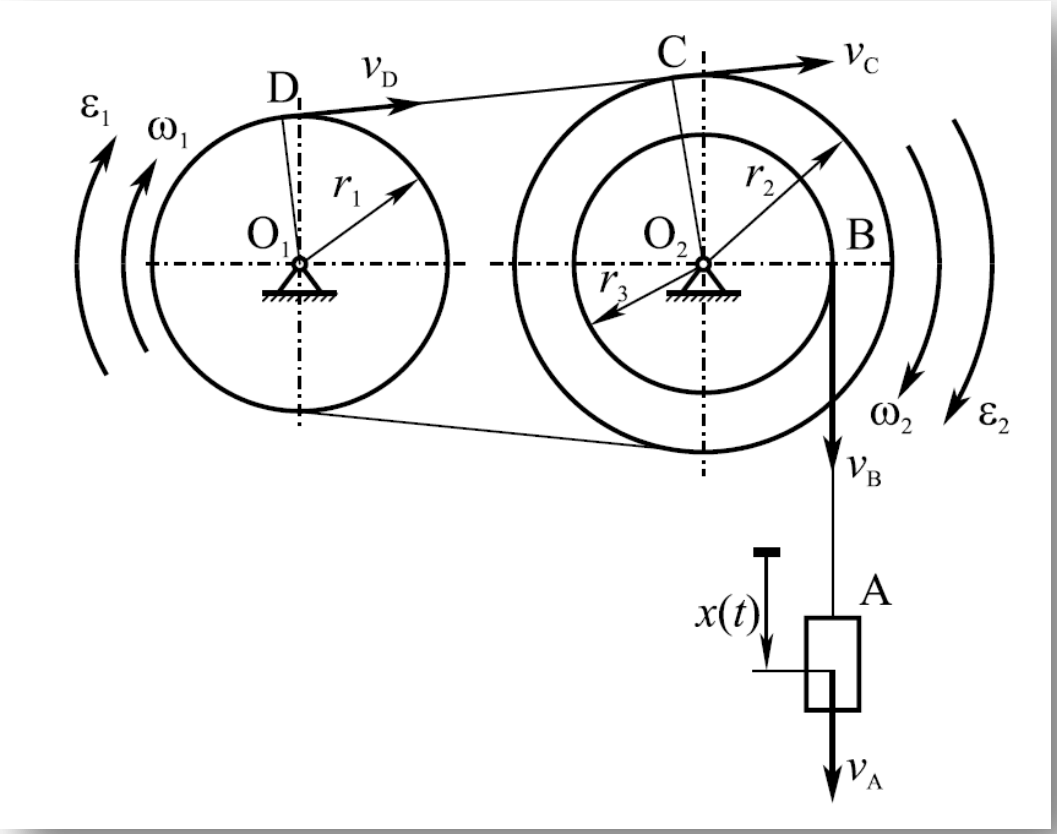
$\varphi(600) = 1200\pi$

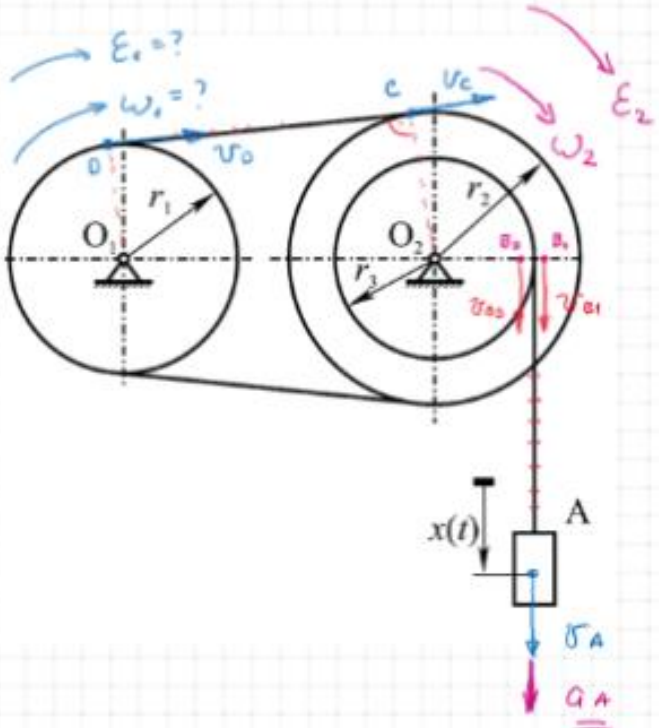
$\frac{1200\pi}{2\pi} = 600 \quad obrt.$

Zadatak 6

Zadatak 2.17 Disk, poluprečnika r_1, obrće se oko nepokretne ose. Oko njega je omotano idealno uže koje je prebačeno preko većeg cilindra, poluprečnika r_2, doboša koji se obrće oko nepokretne ose. Oko manjeg cilindra doboša, poluprečnika r_3, prebačeno je idealno uže na čijem kraju se nalazi prizma A. Prizma se kreće pravolinijski po zakonu $x(t) = a\frac{t^2}{2} + v_0 t$. Odrediti ugaonu brzinu i ugaono ubrzanje diska. Užad ne proklizavaju.




C
v_C
D
v_D
ε_1
ω_1
r_1
O_1
r_2
O_2
B
r_3
ω_2
ε_2
v_B
$x(t)$
A
v_A



$$x(t) = a\frac{t^2}{2} + v_0 t$$

$$v_A = \dot{x} = at + v_0$$

$$a_A = \ddot{x} = a = const$$

$$v_{B3} = v_{B1} = v_A = at + v_0$$

$$v_{B3} = r_3 \omega_2$$

$$r_3 \omega_2 = at + v_0$$

$$\omega_2 = \frac{at + v_0}{r_3} \Big/ \frac{d}{dt} \rightarrow \varepsilon_2 = \frac{a}{r_3}$$

$$v_C = r_2 \omega_2 = r_2 \frac{at + v_0}{r_3}$$

$$v_0 = v_C = r_2 \frac{at + v_0}{r_3}$$

$$v_D = r_1 \omega_1$$

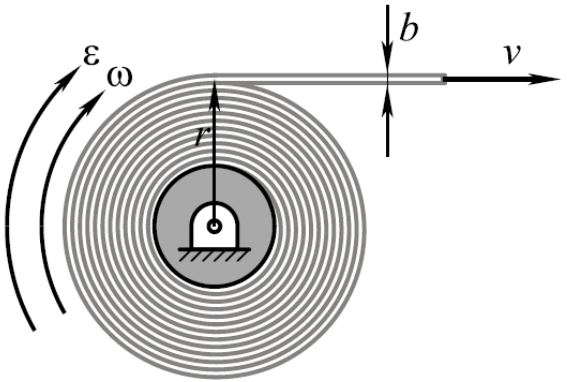
$$r_1 \omega_1 = r_2 \frac{at + v_0}{r_3}$$

$$\omega_1 = \frac{r_2}{r_1} \frac{at + v_0}{r_3} \Big\} \Big/ \frac{d}{dt}$$

$$\varepsilon_1 = \frac{r_2}{r_1} \frac{a}{r_3}$$

Zadatak 7

Zadatak 2.15 U procesu neprekidne štampe papir se uvlači u presu konstantnom brzinom v. Ako je r trenutni poluprečnik rolne papira, a b debljina papira, odrediti kako se menjaju ugaona brzina i ugaono ubrzanje rolne papira u funkciji od r. Kako se ove veličine menjaju tokom vremena.



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