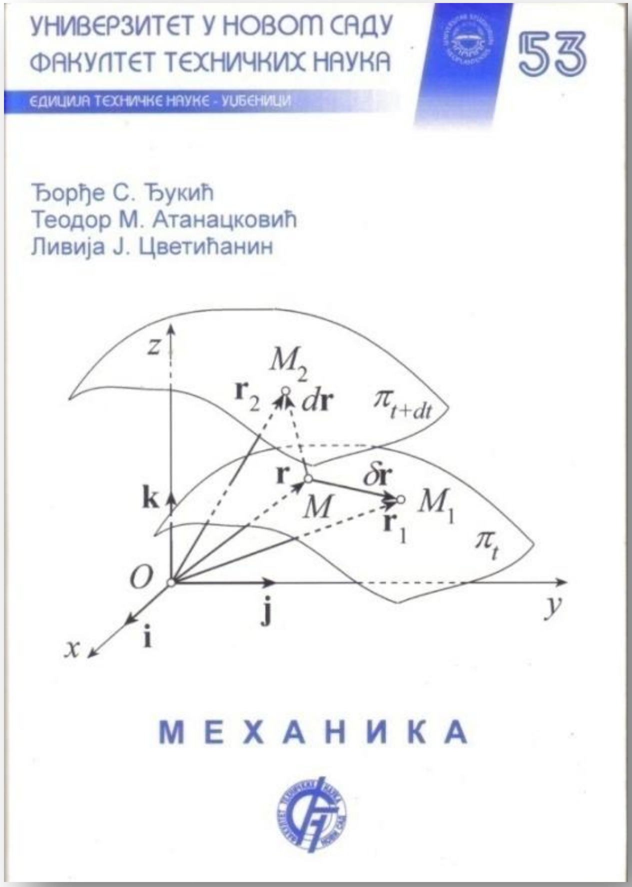
Dinamika – vežbe 5

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.

Literatura


УНИВЕРЗИТЕТ У НОВОМ САДУ
ФАКУЛТЕТ ТЕХНИЧКИХ НАУКА
ЕДИЦИЈА ТЕХНИЧКЕ НАУКЕ - УЏБЕНИЦИ
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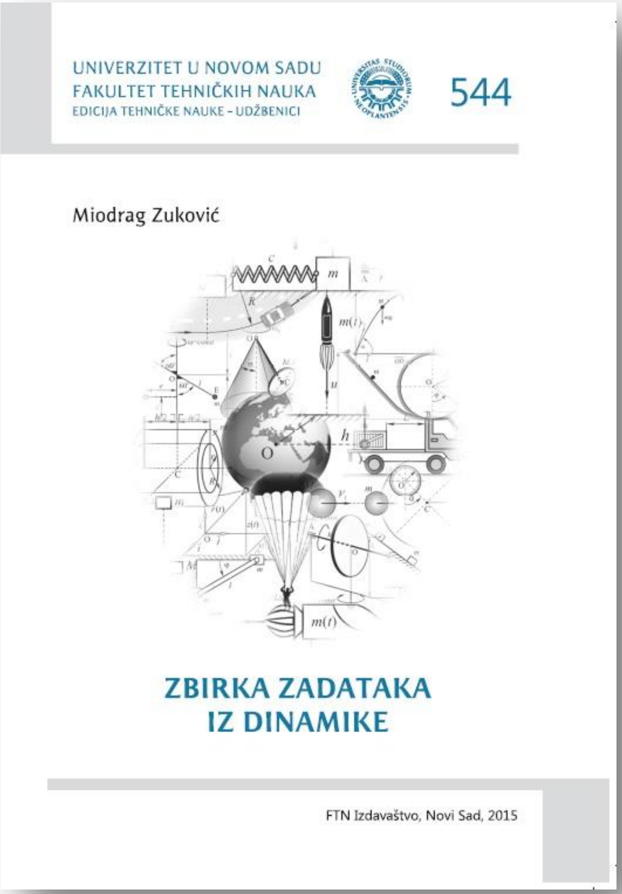
МЕХАНИКА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FAKULTET TEHNIČKIH NAUKA
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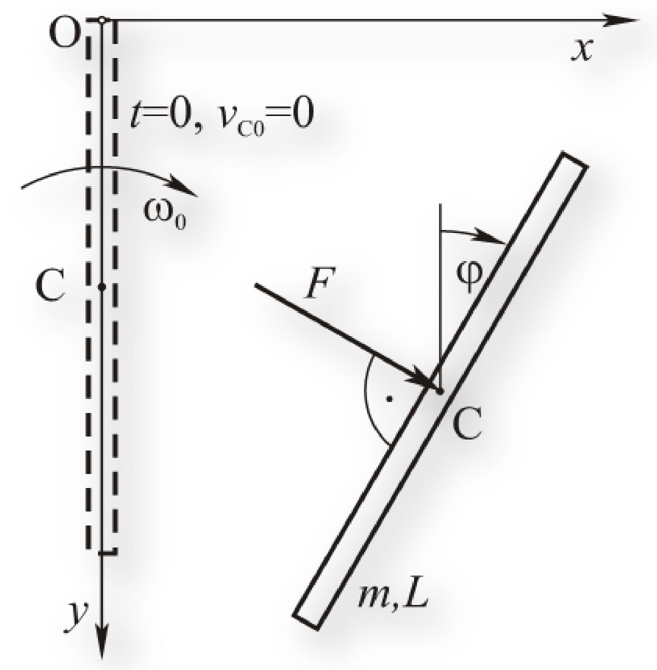
Miodrag Zuković
ZBIRKA ZADATAKA
IZ DINAMIKE
FTN Izdavaštvo, Novi Sad, 2015

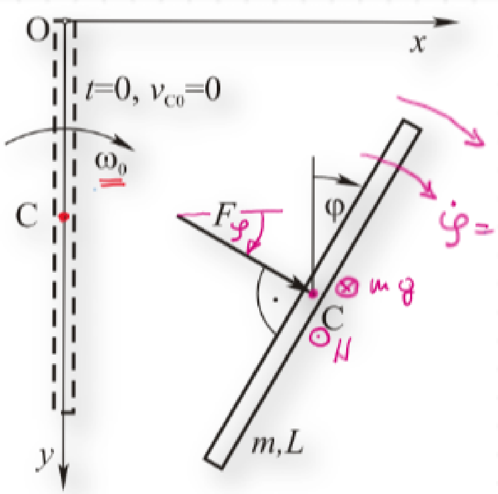
Zadatak 1

Štap, mase m i dužine L, kreće se po horizontalnom glatkom stolu, pri čemu na njega deluje sila konstantnog intenziteta F, čija napadna linija stalno prolazi kroz centar štapa i upravna je na njega. Odrediti kretanje štapa, ($x_C(t), y_C(t), \varphi(t)$), ako se on u početnom trenutku nalazio na y osi i ako mu je brzina centra bila jednaka nuli, a ugaona brzina ω_0.



ШТАП → РАВАНСКО КРЕТ.

ХОР. РАВАН



$\ddot\varphi = \varepsilon$

$\dot\varphi = \omega$

T1 $\quad m\cdot\vec a_c = \sum \vec F_i^s$

$$m\vec a_c = \vec F + m\vec g + \vec N \qquad /\cdot\vec i \ /\cdot\vec j$$

(1) $m\ddot x_c = F\cos\varphi$

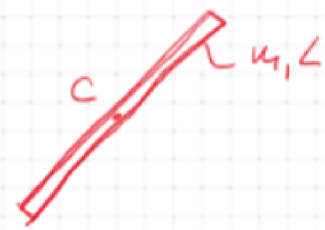
(2) $m\ddot y_c = F\sin\varphi$

T2 $\quad \dfrac{d\vec L_c}{dt} = \vec M_{Fc}^s \qquad /\cdot\vec k$

(3) $J_c\,\varepsilon = \sum M_c^s$

$$\frac{mL^2}{12}\cdot\ddot\varphi = 0$$

ШТАП



$$J_c = \frac{mL^2}{12}$$

(3) → $\ddot{\varphi} = 0$ → $\dot{\varphi} = const = \dot{\varphi}(0) = \omega_0$

$\dot{\varphi} = \omega_0$ → $\frac{d\varphi}{dt} = \omega_0$ → $\int_{\varphi(0)=0}^{\varphi} d\varphi = \omega_0 \int_0^t dt$

$$\boxed{\varphi_{(t)} = \omega_0 \cdot t}$$

(1) $m\ddot{x}_c = F\cos(\omega_0 t)$ → $\frac{d\dot{x}_c}{dt} = \frac{F}{m}\cos(\omega_0 t)$

$$\int_{\dot{x}_c(0)=0}^{\dot{x}_c} d\dot{x}_c = \frac{F}{m}\int_0^t \cos(\underbrace{\omega_0 t}_{u})\,dt \rightarrow \dot{x}_c\Big|_0^{\dot{x}_c} = \frac{F}{m\omega_0}\sin(\omega_0 t)\Big|_0^t \rightarrow$$

$$\dot{x}_c = \frac{F}{m\omega_0}\left(\sin(\omega_0 t) - \overset{0}{\cancel{\sin 0}}\right) \rightarrow \frac{dx_c}{dt} = \frac{F}{m\omega_0}\sin(\omega_0 t)$$

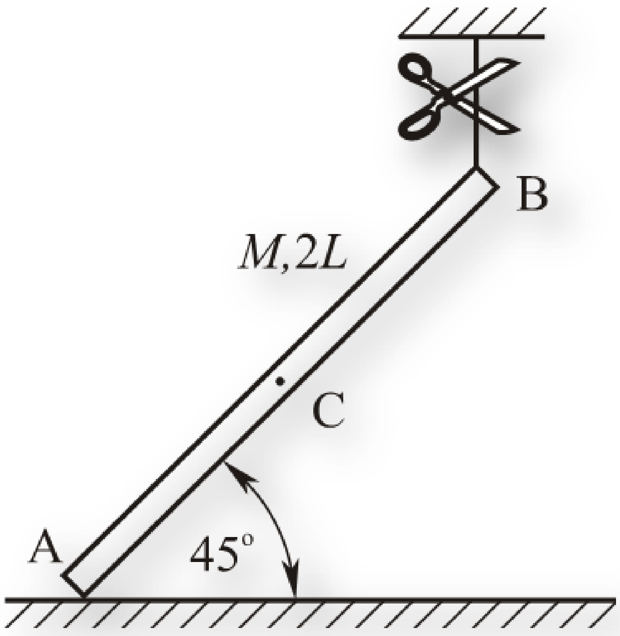
$$\int_{x_c(0)=0}^{x_c} dx_c = \frac{F}{m\omega_0}\int_0^t \sin(\underbrace{\omega_0 t}_{u})\,dt \rightarrow x_c\Big|_0^{x_c} = -\frac{F}{m\omega_0^2}\cos(\omega_0 t)\Big|_0^t$$

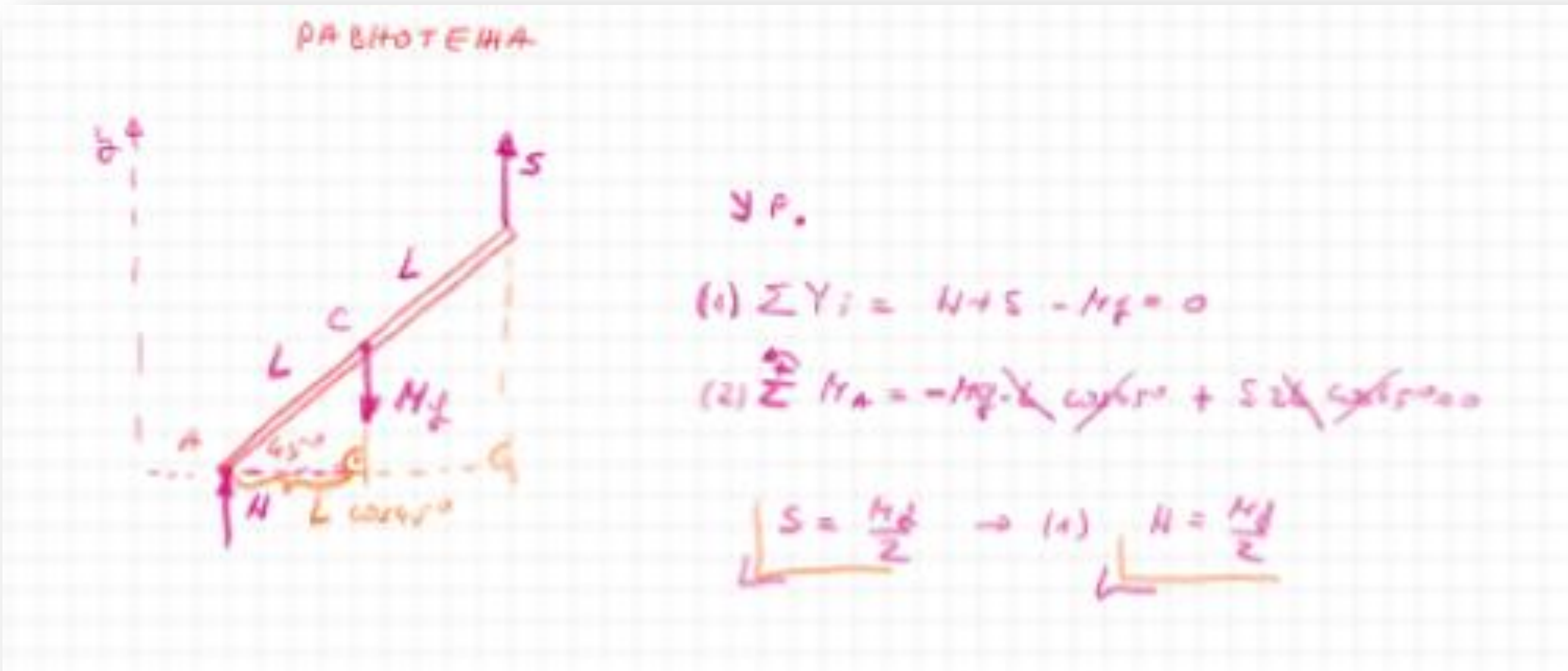
$$x_{c(t)} = -\frac{F}{m\omega_0^2}\left(\cos(\omega_0 t) - \overset{1}{\cancel{\cos 0}}\right)$$

(2) $\int$ → $\dot{y}_c$ $\int$ → y_c

Zadatak 2

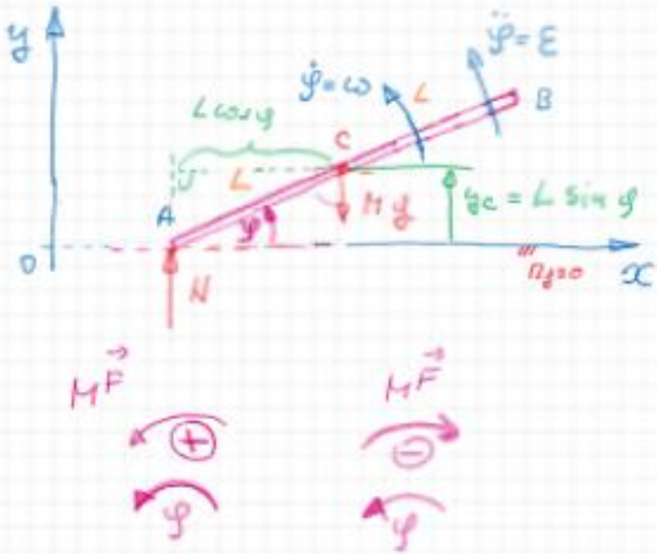
Štap, mase M i dužine $2L$, oslonjen je krajem A o glatki horizontalni pod, a krajem B vezan užetom za tavanicu. U datom, ravnotežnom, položaju štap gradi ugao 45° sa podom. U jednom trenutku pukne uže i štap započinje kretanje u vertikalnoj ravni, u homogenom polju sile zemljine teže. Odrediti reakciju poda neposredno nakon presecanja užeta, kao i brzinu centra štapa u funkciji položaja, ugla φ koji štap u proizvoljnom položaju gradi sa podom.




РАВНОТЕЖА
S
L
C
L
Mg
A
45°
N
L cos45°
УР.
(1) ΣYi = N+S − Mg = 0
(2) ΣM_A = −Mg·L cos45° + S 2L cos45° = 0
S = Mg/2 → (1) N = Mg/2

$N_0 = ?$

$v_C(\varphi) = ?$



ШТАП — РАВ. КР.

T1. $\boxed{M\cdot\vec{a}_C = M\vec{g} + \vec{N}} \quad /\cdot\vec{i}\,/\cdot\vec{j}$

$$(1)\quad M\ddot{x}_C = 0$$

$$(2)\quad M\ddot{y}_C = -Mg + N$$

T2 $\quad (3)\quad J_C\cdot\varepsilon = \Sigma M_C$

$$(3)\quad \frac{M(2L)^2}{12}\ddot{\varphi} = -N\cdot L\cos\varphi$$

$$x_C,\ y_C,\ \varphi,\ N = ?$$

ДОП. ЈЕДН.

$$(4)\quad y_C = L\sin\varphi \quad /\frac{d}{dt}$$

$$(4)'\quad \dot{y}_C = L\underbrace{\dot{\varphi}}_{\omega}\cdot\underbrace{\cos\varphi}_{v} \quad /\frac{d}{dt}$$

$$(4)''\quad \ddot{y}_C = L\ddot{\varphi}\cdot\cos\varphi - L\dot{\varphi}^2\sin\varphi$$

$N_0 = ?$ КРЕТАЊЕ

$t = 0 \rightarrow \varphi_0 = 45^\circ$

$$\dot{x}_{c0} = \dot{y}_{c0} = \dot{\varphi}_0 = 0$$

$$\ddot{x}_{c0} \neq 0, \quad \ddot{y}_{c0} \neq 0, \quad \ddot{\varphi}_0 = 0$$

$$\left.\begin{aligned}
&(1) \quad M\ddot{x}_{c0} = 0 \\
&(2) \quad M\ddot{y}_{c0} = -Mg + N_0 \\
&(3) \quad \frac{M(2L)^2}{12}\ddot{\varphi}_0 = -N_0 L \cos 45^\circ \\
&(4)'' \quad \ddot{y}_{c0} = L\ddot{\varphi}_0 \cos 45^\circ - L\cancel{\dot{\varphi}_0}^{\,0} \sin 45^\circ
\end{aligned}\right\} \rightarrow
\begin{aligned}
&\ddot{x}_{c0} = 0 \\
&\ddot{y}_{c0} = \frac{2}{5}g - g \neq 0 \\
&\ddot{\varphi}_0 = \ldots \\
&\boxed{N_0 = \frac{2}{5}Mg}
\end{aligned}$$

$$v_c(\varphi) = ?$$

I $\quad (1), \ldots (4)'' \quad \int \rightarrow v_c(\varphi)$

II

$$E_k + \Pi = \cancel{E_{k0}}^{\,0} + \Pi_0$$

$$\Pi = Mg \cdot L \sin\varphi$$

$$\Pi_0 = Mg \cdot L \sin 45^\circ$$

$$\underbrace{E_k = \frac{1}{2} M v_c^2 + \frac{1}{2} J_c \omega^2}_{\text{КЕНИГОВА Т.}}$$

$$\omega = \dot{\varphi}$$

$$(1) \quad \ddot{x}_c = 0 \rightarrow \dot{x}_c = const = \dot{x}_c(0) = 0$$

$$\dot{x}_c = 0$$

$$v_c^2 = \cancel{\dot{x}_c^2}^{\,0} + \dot{y}_c^2 = \dot{y}_c^2$$

$$(4)' \quad \dot{y}_c^2 = L^2 \dot{\varphi}^2 \cos^2\varphi$$

$$v_c^2 = L^2 \omega^2 \cos^2\varphi$$

$$\omega^2 = \frac{v_c^2}{L^2 \cos^2\varphi}$$

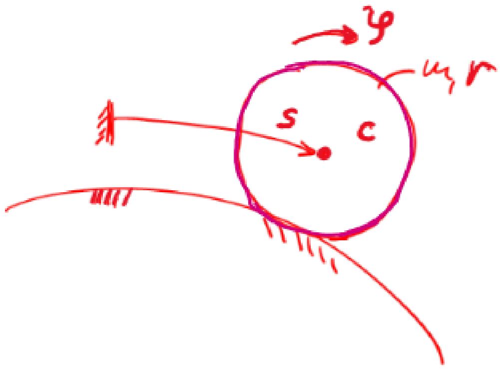
$$E_k = \frac{1}{2} M v_c^2 + \frac{1}{2} \frac{M(2L)^2}{12} \frac{v_c^2}{L^2 \cos^2\varphi}$$

$$\text{II} \rightarrow \frac{1}{2} M \left(1 + \frac{(2L)^2}{12} \cdot \frac{1}{L^2 \cos^2\varphi}\right) v_c^2 + MgL\sin\varphi = MgL\sin 45^\circ$$

$$v_c^2 = \ldots\ldots$$

$$v_c =$$

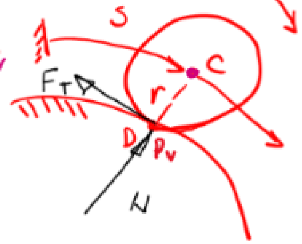
Kotrljanje bez proklizavanja i sa proklizavanjem



s, φ

КБК

1 ст. сл.



φ $\quad \dot{\varphi} = \omega$

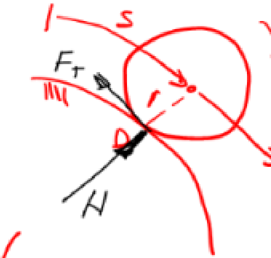
$\underline{v_C = \dot{s}}$

$v_D = 0 \rightarrow D \equiv P_v \rightarrow v_C = \boxed{\dot{s} = R\dot{\varphi}} = R\omega$ (*)

$F_T < \mu N$

КСК

2 ст. сл.



φ $\quad \dot{\varphi} = \omega$

$v_C = \dot{s}$

$v_C \neq r\omega$

$v_D \neq 0 \rightarrow D \neq P_v \rightarrow \dot{s} \neq r\dot{\varphi}$

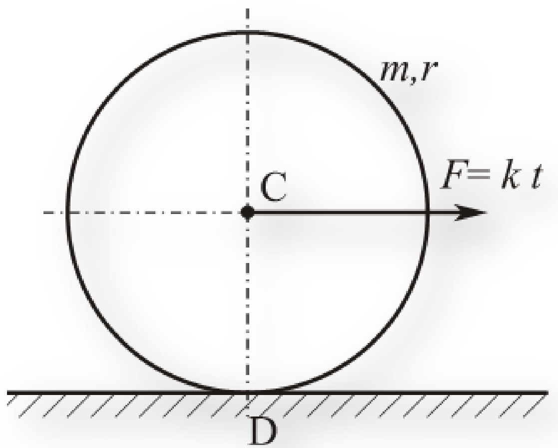
$\boxed{F_T = \mu N}$ (**)

$$dA^{\vec{F}_T} = \vec{F}_T \cdot d\vec{r}_D = 0$$

$$dA^{\vec{F}_T} = \vec{F}_T \cdot d\vec{r}_D \neq 0$$

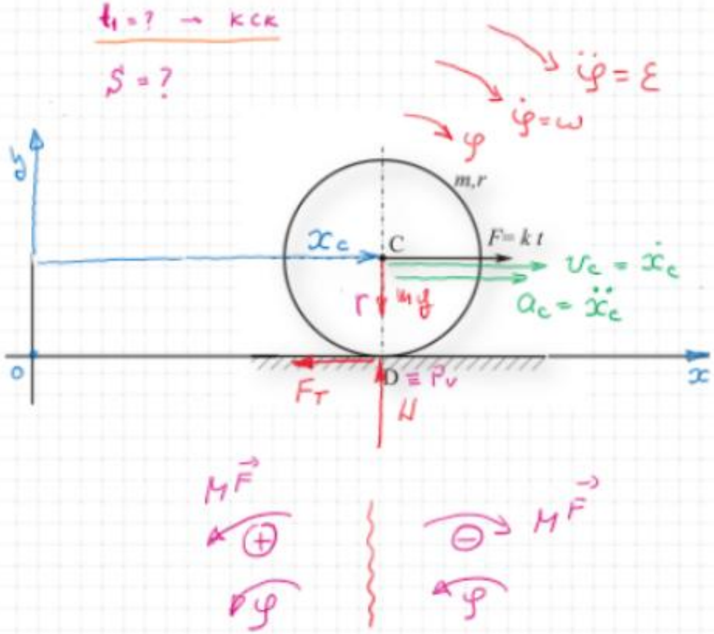
Zadatak 3

Disk, mase m i poluprečnika r, započinje kretanje po horizontalnom podu kotrljanjem bez klizanja iz stanja mirovanja. Na centar diska deluje horizontalna sila, stalnog smera, čiji se intenzitet tokom vremena menja po zakonu $F = kt$, $k = const > 0$. Odrediti trenutak t_1 u kome će disk početi da proklizava i put S koji će centar diska preći do tog trenutka. Koeficijent trenja između diska i poda je μ.



$t_1 = ?$ → КСК

$S = ?$



ДИСК → РАВ. КР

КБК — $t_0 = 0$ — $\boxed{v_C = r\omega}$, $F_T < \mu N$ — КБК

КСК — $t_1 = ?$ — $v_C \neq r\omega$, $F_T = \mu N$ — t

T1 $\quad m\vec{a}_C = \vec{F} + m\vec{g} + \vec{N} + \vec{F}_T \;/\cdot\vec{i}\,/\vec{j}$

(1) $\quad m\ddot{x}_C = F - F_T$

(2) $\quad 0 = -mg + N$

ДИСК

$C\odot$ $\; m, r$

$J_C = \frac{mr^2}{2}$

T2 $\quad$ (3) $\quad J_C\,\varepsilon = \Sigma M_C$

(3) $\quad \frac{mr^2}{2}\ddot{\varphi} = +F_T \cdot r$

$x_C, \varphi, N, F_T = ?$

ДОП. ЈЕДН. → КБК → $v_D = 0$, $D \equiv P_v$ → $v_C = r\omega$

(4) $\quad \dot{x}_C = r\dot{\varphi} \quad / \frac{d}{dt}$

$\ddot{x}_C = r\ddot{\varphi}$

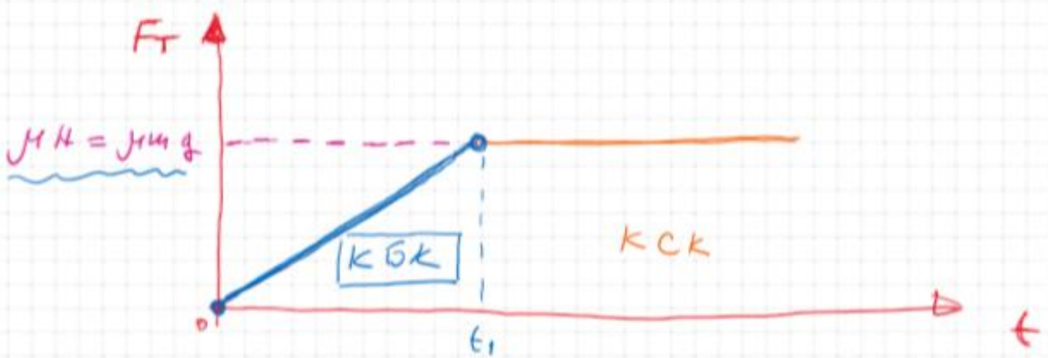
$(2) \rightarrow N = mg$ ★

$(4) \rightarrow \ddot{\varphi} = \frac{\ddot{x}_c}{r} \rightarrow (3) \quad \frac{m\cancel{r^2}}{2}\,\frac{\ddot{x}_c}{\cancel{r}} = F_T \cdot \cancel{r} \rightarrow \ddot{x}_c = \frac{2F_T}{m}$ *

$(1) \rightarrow \cancel{m} \cdot \frac{2F_T}{\cancel{m}} = F - F_T \rightarrow \boxed{F_T = \frac{F}{3} = \frac{kt}{3}}$ ✿

$t_1 = ? \rightarrow KCK \rightarrow F_T(t_1) = \mu N(t_1)$

★ ✿ $\rightarrow \frac{kt_1}{3} = \mu mg \rightarrow \boxed{t_1 = \frac{3\mu mg}{k}}$



$$S = ?$$

$$\circledast \rightarrow * \rightarrow \ddot{x}_c = \frac{2}{m} F_r$$

$$\ddot{x}_c = \frac{2}{m} \frac{k t}{3}$$

$$\boxed{\ddot{x}_c = \frac{2k}{3m} t}$$

$$\frac{d\dot{x}_c}{dt} = \frac{2k}{3m} t \rightarrow \int_{\dot{x}_c(0)=0}^{\dot{x}_c} d\dot{x}_c = \frac{2k}{3m} \int_0^t t\, dt \rightarrow \dot{x}_c = \frac{\cancel{2}k}{3m} \frac{t^2}{\cancel{2}}$$

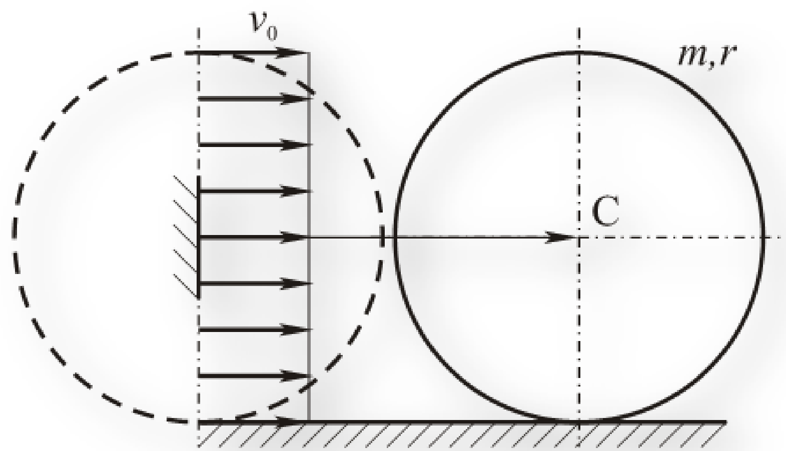
$$\frac{dx_c}{dt} = \frac{k}{3m} t^2 \rightarrow \int_{x_c(0)=0}^{x_c} dx_c = \frac{k}{3m} \int_0^t t^2\, dt$$

$$\boxed{x_c = \frac{k}{3m} \frac{t^3}{3}} \rightarrow S = x_c(t_1) = \frac{k}{9m} t_1^{\,3}$$

$$S = \dots$$

Zadatak 4

Disk, mase m i poluprečnika r, kreće se po horizontalnom podu. U početnom trenutku svim tačkama diska saopštena je u horizontalnom pravcu ista brzina, intenziteta v_0. Odrediti trenutak t_1 u kome će disk početi da se kotrlja bez klizanja. Koliki put S pređe centar diska do tog trenutka. Koeficijent trenja između diska i poda je μ.



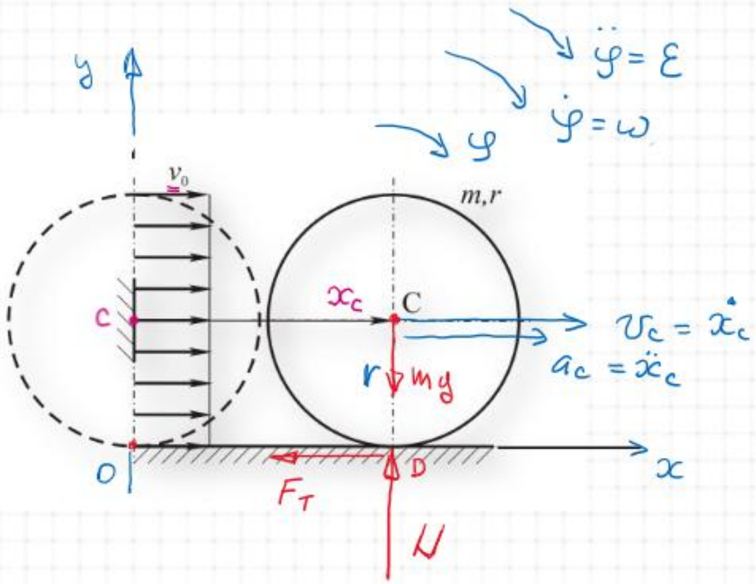
$t_1 = ?$ → КБК

КСК: $t=0$, $F_T = \mu N$, $v_c \neq r\omega$ — КБК: $t_1 = ?$, $F_T < \mu N$, $v_c = r\omega$ — t

$t=0$

$\dot{x}_c(0) = v_c(0) = v_0$

$\dot{\varphi}(0) = \omega(0) = 0$



РАВ. КР. → КСК

(1) $m\ddot{x}_c = -F_T$

(2) $0 = N - mg$

(3) $J_c\,\varepsilon = \sum M_c$

$\frac{mr^2}{2}\ddot{\varphi} = F_T \cdot r$

$x_c, \varphi, N, F_T = ?$

2 СТ. СЛ.

ДОП. ЈЕДН. → КСК

(4) $F_T = \mu N$

$$(2) \rightarrow N = mg \rightarrow (4)\ \underset{\sim\sim\sim\sim\sim}{F_T = \mu m g}$$

$$(1) \rightarrow \cancel{m}\,\ddot{x}_c = -\mu \cancel{m} g \rightarrow \frac{d\dot{x}_c}{dt} = -\mu g \rightarrow \int_{\dot{x}_c(0)=v_0}^{\dot{x}_c} d\dot{x}_c = -\mu g \int_0^t dt$$

$$\dot{x}_c \Big|_{v_0}^{\dot{x}_c} = -\mu g \cdot t\Big|_0^t \rightarrow \dot{x}_c - v_0 = -\mu g t$$

$$\boxed{\dot{x}_c = v_0 - \mu g t}\ ^{*} \qquad \int \rightarrow x_c \rightarrow S = x_c(t_1) = \ldots$$

$$(3) \rightarrow \frac{\cancel{m} r^{\cancel{2}}}{2}\ddot{\varphi} = \mu \cancel{m} g \cdot \cancel{r}$$

$$\frac{d\dot{\varphi}}{dt} = \frac{2\mu g}{r} \rightarrow \int_{\dot{\varphi}(0)=0}^{\dot{\varphi}} d\dot{\varphi} = \frac{2\mu g}{r}\int_0^t dt \rightarrow \dot{\varphi}\Big|_0^{\dot{\varphi}} = \frac{2\mu g}{r}\, t\Big|_0^t$$

$$\boxed{\dot{\varphi} = \frac{2\mu g}{r}\, t}\ ^{**}$$

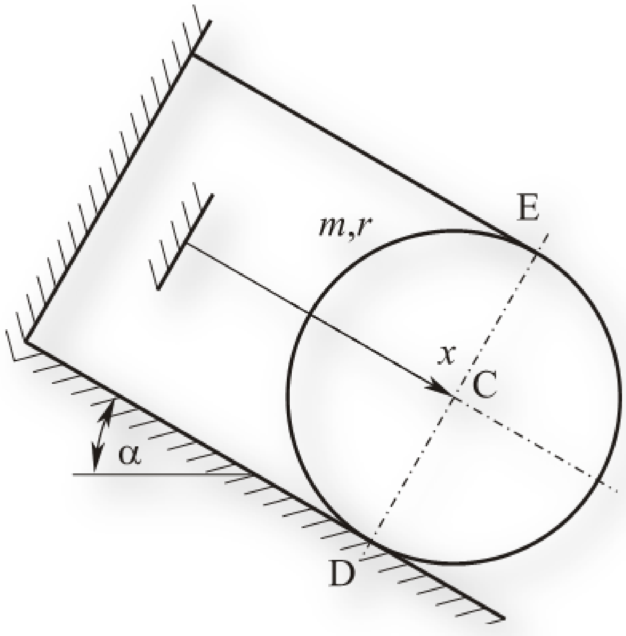
$$t_1 = ? \rightarrow \text{KБK} \rightarrow v_c(t_1) = r\,\omega(t_1)$$

$$\dot{x}_c(t_1) = r\,\dot{\varphi}(t_1)$$

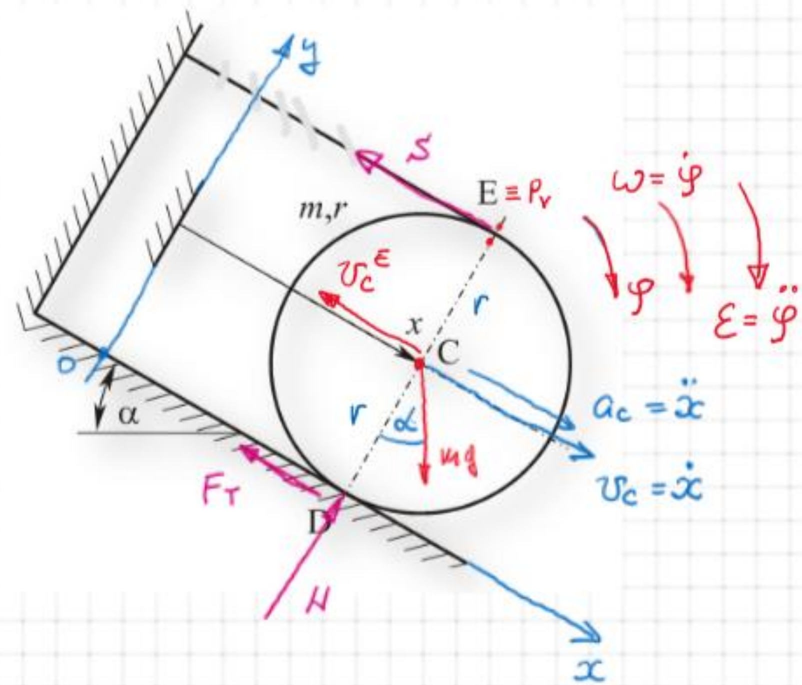
$$\left.\begin{matrix} * \\ ** \end{matrix}\right\} \rightarrow v_0 - \mu g t_1 = \cancel{r} \cdot \frac{2\mu g}{\cancel{r}}\, t_1 \rightarrow \boxed{t_1 = \frac{v_0}{3\mu g}}$$

Zadatak 5

Disk, mase m i radijusa r, nalazi se na hrapavoj strmoj ravni nagibnog ugla α. Oko diska je omotano idealno uže koje ne proklizava u odnosu na njega. Drugi kraj užeta vezan je za zid, tako da je deo užeta između zida i diska paralelan sa strmom ravni. Disk kretanje započinje iz stanja mirovanja – pod dejstvom sile težine, savlađujući trenje (koeficijent trenja je μ). Odrediti kretanje centra diska $x(t)$ i silu u užetu S.



$x_c(t) = x(t) = ?\ ,\quad S = ?$



T1 $\quad m\vec{a}_c = m\vec{g} + \vec{S} + \vec{N} + \vec{F}_T \quad /\cdot\vec{i}\,/\cdot\vec{j}$

(1) $\quad m\ddot{x} = mg\sin\alpha - S - F_T$

(2) $\quad 0 = -mg\cos\alpha + N$

T2 $\quad$ (3) $\quad J_c\,\varepsilon = \sum M_c$

(3) $\quad \dfrac{mr^2}{2}\ddot{\varphi} = F_T\cdot r - S\cdot r$

$x,\ \varphi,\ S,\ N,\ F_T = ?$

D $\to$ KCK $\to$ (4) $F_T = \mu N$

E $\to$ KBK $\to$ $\vec{v}_C = \cancel{\vec{v}_E}^{\,0} + \vec{v}_C^{\,E} \to \dot{x}\,\vec{i} = -r\dot{\varphi}\,\vec{i}$

$E \equiv P_v$

$$v_C^E = r\omega = r\dot{\varphi}$$

$$(5) \quad \dot{x} = -r\dot{\varphi} \qquad / \frac{d}{dt}$$

$$\ddot{x} = -r\ddot{\varphi}$$

$$(5) \to \ddot{\varphi} = -\frac{\ddot{x}}{r} \to (3) \quad \frac{m\cancel{r}^2}{2}\left(-\frac{\ddot{x}}{\cancel{r}}\right) = F_T \cdot \cancel{r} - S \cdot \cancel{r} \to$$

$$* \boxed{S = F_T + \frac{m}{2}\ddot{x} =} \to (1)$$

$$(1) \quad m\ddot{x} = mg\sin\alpha - \left(F_T + \frac{m}{2}\ddot{x}\right) - F_T$$

$$\boxed{\left(m + \frac{m}{2}\right)\ddot{x} = mg\sin\alpha - 2F_T} \; **$$

$$(2) \rightarrow N = mg\cos\alpha \rightarrow (4)\ F_T = \mu m g \cos\alpha$$

$$** \rightarrow \frac{3}{2}\cancel{m}\ddot{x} = \cancel{m} g \sin\alpha - 2\mu \cancel{m} g \cos\alpha$$

$$\star \quad \boxed{\ddot{x} = \frac{2}{3} g(\sin\alpha - 2\mu\cos\alpha) = C > 0}$$

$$\left.\begin{matrix}\star \\ F_T\end{matrix}\right\} \rightarrow * \quad S = \mu m g \cos\alpha + \frac{m}{\cancel{2}}\frac{\cancel{2}}{3} g(\sin\alpha - 2\mu\cos\alpha)$$

$$S = \ldots\ldots$$

$$\star \quad \ddot{x} = C \ \int\rightarrow \ \dot{x} = Ct + c_1 \ \int \rightarrow x = C\frac{t^2}{2} + c_1 t + c_2$$

ПУ
$$\left.\begin{matrix}\dot{x}(0) = 0 \\ x(0) = 0\end{matrix}\right\} \rightarrow \begin{matrix}c_1 = 0 \\ c_2 = 0\end{matrix}$$

Dinamika – vežbe 5

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.