Dinamika
Dinamika materijalne tačke – Zakoni kretanja materijalne tačke, ...

Kinematika i dinamika

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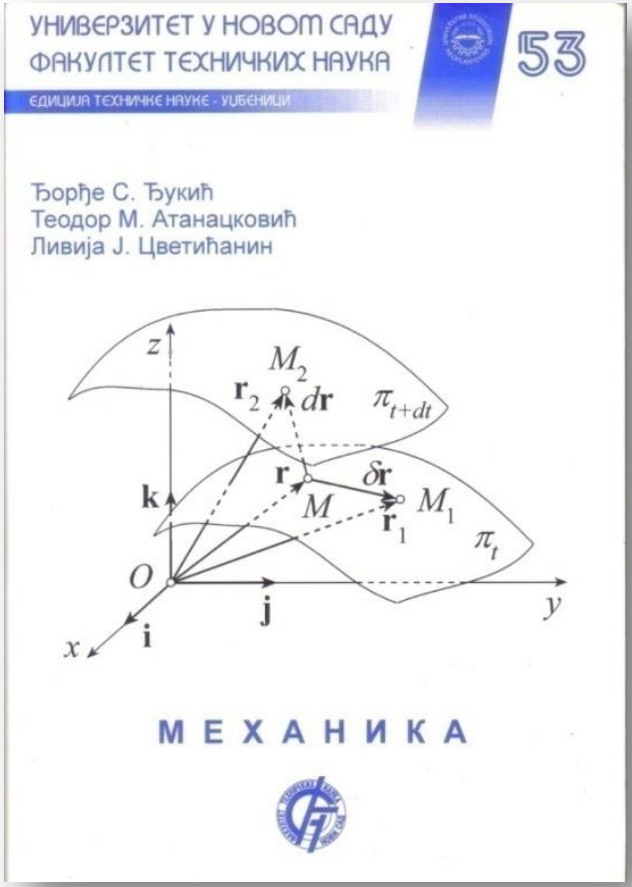
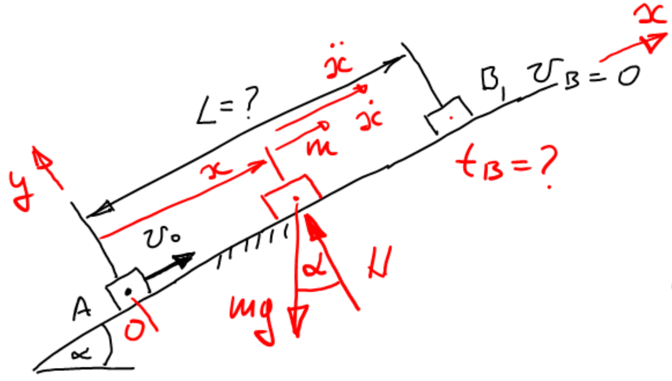
Novi Sad, 2021.

Literatura

- Đorđe S. Đukić, Teodor M. Atanacković, Livija J. Cvetićanin: Mehanika, Fakultet tehničkih nauka u Novom Sadu, Novi Sad, 2003.





II 1b.3. $m\vec{a} = m\vec{g} + \vec{N}$

$$m(\ddot{x}\,\vec{i}) = -mg\sin\alpha\,\vec{i} - mg\cos\alpha\,\vec{j} + N\vec{j} \;/\cdot\vec{i}\;/\cdot\vec{j}$$

$$\boxed{(1)\quad \cancel{m}\,\ddot{x} = -\cancel{m}\,g\sin\alpha} \;*$$

$$(2)\qquad 0 = -mg\cos\alpha + N \;\rightarrow\; N = mg\cos\alpha$$

РЕПАРАМЕТРИЗАЦИЈА

$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{dx}{dt}\cdot\frac{d\dot{x}}{dx} = \dot{x}\,\frac{d\dot{x}}{dx}$$

$$(1)\quad \ddot{x} = -g\sin\alpha = const < 0$$

$$(1) \rightarrow \quad \dot{x}\,\frac{d\dot{x}}{dx} = -g\sin\alpha \;\rightarrow\; \int_{\dot{x}(0)=v_0}^{\dot{x}(t_B)=0} \dot{x}\,d\dot{x} = -g\sin\alpha \int_{x(0)=0}^{x(t_B)=L} dx$$

$$\left.\frac{\dot{x}^2}{2}\right|_{v_0}^{0} = -g\sin\alpha\; x\Big|_{0}^{L} \;\rightarrow\; \frac{\cancel{0^2}}{2} - \frac{v_0^2}{2} = -g\sin\alpha\cdot(L-\cancel{0})$$

$$L = \frac{v_0^2}{2g\sin\alpha}$$

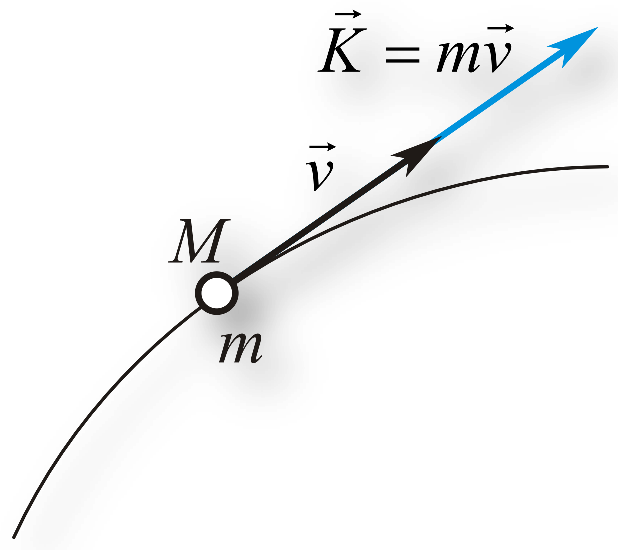
Šta ćemo naučiti?

20. **Mere kretanja tačke (količina kretanja, moment količine kretanja i kinetička energija).**
21. **Impuls sile.**
22. **Rad sile. Snaga sile. Potencijalna energija sile.**
23. **Zakon promene količine kretanja tačke.**
24. **Zakon promene momenta količine kretanja tačke.**
25. **Zakon promene kinetičke energije tačke.**
26. **Zakon održanja totalne mehaničke energije tačke.**

20. Mere kretanja tačke

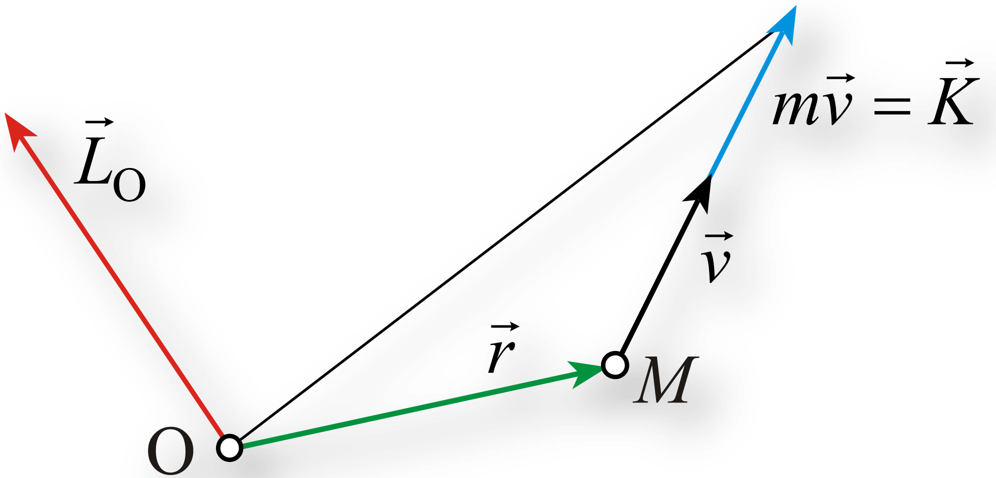
- **Količina kretanja**

$$\boxed{\vec{K} = m\vec{v}}$$



- **Moment količine kretanja**

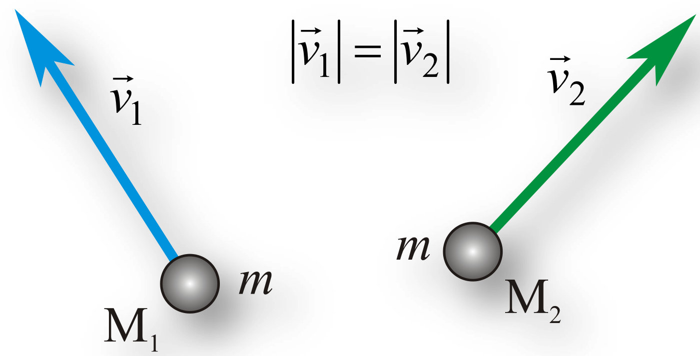
$$\boxed{\vec{L}_{\mathrm{O}} = \vec{r} \times \vec{K} = \vec{r} \times m\vec{v}}$$



20. Mere kretanja tačke

- **Kinetička energija**

$$\boxed{E_K = \frac{1}{2} m v^2 = \frac{1}{2} m(\vec{v} \cdot \vec{v})}$$



Kinetička energija (Dekartov, prirodni i polarni koordinatni sistem):

$$E_K = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$$E_K = \frac{1}{2} m \dot{s}^2$$

$$E_K = \frac{1}{2} m(\dot{r}^2 + r^2 \dot{\varphi}^2)$$

$$\begin{matrix} \vec{v}_1 \neq \vec{v}_2 \\ |\vec{v}_1| = |\vec{v}_2| \end{matrix} \Rightarrow$$

$$\vec{K}_1 = m\vec{v}_1 \neq m\vec{v}_2 = \vec{K}_2$$

$$E_{k1} = \frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 = E_{k2}$$

21. Impuls sile

- Elementarni impuls sile

$$\boxed{d\vec{I} = \vec{F}dt}$$

- Impuls sile

$$\boxed{\vec{I}_{01} = \int_{t_0}^{t_1} \vec{F}dt}$$

$$\vec{I}_{01} = \int_{t_0}^{t_1} \vec{F}dt = \int_{t_0}^{t_1} \left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right)dt = I_{01x}\vec{i} + I_{01y}\vec{j} + I_{01z}\vec{k}$$

$$I_{01x} = \int_{t_0}^{t_1} F_x dt, I_{01y} = \int_{t_0}^{t_1} F_y dt, I_{01z} = \int_{t_0}^{t_1} F_z dt$$

$$I_{01x} = \sqrt{\left(I_{01x}\right)^2 + \left(I_{01y}\right)^2 + \left(I_{01z}\right)^2}, \quad \cos\alpha_{\vec{I}_{01}} = \frac{I_{01x}}{I_{01}}, \ldots$$

21. Impuls sile

- **Primer:** Odrediti impuls konstantne sile na vremenskom intervalu t_0-t_1.

- Kada se impuls može izračunati sile bez poznavanja kratanja?

$$\vec{F} = \vec{F}(t, \vec{r}, \vec{v})$$

$$1. \quad \vec{F} = \overrightarrow{const} \rightarrow \vec{I}_{01} = \int_{t_0}^{t_1} \vec{F} dt = \vec{F} \int_{t_0}^{t_1} dt = \vec{F}(t_1 - t_0)$$

$$2. \quad \vec{F} = \vec{F}(t) \rightarrow \vec{I}_{01} = \int_{t_0}^{t_1} \vec{F}(t) dt$$

ПР. 1 $\qquad \vec{F} = \overrightarrow{const}$

$$d\vec{I} = \vec{F}\,dt \quad , \quad \vec{I}_{01} = \int_{t_0}^{t_1} d\vec{I} = \int_{t_0}^{t_1} \vec{F}\,dt = \vec{F}\int_{t_0}^{t_1} dt = \vec{F}\cdot t\Big|_{t_0}^{t_1}$$

$$\vec{I}_{01} = \vec{F}\cdot(t_1 - t_0) = \vec{F}\cdot\Delta t$$

ПР. 2 $\qquad \underbrace{\vec{F} = 1\,\vec{i} + t\,\vec{j}}$

$$d\vec{I} = \vec{F}\cdot dt = (1\,\vec{i} + t\,\vec{j})\,dt \rightarrow \vec{I}_{01} = \int_{t_0}^{t_1} d\vec{I} = \int_{t_0}^{t_1} (1\,\vec{i}\,dt + t\,\vec{j}\,dt)$$

$$\vec{I}_{01} = \left(\int_{t_0}^{t_1} dt\right)\vec{i} + \left(\int_{t_0}^{t_1} t\,dt\right)\vec{j} = t\Big|_{t_0}^{t_1}\vec{i} + \frac{t^2}{2}\Big|_{t_0}^{t_1}\vec{j}$$

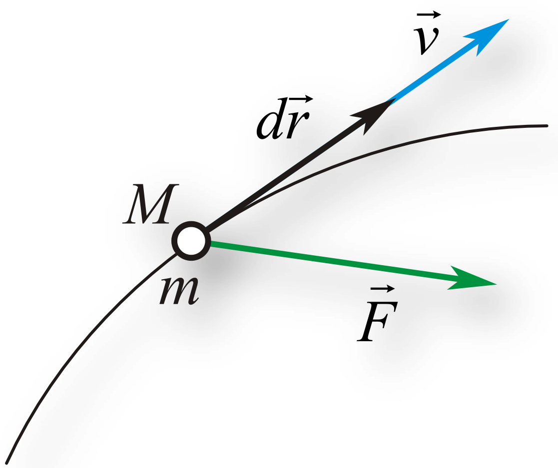
$$= \underbrace{(t_1 - t_0)}_{I_{01x}}\,\vec{i} + \frac{1}{2}\underbrace{(t_1^2 - t_0^2)}_{I_{01y}}\,\vec{j}$$

22. Rad sile

Rad sile

- Elemantarni rad sile

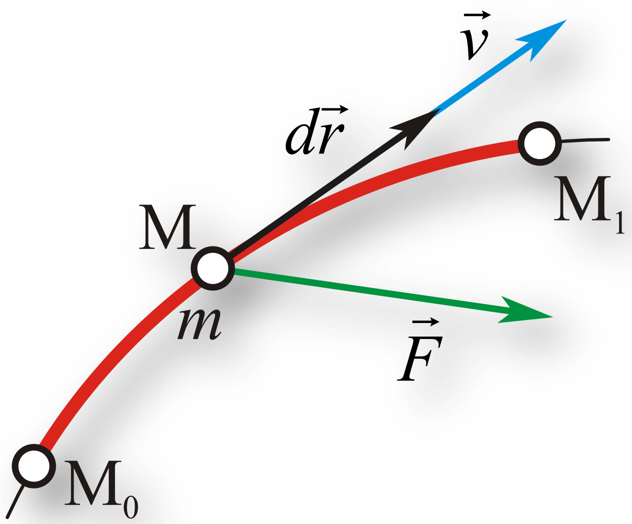
$$\boxed{dA = \vec{F} \cdot d\vec{r}}$$



$$dA = \left|\vec{F}\right|\left|d\vec{r}\right| \cos \angle(\vec{F}, d\vec{r})$$

- Rad sile

$$\boxed{A_{01} = \int_0^1 \vec{F} \cdot d\vec{r}}$$



Rad sile

- **Primer:** Odrediti rad konstantne sile na pomeranju taćke iz položaja M_0 u položaj M_1.

- Kada se može izračunati rad sile bez poznavanja kratenja?

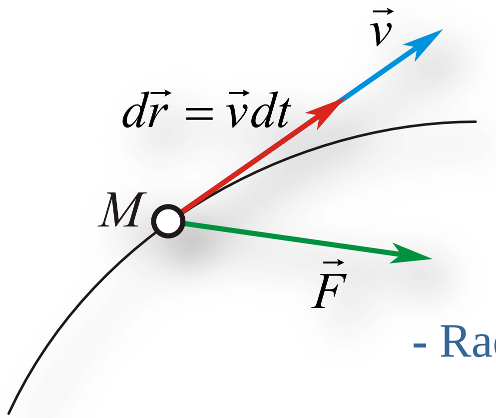
$$\vec{F} = \vec{F}(t, \vec{r}, \vec{v})$$

$$1. \quad \vec{F} = \overrightarrow{const} \to A_{01} = \int_{M_0}^{M_1} \vec{F} \cdot d\vec{r} = \vec{F} \cdot \int_{\vec{r}_0}^{\vec{r}_1} d\vec{r} = \vec{F} \cdot (\vec{r}_1 - \vec{r}_0)$$

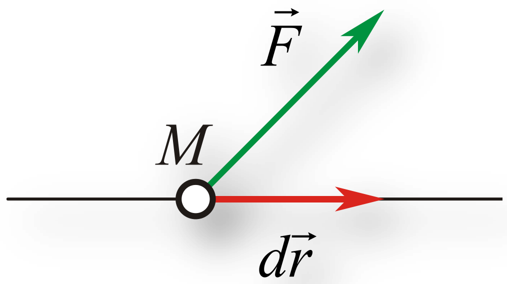
$$2. \quad \vec{F} = \vec{F}(\vec{r}) \to A_{01} = \int_{M_0}^{M_1} \vec{F}(\vec{r}) \cdot d\vec{r}$$

Rad sile

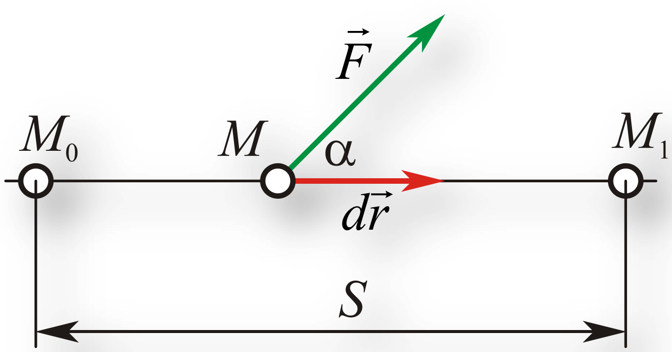
- Krivolinijsko kretanje



- Pravolinijsko kretanje



- Rad konstantne sile pri pravolinijskmo kretanju



$$\vec{F} = \overrightarrow{const}; F = const, \alpha = const$$

$$A_{01} = \int_{M_0}^{M_1} \vec{F} \cdot d\vec{r} = F \cos\alpha \; S$$

$$\alpha = 0 \rightarrow A_{01} = F \; S$$

$$\alpha = \pi / 2 \rightarrow A_{01} = 0$$

$$\alpha = \pi \rightarrow A_{01} = -F \; S$$

Rad sile

$$d\vec{r} = \vec{v}dt$$

Dekartove koordinate

$$\left.\begin{array}{l} d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k} \\ \vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} \end{array}\right\} \rightarrow A_{01} = \int_0^1 (F_x dx + F_y dy + F_z dz)$$

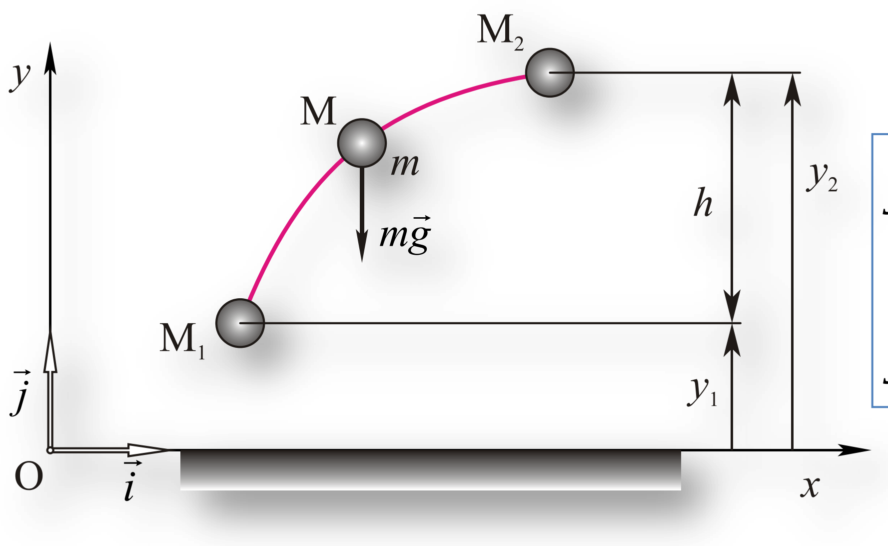
Prirodne koordinate

$$\left.\begin{array}{l} d\vec{r} = ds\vec{t} \\ \vec{F} = F_t\vec{t} + F_n\vec{n} + F_b\vec{b} \end{array}\right\} \rightarrow A_{01} = \int_0^1 F_t ds$$

Dekartove koordinate

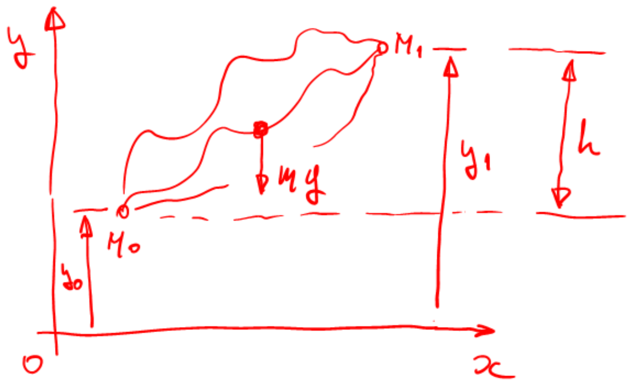
Rad sile - rad sile težine

$$\left.\begin{array}{l} d\vec{r} = dx\vec{i} + dy\vec{j} \\ \vec{F} = m\vec{g} = -mg\vec{j} \end{array}\right\} \to A_{12} = \int_{y_1}^{y_2} -mgdy = -mg(y_2 - y_1)$$



$$\left.\begin{array}{c} y_2 > y_1, y_2 - y_1 = h > 0, (\uparrow) \\ y_2 = y_1, (\rightarrow) \\ y_2 < y_1, y_1 - y_2 = h > 0, (\downarrow) \end{array}\right\} \to \left\{\begin{array}{l} A_{12} = -mgh \\ A_{12} = 0 \\ A_{12} = mgh \end{array}\right.$$

Rad sile - rad sile težine



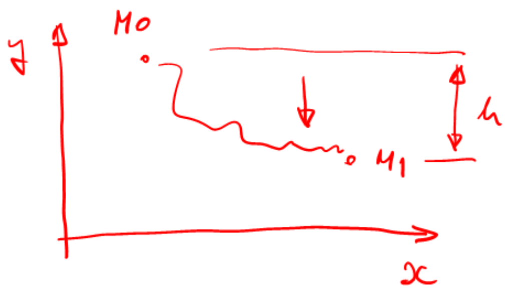
$$dA^{m\vec{g}} = m\vec{g}\cdot d\vec{r}$$

$$= (-mg\vec{j})\cdot(dx\,\vec{i} + dy\,\vec{j})$$

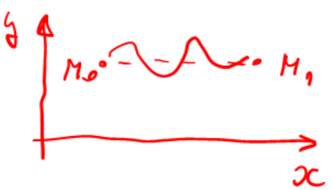
$$= -mg\,dy$$

$$A_{01}^{m\vec{g}} = \int_0^1 dA^{m\vec{g}} = -mg\int_{y_0}^{y_1} dy = -mg\,y\Big|_{y_0}^{y_1}$$

$$= -mg\,(y_1 - y_0) = -mgh$$



$$A_{01}^{m\vec{g}} = +mgh$$



$h = 0$

$$A_{01}^{m\vec{g}} = 0$$

Snaga sile

$$P = \frac{dA}{dt} = \vec{F} \cdot \vec{v} \quad \left[\frac{\mathrm{J}}{\mathrm{s}} = \frac{\mathrm{Nm}}{\mathrm{s}} \right] = [\mathrm{W}]$$

1 hp = 746 W = 0.746 kW

Primer: Dizač podiže teg mase 200kg brzinom 2m/s. Kolika je snaga dizača - sile kojom on podiže teg.

$$P = \vec{F} \cdot \vec{v} = mgv \approx 4000\mathrm{W}$$

Umesto zahvaljivanja na pažnji...

- Jugo 45

- Traktor IMT 539

Ko ima veću snagu?

33 kW (45 hp)

29,5 kW (39 hp)

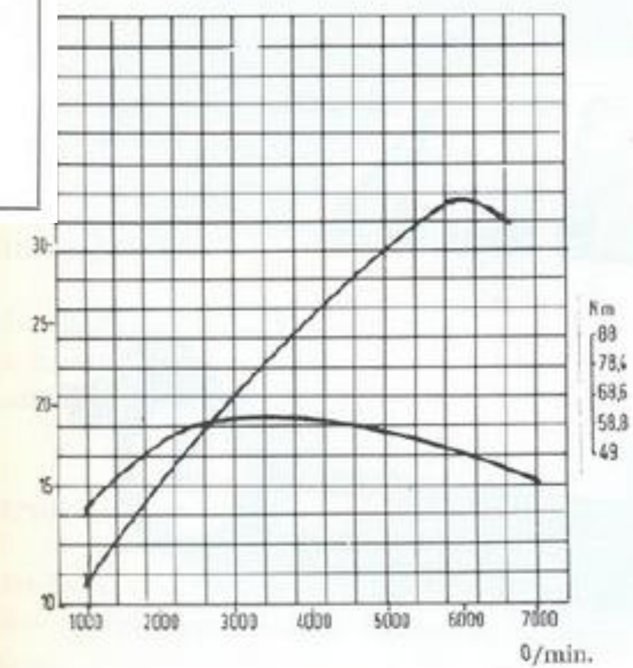
1 kW= 1.34 hp

1 hp= 0,7457 kW

Najprodavaniji model IMT-a.
Glavna razlika u odnosu na 533 jeste motor od 39 KS, ...

PODACI ZA IDENTIFIKACIJU

N A Z I V	Tip šasije	Tip motora	
		900 cm³	1100 cm³
JUGO 45 — 55	VX 1 145 A 00	100 GL 064	128 A. 064
MOTOR			
Smeštaj		napred, popreko	
Ciklus		Otto, četvorotaktni	
Broj i raspored cilindara		4, u liniji	
Prečnik cilindara . . . . mm		65	80
Hod klipova . . . . mm		68	55,5
Ukupna radna zapremina . . . . cm³		903	1116
Stepen kompresije		9	9,2
Maksimalna snaga (DIN) . . . . kW		33,1	40,4
Broj obrtaja pri kome motor daje maksimalnu snagu . . . . o/min		5.800	6.000
Max. obrtni moment (DIN) . . . . Nm		62,8	77,4
Broj obrtaja pri kome motor daje max. obrtni moment . . . o/min		3.300	3.000



Sl. 1 — Karakteristične krive za motore tip 100 GL 064 snimljene DIN metodom

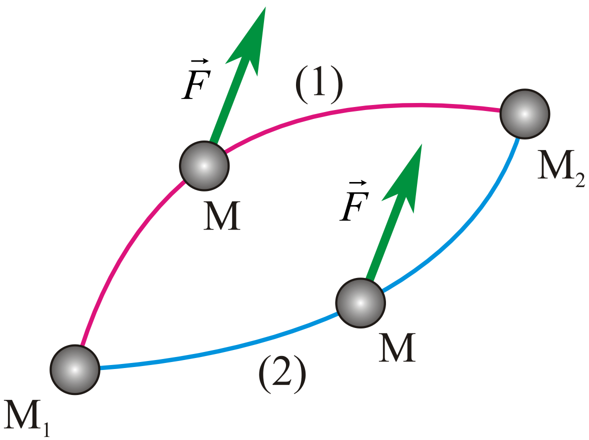
1.3. TEHNIČNI PODACI

Motor

Proizvođač	Industrija motora Rakovica
Tip	M33/T-LP četvorotaktni dizel motor
Broj cilindara	3
Prečnik cilindara	91,4 mm
Hod klipa	127 mm
Radna zapremnina	2500 cm^3
Stepen kompresije	17,4:1
Red paljenja	1—2—3
***Snaga motora na zamajcu** pri 2000 min^{-1}	**28,7 KW (39 KS)**
Snaga na priključnom vratilu pri 2000 min^{-1}	**26,5 KW (36,5 KS)**
***Maksimalni obrtni momenat pri 1300 min^{-1}**	**15,3 daNm (kpm)**
Košuljice cilindara	Zamenjive, suve, livene
Podmazivanje	Uljem pod pritiskom, pomoću rotacione pumpe

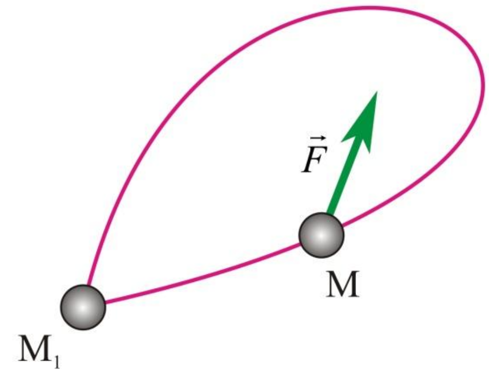
Potencijalna energija sile

- **Potencijalne sile**



$$A_{12}^{(1)} = \int_1^2 \vec{F} \cdot d\vec{r}_1 = \int_1^2 \vec{F} \cdot d\vec{r}_2 = A_{12}^{(2)}$$

$$A_{11} = \oint \vec{F} \cdot d\vec{r} = 0$$



- **Potencijalna energija-rad potencijalne sile**

$$\Pi = \Pi(\vec{r}) = \Pi(x, y, z)$$

$$dA = -d\Pi$$

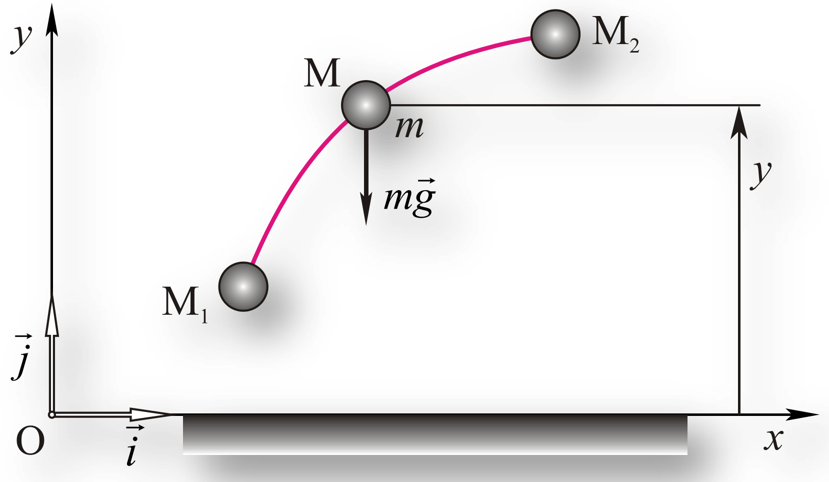
$$A_{01} = \Pi_0 - \Pi_1$$

- **potencijalne sile**: gravitaciona sila, sila težine, sila u opruzi,...

-**nepotencijalne sile**: sila trenja, sila fluidnog otpora,...

Potencijalne sile – potencijalna energija

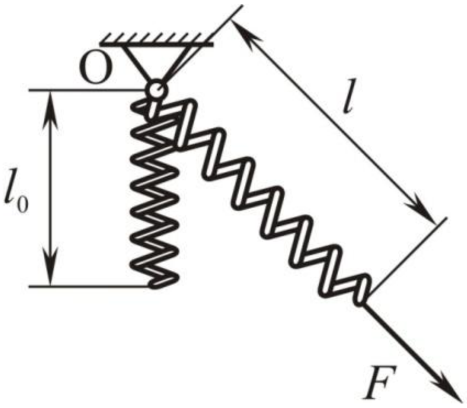
Sila težine



$$\Pi_g = mgy, \quad \Pi_g = \pm mgd$$

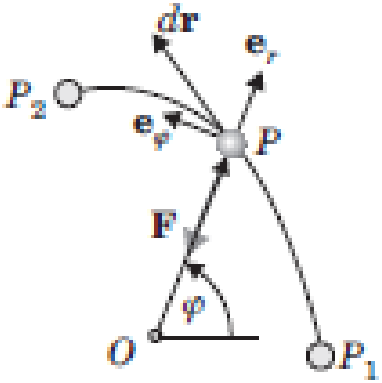
Sila u opruzi

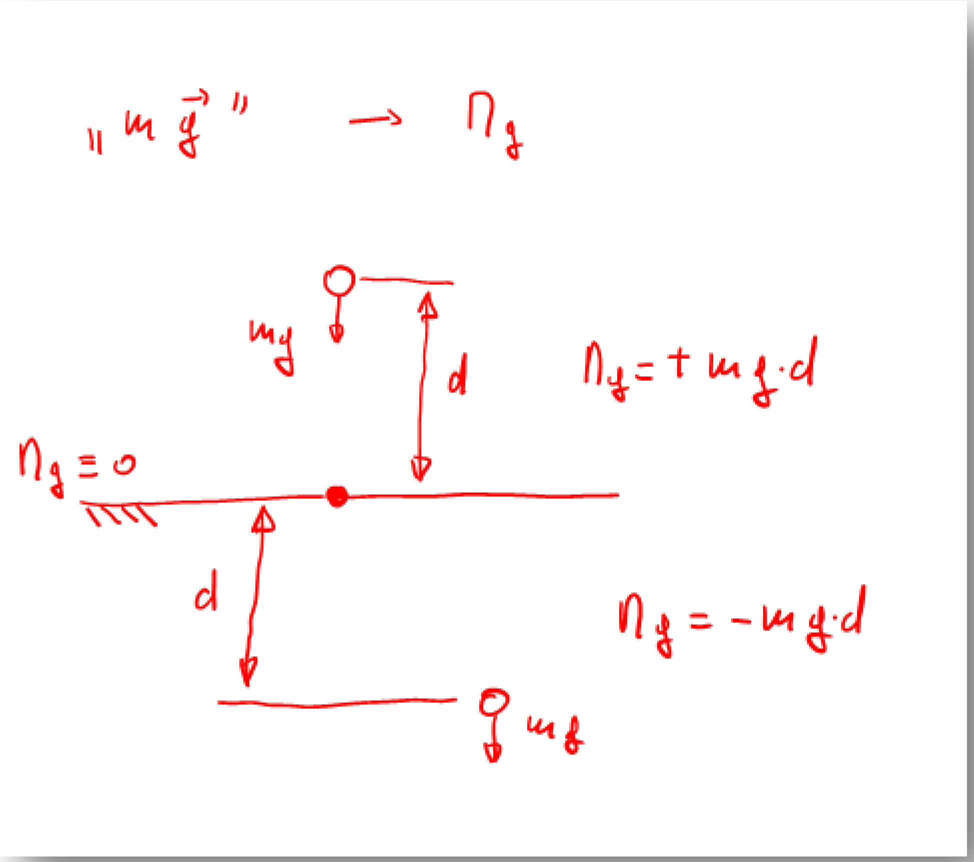
$$F_c = c\Delta l, \Pi_c = \frac{1}{2}c(\Delta l)^2$$



Gravitaciona sila

$$F = -\frac{\gamma}{r^2}, \quad \Pi = -\frac{\gamma}{r}$$




„mg⃗" → Πg
mg
d
Πg = + mg·d
Πg = 0
d
Πg = − mg·d
mg

Uslovi potencijalnosti sile

$$\left.\begin{array}{r} dA = \vec{F}d\vec{r} = F_x dx + F_y dy + F_z dz \\ d\Pi = d\Pi(x,y,z) = \dfrac{\partial \Pi}{\partial x}dx + \dfrac{\partial \Pi}{\partial y}dy + \dfrac{\partial \Pi}{\partial z}dz \end{array}\right\} \rightarrow dA = -d\Pi \rightarrow$$

$$F_x dx + F_y dy + F_z dz = -\left(\frac{\partial \Pi}{\partial x}dx + \frac{\partial \Pi}{\partial y}dy + \frac{\partial \Pi}{\partial z}dz\right) \rightarrow$$

$$\boxed{F_x = -\frac{\partial \Pi}{\partial x}, F_y = -\frac{\partial \Pi}{\partial y}, F_z = -\frac{\partial \Pi}{\partial z}}$$

$$\boxed{\vec{F} = -\vec{\nabla}\Pi}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

Uslovi potencijalnosti sile

$$\vec{\nabla}\times\vec{F}=\vec{\nabla}\times\left(-\vec{\nabla}\Pi\right)=-\left(\vec{\nabla}\times\vec{\nabla}\right)\Pi=0$$

$$\vec{\nabla}\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ F_x & F_y & F_z\end{vmatrix}=0$$

$$\boxed{\frac{\partial F_y}{\partial x}=\frac{\partial F_x}{\partial y},\ \frac{\partial F_z}{\partial y}=\frac{\partial F_y}{\partial z},\ \frac{\partial F_x}{\partial z}=\frac{\partial F_z}{\partial x}}$$

Uslovi potencijalnosti sile

$$dA = -d\Pi$$

$$\left.\begin{aligned} dA = \vec{F}d\vec{r} = F_x dx + F_y dy \\ d\Pi = d\Pi(x,y) = \frac{\partial \Pi}{\partial x}dx + \frac{\partial \Pi}{\partial y}dy \end{aligned}\right\} \rightarrow F_x dx + F_y dy = -\left(\frac{\partial \Pi}{\partial x}dx + \frac{\partial \Pi}{\partial y}dy\right) \rightarrow$$

$$F_x = -\frac{\partial \Pi}{\partial x}, F_y = -\frac{\partial \Pi}{\partial y}$$

$$\left.\begin{aligned} \frac{\partial F_x}{\partial y} = -\frac{\partial}{\partial y}\frac{\partial \Pi}{\partial x} = -\frac{\partial^2 \Pi}{\partial y \partial x} \\ \frac{\partial F_y}{\partial x} = -\frac{\partial}{\partial x}\frac{\partial \Pi}{\partial y} = -\frac{\partial^2 \Pi}{\partial x \partial y} \end{aligned}\right\} \rightarrow \quad \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

Opšti zakoni dinamike materijalne tačke

23. Zakon promene količine kretanja

$$\frac{d\vec{K}}{dt} = \vec{F} \rightarrow d\vec{K} = \vec{F}dt \rightarrow \quad \boxed{d\vec{K} = d\vec{I}}$$

$$\boxed{\vec{K}_1 - \vec{K}_0 = \vec{I}_{01}}$$

Zakon o promeni količine kretanja

кол. кр.

$$\vec{K} = m\vec{v}$$

II Њ.З.

$$m\vec{a} = \vec{F}$$

$$m\frac{d\vec{v}}{dt} = \vec{F}$$

$$m = const$$

$$\frac{d}{dt}(m\vec{v}) = \vec{F}$$

$$\frac{d\vec{K}}{dt} = \vec{F}, \quad \vec{F} = \vec{F}_r = \sum \vec{F}_i$$

$$d\vec{K} = \vec{F}dt$$

$$* \quad d\vec{K} = d\vec{I} \quad / \int_0^1$$

$$\vec{K}_1 - \vec{K}_0 = \int_0^1 d\vec{I}$$

$$* \quad \boxed{\vec{K}_1 - \vec{K}_0 = \vec{I}_{01}}$$

$$m\vec{v}_1 - m\vec{v}_0 = \vec{I}_{01}$$

први инт.

- **Primer:** Materijalna tačka, mase m=1kg, kreće se pravolinijski pod dejstvom sile $\vec{F}=10(1-t)\vec{i}$. Odrediti trenutak t_1 u kome tačka menja smer kretanja - $\dot{x}(0)=20\text{m/s}$.

$$\vec{k}_1-\vec{k}_0=\vec{I}_{01}\rightarrow m\vec{v}_1-m\vec{v}_0=\int_{t_0=0}^{t_1}\vec{F}dt$$

$$1(0\vec{i})-1(20\vec{i})=\int_0^{t_1}10(1-t)\vec{i}\,dt\rightarrow-20\vec{i}=(10t_1-5t_1^{\,2})\vec{i}$$

$$5t_1^{\,2}-10t_1-20=0\rightarrow t_1=1+\sqrt{5}$$

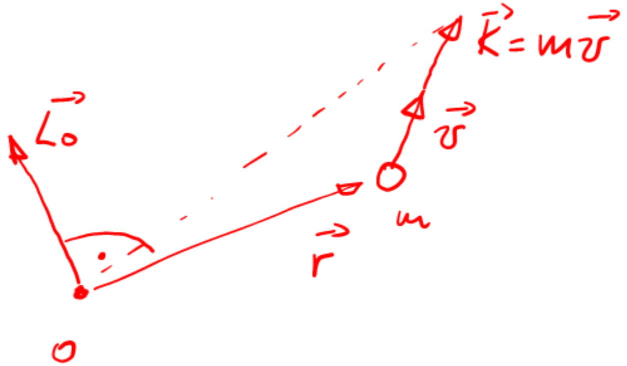
24. Zakon promene momenta količine kretanja

$$m\vec{a} = \vec{F} \rightarrow \vec{r} \times m\vec{a} = \vec{r} \times \vec{F} \rightarrow$$

$$\boxed{\dot{\vec{L}}_O = \vec{M}_O^{\vec{F}}}$$

$$\dot{\vec{L}}_O = \frac{d}{dt}(\vec{r} \times m\vec{v}) = \vec{v} \times m\vec{v} + \vec{r} \times m\vec{a} = \vec{r} \times m\vec{a}$$

24. Zakon promene momenta količine kretanja



$$\vec{L}_O = \vec{r} \times \vec{K}$$
$$= \vec{r} \times m\vec{v}$$

$$\frac{d\vec{L}_O}{dt} = \frac{d}{dt}\left(\vec{r} \times m\vec{v}\right) = \frac{d\vec{r}}{dt} \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= \cancelto{0}{\vec{v} \times m\vec{v}} + \underline{\vec{r} \times m\vec{a}}$$

Iz. 3. $\quad \vec{r} \times \big| \quad m\vec{a} = \vec{F}$

$$\vec{r} \times m\vec{a} = \vec{r} \times \vec{F}$$

$$\boxed{\frac{d\vec{L}_O}{dt} = \vec{M}_O^{\vec{F}}}$$

25. Zakon promene kinetičke energije tačke

$$m\vec{a} = \vec{F} \rightarrow m\frac{d\vec{v}}{dt} = \vec{F} \rightarrow m\frac{d\vec{v}}{dt} \cdot d\vec{r} = \vec{F} \cdot d\vec{r} \rightarrow$$

$$md\vec{v} \cdot \vec{v} = \vec{F} \cdot d\vec{r} \rightarrow d(\frac{1}{2}m\vec{v} \cdot \vec{v}) = \vec{F} \cdot d\vec{r}$$

$$\boxed{dE_K = dA}$$

$$\boxed{E_{K1} - E_{K0} = A_{01}}$$

25. Zakon promene kinetičke energije tačke

$$\boxed{E_k = \frac{1}{2} m v^2}$$

II Њ.З. $\quad m\vec{a} = \vec{F}$

$$m \frac{d\vec{v}}{dt} = \vec{F} \quad / \cdot d\vec{r} \quad \rightarrow \quad m\, d\vec{v}\, \frac{d\vec{r}}{dt} = \vec{F} \cdot d\vec{r}, \qquad m\, d\vec{v} \cdot \vec{v} = dA$$

$$d(\vec{v} \cdot \vec{v}) = d\vec{v} \cdot \vec{v} + \vec{v} \cdot d\vec{v} = 2\, d\vec{v} \cdot \vec{v} \quad \rightarrow \quad d\vec{v} \cdot \vec{v} = \frac{1}{2} d(\vec{v} \cdot \vec{v})$$

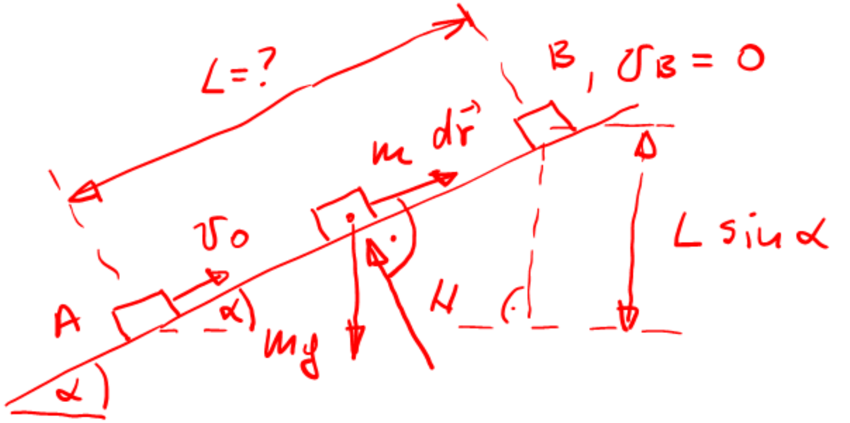
$$\frac{1}{2} m\, d(\vec{v} \cdot \vec{v}) = dA \quad \rightarrow \quad d\left(\frac{1}{2} m v^2\right) = dA \quad \rightarrow \quad \boxed{dE_k = dA}\ ^*$$

$$* \int_0^1 \rightarrow \boxed{E_{k1} - E_{k0} = A_{01}}$$

$$\Delta E_{k01} = A_{01}$$

$$\boxed{\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 = A_{01}}$$ први интеграл

$$\frac{dE_k}{dt} = \frac{dA}{dt} \rightarrow \boxed{\frac{dE_k}{dt} = P}$$



$$dA^{\vec{N}} = \underbrace{\vec{N} \cdot d\vec{r}}_{\perp} = 0$$

$$\vec{v} = \frac{d\vec{r}}{dt} \rightarrow \underline{\underline{d\vec{r} = \vec{v}\,dt}}$$

$$E_{kB} - E_{kA} = \cancel{A_{AB}^{\vec{N}}}^{\,0} + A_{AB}^{m\vec{g}}$$

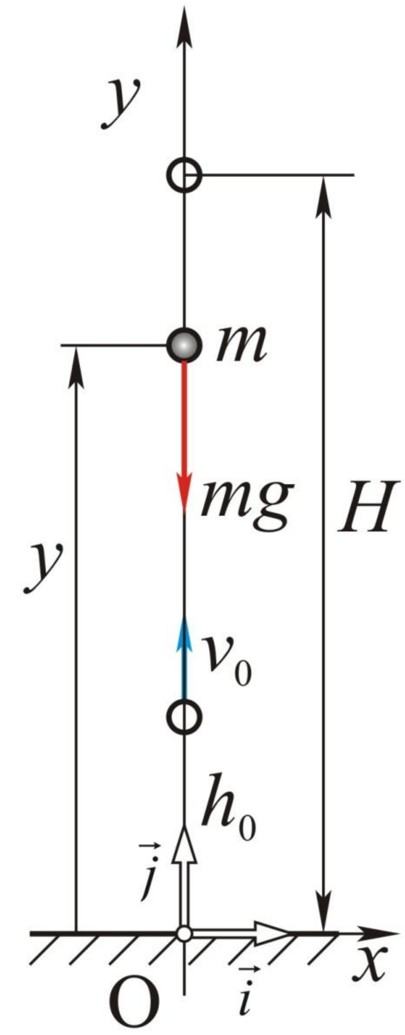
$$\underbrace{\cancel{\tfrac{1}{2} m v_B^2}_{\,0} - \tfrac{1}{2} m v_0^2 = -mg \cdot L \sin\alpha} \rightarrow L = \frac{v_0^2}{2g \sin\alpha}$$

- **Primer**: Materijalna tačka, mase *m*, kreće se u homogenom polju sile zemljine teže. Sa visine h_0 iznad površine Zemlje bačena je vertikalno uvis brzinom v_0. Zanemarujući sve sile osim sile težine odrediti visinu leta *H*.

$$E_{K1} - E_{K0} = A_{01}^{m\vec{g}}$$

$$\frac{1}{2}m(0)^2 - \frac{1}{2}m{v_0}^2 = -mg(H - h_0)$$

$$H = \frac{{v_0}^2}{2g} + h_0$$



26. Zakon održanja totalne mehaničke energije tačke

$$\boxed{E = E_K + \Pi}$$

$$\left.\begin{aligned} dE_K &= dA = dA^{pot} + dA^{nep} \\ dA^{nep} &= 0 \\ dA &= -d\Pi \end{aligned}\right\} \rightarrow dE_K = -d\Pi \rightarrow d(E_K + \Pi) = 0 \rightarrow$$

$$\boxed{E_K + \Pi = E = const}$$

$$\boxed{E_{K1} + \Pi_1 = E_{K0} + \Pi_0}$$

26. Zakon održanja totalne mehaničke energije tačke

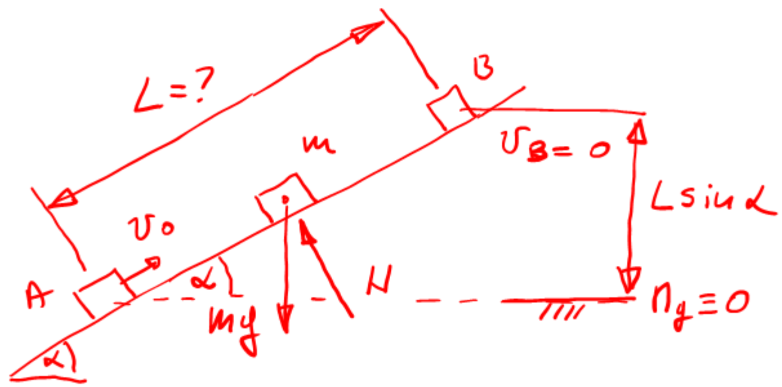
$$E = E_k + \Pi$$

$$dE_k = dA, \quad dE_k = dA^{POT.} + dA^{NEPOT.}, \quad dE_k = -d\Pi + dA^{NEPOT.}$$

$$dA^{POT.} = -d\Pi \qquad d(E_k + \Pi) = dA^{NEPOT.}$$

$$\text{спец.} \quad dA^{NEPOT.} = 0 \quad \longrightarrow \quad d(E_k + \Pi) = 0 \quad \longrightarrow \quad \boxed{E_k + \Pi = const}$$

$$\boxed{E_{k1} + \Pi_1 = E_{k0} + \Pi_0}$$



$$dA^{\vec N} = 0$$

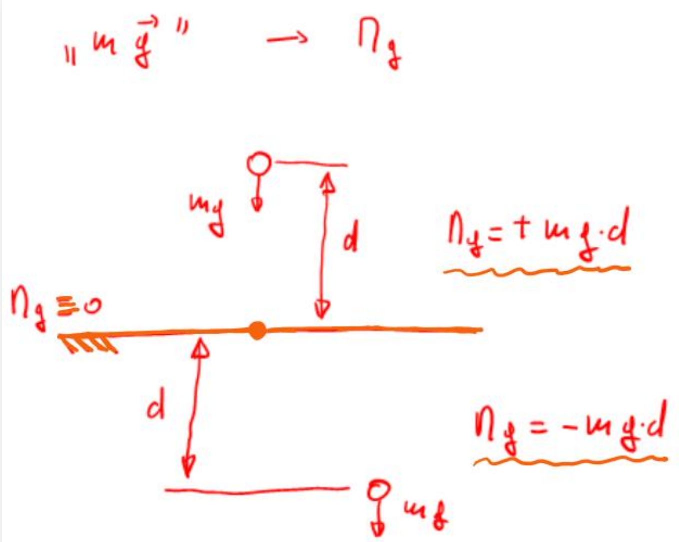
$m\vec g$ – пот. сила

$$E = E_k + \Pi = const$$

$$\underbrace{\cancel{E_{kB}}}_{0} + \Pi_B = E_{kA} + \overbrace{\cancel{\Pi_A}}^{0}$$

$$+mg\cdot L\sin\alpha = \frac{1}{2} m v_0^2$$

$$L = \frac{v_0^2}{2g\sin\alpha}$$



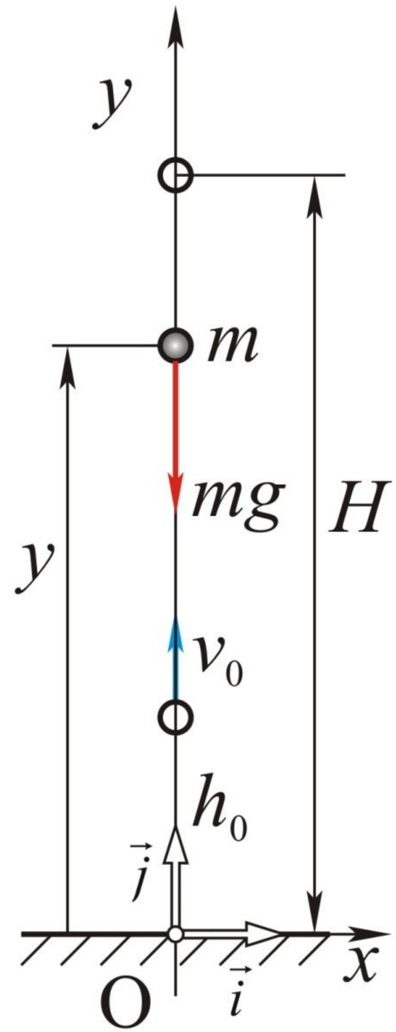
Primer

- Materijalna tačka, mase m, kreće se u homogenom polju sile zemljine teže. Sa visine h_0 iznad površine Zemlje bačena je vertikalno uvis brzinom v_0. Zanemarujući sve sile osim sile težine odrediti visinu leta H.

$$E_{K1} + \Pi_1 = E_{K0} + \Pi_0$$

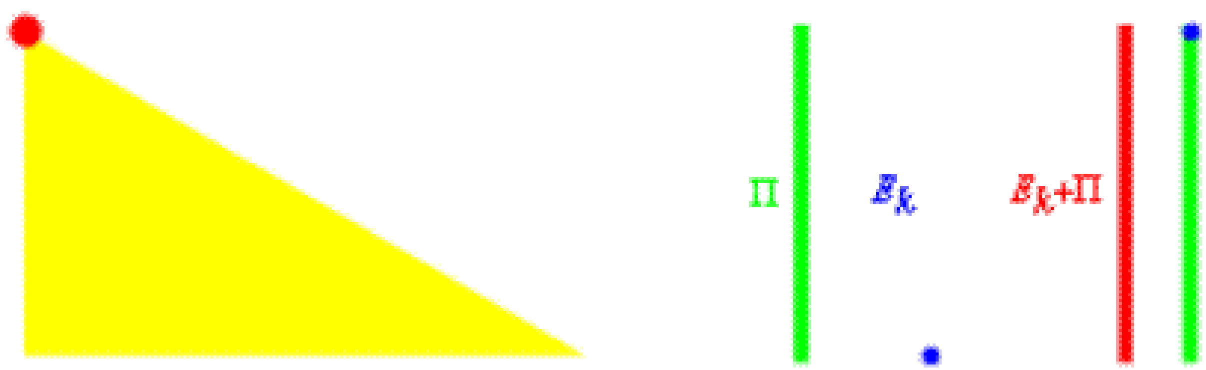
$$\frac{1}{2} m(0)^2 + mgH = \frac{1}{2} m{v_0}^2 + mgh_0$$

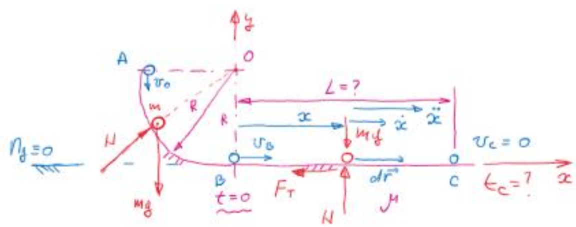
$$H = \frac{{v_0}^2}{2g} + h_0$$



Primer

- Materijalna tačka, mase m, kreće bez početne brzine niz glatku strmu ravan nagibnog ugla α. Kolika je brzina tačke u podnožju strme ravni, ako je dužina strme ravni L.





„A – B"

$$E_{kB} - E_{kA} = \cancel{A_{AB}^{\vec N}}^{\,0} + A_{AB}^{m\vec g}$$

$$\frac{1}{2}\cancel{m} v_B^2 - \frac{1}{2}\cancel{m} v_0^2 = +\cancel{m} g R$$

$$v_B^2 = v_0^2 + 2gR$$

$$v_B = \sqrt{v_0^2 + 2gR}$$

$$E_{kB} + \cancel{\Pi_B}^{\,0} = E_{kA} + \Pi_A$$

$$\frac{1}{2} m v_B^2 + 0 = \frac{1}{2} m v_0^2 + mg\cdot R$$

$$v_B = \sqrt{v_0^2 + 2gR}$$

„B – C"

$$E_{kC} + \Pi_C \neq E_{kB} + \Pi_B$$

I $\quad m\vec a = m\vec g + \vec N + \vec F_T \quad /\cdot\vec i \quad /\cdot\vec j$

(1) $m\ddot x = -F_T$

(2) $0 = N - mg$

(3) $F_T = \mu N$

(2) → $N = mg = const$

(3) → $F_T = \mu m g = const$

(1) → $m\ddot x = -\mu m g$ → * $\boxed{\ddot x = -\mu g = const < 0}$

РЕПАРАМЕТРИЗАЦИЈА

$$\ddot x = \frac{d\dot x}{dt} = \frac{dx}{dt}\,\frac{d\dot x}{dx} = \dot x\,\frac{d\dot x}{dx}$$

$$* \to \quad \dot x\,\frac{d\dot x}{dx} = -\mu g \quad \to \quad \int_{\dot x(0)=v_B}^{\dot x(t_C)=v_C=0} \dot x\, d\dot x = -\mu g \int_{x(0)=0}^{x(t_C)=L} dx$$

$$\frac{\dot x^2}{2}\Big|_{v_B}^{0} = -\mu g\, x\Big|_{0}^{L} \quad \to \quad \frac{0^2}{2} - \frac{v_B^2}{2} = -\mu g(L-0)$$

$$\boxed{L = \frac{v_B^2}{2\mu g} = \frac{v_0^2 + 2gR}{2\mu g}}$$

I $\quad \cancel{E_{kC}}^{\,0} - E_{kB} = \cancel{A_{BC}^{\vec N}}^{\,0} + \cancel{A_{BC}^{m\vec g}}^{\,0} + A_{BC}^{\vec F_T}$

$$\boxed{-\frac{1}{2} m v_B^2 = -F_T\cdot L}$$

$$F_T = \mu m g = const$$

$$-\frac{1}{2} m v_B^2 = -\mu m g\cdot L \quad \to \quad L = \dots$$

$$dA^{\vec F_T} = \vec F_T\cdot d\vec r = (-\mu m g\,\vec i)\cdot(dx\,\vec i + \cancel{dy\,\vec j}^{\,0}) = -\mu m g\, dx$$

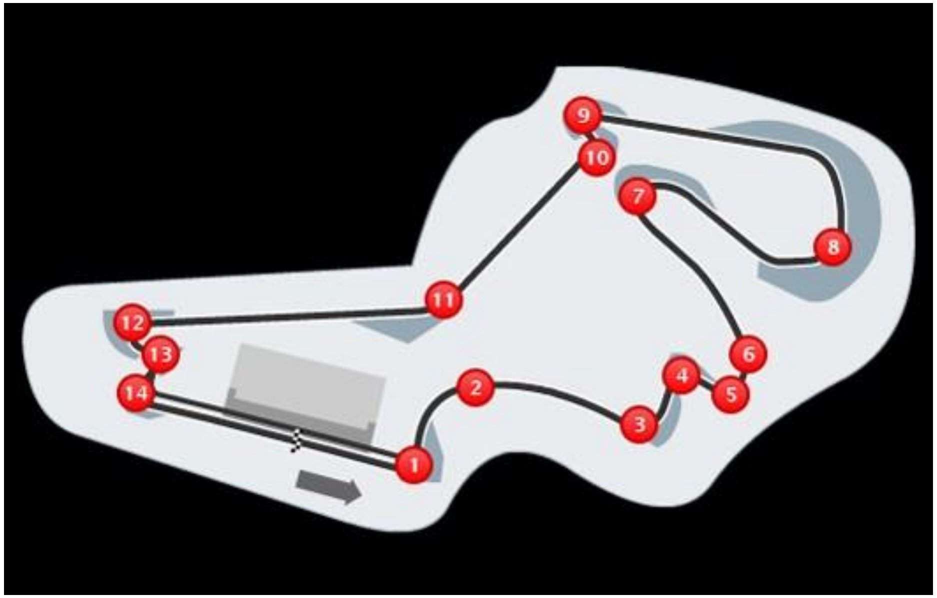
$$A_{BC}^{\vec F_T} = -\mu m g \int_{x_B=0}^{x_C=L} dx = -\mu m g\cdot L$$

Umesto zahvaljivanja na pažnji...

Velika nagrada Turske, poslednji put?

Svaka priča o stazi u Turskoj počinje sa fantastičnom Krivinom 8…

Ovaj levi zavoj na stazi u Turskoj ima tri temena, i **vozači trpe silu do čak 5G**, u ukupnom trajanju dužem od 7 sekundi.



Šta znači:
“vozači trpe silu do čak 5G”?

http://www.rts.rs/page/sport/sr/story/133/auto-moto/887258/velika-nagrada-turske-poslednji-put.html

Šta smo naučili?

20. Mere kretanja tačke (količina kretanja, moment količine kretanja i kinetička energija).
21. Impuls sile.
22. Rad sile. Snaga sile. Potencijalna energija sile.
23. Zakon promene količine kretanja tačke.
24. Zakon promene momenta količine kretanja tačke.
25. Zakon promene kinetičke energije tačke.
26. Zakon održanja totalne mehaničke energije tačke.

Dinamika
Dinamika materijalne tačke – Zakoni kretanja materijalne tačke, ...

Kinematika i dinamika

Miodrag Zuković

Novi Sad, 2021.