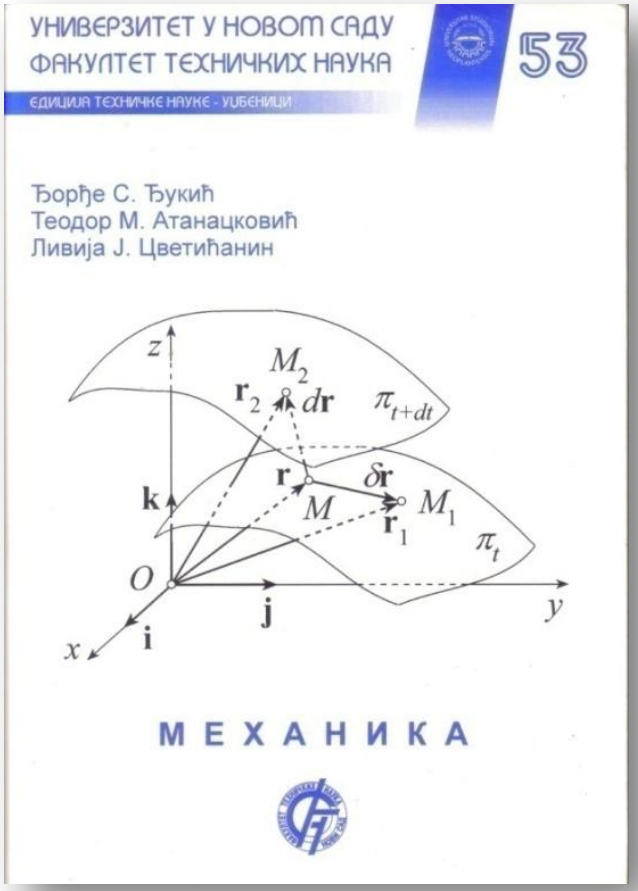
Mehanika 2 (Kinematika)
Vežbe 3

Miodrag Zuković

Novi Sad, 2023.

Literatura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ФАКУЛТЕТ ТЕХНИЧКИХ НАУКА
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ЕДИЦИЈА ТЕХНИЧКЕ НАУКЕ - УЏБЕНИЦИ
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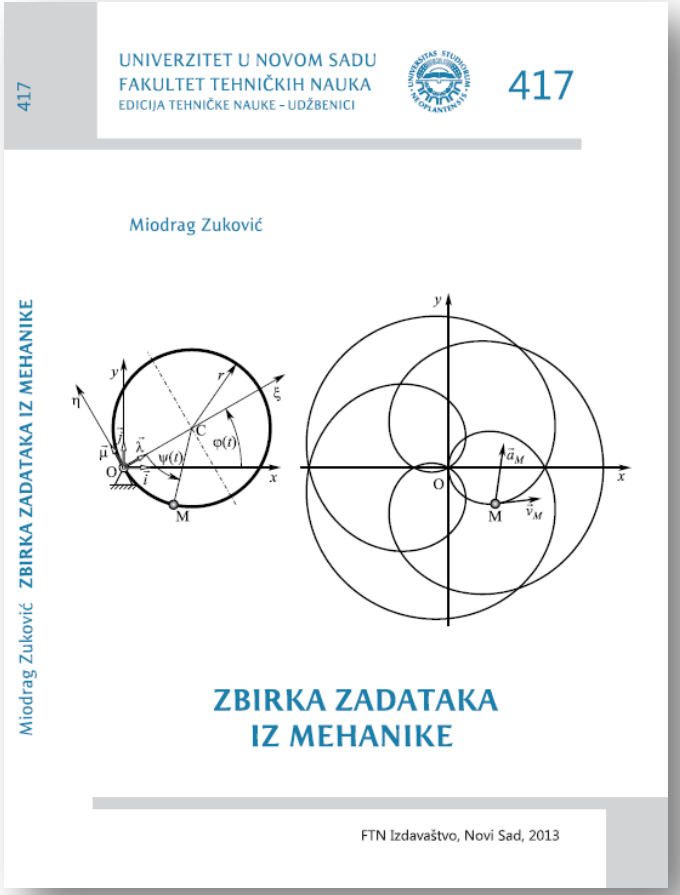
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UNIVERZITET U NOVOM SADU
FAKULTET TEHNIČKIH NAUKA
EDICIJA TEHNIČKE NAUKE - UDŽBENICI
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Miodrag Zuković ZBIRKA ZADATAKA IZ MEHANIKE
ZBIRKA ZADATAKA
IZ MEHANIKE
FTN Izdavaštvo, Novi Sad, 2013

Šta ćemo naučiti?

7. Kinematika tačke - prirodni koordinatni sistem
8. Kretanje tačke po kružnici

Zadatak 1

1. Kretanje tačke u ravni zadato je parametarskim jednačinama kretanja:

$$x(t) = 4 + 4\cos t$$

$$y(t) = 2 + 2\sin t$$

Odrediti i nacrtati trajektoriju kretanja tačke. Odrediti položaj, brzinu, ubrzanje tačke i poluprečnik krivine trajektorije u trenutku t_1 u kom se tačka prvi put nađe na y osi.

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$t_1 = ?$ → HA y OSI

$x(t_1) = 0 \rightarrow x(t_1) = 4 + 4\cos t_1 = 0$

$4\cos t_1 = -4$

$\cos t_1 = -1$

$t_1 = \pi, 3\pi, \ldots$

$t_1 = \pi$

(1) $x = 4 + 4\cos t$

(2) $y = 2 + 2\sin t$

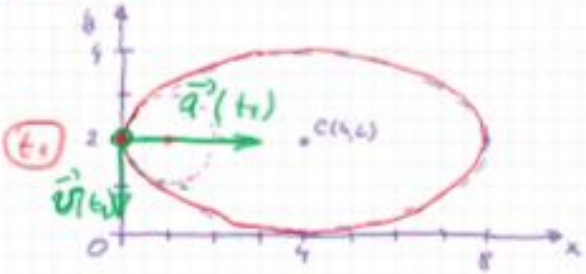
} el. t →

$\frac{x-4}{4} = \cos t \,/^2$

$\frac{y-2}{2} = \sin t \,/^2$

$\frac{(x-4)^2}{4^2} + \frac{(y-2)^2}{2^2} = 1$ JA

ELIPSA



OK $t \geq 0$

$-1 \leq \frac{\cos t}{\sin t} \leq 1$

(1) → $0 \leq x \leq 8$

(2) → $0 \leq y \leq 4$

(1) $x(t) = 4 + 4\cos t$

(2) $y(t) = 2 + 2\sin t$

$\dot{x}(t) = -4\sin t$

$\dot{y}(t) = 2\cos t$

$v(t) = \sqrt{16\sin^2 t + 4\cos^2 t}$

$\ddot{x}(t) = -4\cos t$

$\ddot{y}(t) = -2\sin t$

$a(t) = \sqrt{16\cos^2 t + 4\sin^2 t}$

$t_1 = \pi$ $\quad \dot{x}(\pi) = -4\sin\pi = 0$ $\quad \ddot{x}(\pi) = -4\cos\pi = 4$

$\dot{y}(\pi) = 2\cos\pi = -2$ $\quad \ddot{y}(\pi) = -4\sin\pi = 0$

$v(\pi) = \sqrt{0^2 + (-2)^2} = 2$ $\quad a(\pi) = \sqrt{4^2 + 0^2} = 4$

$a_T^2 = \dot{v}^2$ $\quad v(t) = \sqrt{16\sin^2 t + 4\cos^2 t}$

$$\dot{v}(t) = \frac{16 \cdot 2\sin t\cos t + 4 \cdot 2\cos t \cdot (-\sin t)}{2\sqrt{16\sin^2 t + 4\cos^2 t}}$$

$t_1 = \pi$ $\quad \dot{v}(\pi) = \frac{16\sin\pi\cos\pi + 4\cos\pi(-\sin\pi)}{\sqrt{\quad}} = 0 \rightarrow$

$a_T(\pi) = 0$

$a_N(\pi) = \sqrt{a^2(\pi) - a_T^2(\pi)} = \sqrt{4^2 - 0^2} = 4$

$$R_K(\pi) = \frac{v^2(\pi)}{a_N(\pi)} = \frac{2^2}{4} = 1 \text{ m}$$

Zadatak 2

1. Kretanje tačke u ravni zadato je parametarskim jednačinama kretanja:

$$x(t) = 3t$$

$$y(t) = t^2 - 1$$

Odrediti i nacrtati trajektoriju kretanja tačke. Odrediti položaj, brzinu, ubrzanje tačke i poluprečnik krivine trajektorije u trenutku t_1 u kom se tačka nađe na x osi.

1. Kretanje tačke u ravni zadato je parametarskim jednačinama kretanja:

$$x(t) = 3t$$
$$y(t) = t^2 - 1$$

Odrediti i nacrtati trajektoriju kretanja tačke. Odrediti položaj, brzinu, ubrzanje tačke i poluprečnik krivine trajektorije u trenutku t_1 u kom se tačka nađe na x osi.

$t_1 = ?$ → HG x osa

$y(t_1) = 0$

$y(t_1) = t_1^2 - 1 = 0$

$t_1 = 1$, $t_1 = -1 < 0$

$t_1 = 1$

$\dot{x}(1) = 3$

$\dot{y}(1) = 2$

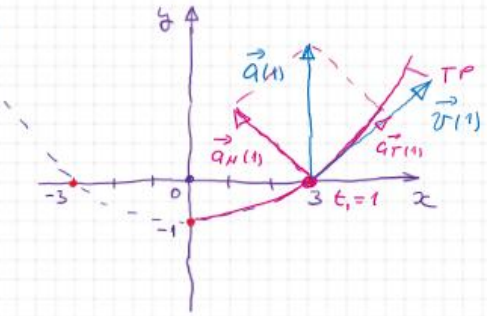
$v(1) = \sqrt{13}$

(1) $x = 3t \longrightarrow t = \frac{x}{3}$

(2) $y = t^2 - 1 \longrightarrow y = \frac{x^2}{9} - 1$ } JN

NAPAĐENA

$y = \frac{1}{9}(x^2 - 9)$



ok $t \geq 0$

(1) → $x \geq 0$

$\dot{x}(t) = 3 = const$

$\dot{y}(t) = 2t$

$* \; v(t) = \sqrt{9 + 4t^2}$

$\ddot{x}(t) = 0 = const$

$\ddot{y}(t) = 2 = const$

$a(t) = \sqrt{0^2 + 2^2} = 2 = const$

$a_N = \frac{v^2}{R_k} \rightarrow R_k = \frac{v^2}{a_N}$

$a_T^2 = \dot{v}^2 \quad * \rightarrow \dot{v}(t) = \frac{4t}{2\sqrt{9 + 4t^2}}$

$\dot{v}(t_1) = \dot{v}(1) = \frac{4}{\sqrt{13}}$

$a_T^2(1) = \dot{v}^2(1) = \left(\frac{4}{\sqrt{13}}\right)^2 = \frac{16}{13}$

$a_N(1) = \sqrt{a_{(1)}^2 - a_T^2(1)} = \sqrt{2^2 - \frac{16}{13}} = \sqrt{4 - \frac{16}{13}}$

$= \sqrt{\frac{36}{13}} = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{3}$

$R_K(1) = \frac{v^2(1)}{a_N(1)} = \frac{(\sqrt{13})^2}{\frac{6}{\sqrt{13}}} = \frac{13\sqrt{13}}{6}$ m

Zadatak 3

1. Kretanje tačke u ravni zadato je parametarskim jednačinama kretanja:

$$x(t) = e^{t}$$

$$y(t) = e^{2t}.$$

Odrediti trajektoriju kretanja tačke i nacrtati je. Odrediti položaj, brzinu i ubrzanje tačke kao i poluprečnik krivine trajektorije u početnom trenutku t_0=0.

1. Kretanje tačke u ravni zadato je parametarskim jednačinama kretanja:

$$x(t) = e^t$$
$$y(t) = e^{2t}.$$

Odrediti trajektoriju kretanja tačke i nacrtati je. Odrediti položaj, brzinu i ubrzanje tačke kao i poluprečnik krivine trajektorije u početnom trenutku $t_0=0$.

(1) $x = e^t$
(2) $y = e^{2t}$ $\Big\}$ $\xrightarrow{e^{\wedge}}$ $e^t = x$

$y = (e^t)^2 \Rightarrow y = x^2$

ok $t \geq 0$

(1) $x = e^t$

$x \geq 1$

(2) $y = e^{2t} \rightarrow y \geq 1$

(1) $x(t) = e^t$ $\Big\}$ $\dot{x}(t) = e^t$ $\Big\}$ $\ddot{x}(t) = e^t$

(2) $y(t) = e^{2t}$ $\Big\}$ $\dot{y}(t) = 2e^{2t}$ $\Big\}$ $\ddot{y}(t) = 4e^{2t}$

$* \quad v(t) = \sqrt{e^{2t} + 4e^{4t}}$ $\qquad a(t) = \sqrt{e^{2t} + 16e^{4t}}$

$t_0 = 0$

$\dot{x}(0) = e^0 = 1$ $\qquad \ddot{x}(0) = 1$

$\dot{y}(0) = 2e^0 = 2$ $\qquad \ddot{y}(0) = 4$

$v(0) = \sqrt{5}$ $\qquad a(0) = \sqrt{17}$

$* \Big| \frac{d}{dt} \rightarrow \quad \dot{v}(t) = \frac{2e^{2t} + 16e^{4t}}{2\sqrt{e^{2t} + 4e^{4t}}}$

$\dot{v}(0) = \frac{1+8}{\sqrt{5}} = \frac{9}{\sqrt{5}} \rightarrow a_T^2(0) = \dot{v}^2(0) = \frac{81}{5}$

$a_N(0) = \sqrt{a^2(0) - a_T^2(0)} = \sqrt{17 - \frac{81}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

$R_k(0) = \frac{v^2(0)}{a_N(0)} = \frac{(\sqrt{5})^2}{\frac{2}{\sqrt{5}}} = \frac{5\sqrt{5}}{2}$

Zadatak 4

1. Kretanje tačke u ravni zadato je parametarskim jednačinama kretanja:

$$x(t) = t^2$$

$$y(t) = -2 + 2t^2.$$

Odrediti trajektoriju kretanja tačke i nacrtati je. Odrediti položaj, brzinu i ubrzanje tačke u trenutku u kom se tačka nađe na osi x. Koliki put je prešla tačka do tog trenutka.

1. Kretanje tačke u ravni zadato je parametarskim jednačinama kretanja:

$$x(t) = t^2$$
$$y(t) = -2 + 2t^2.$$

Odrediti trajektoriju kretanja tačke i nacrtati je. Odrediti položaj, brzinu i ubrzanje tačke u trenutku u kom se tačka nađe na osi x. Koliki put je prešla tačka do tog trenutka.

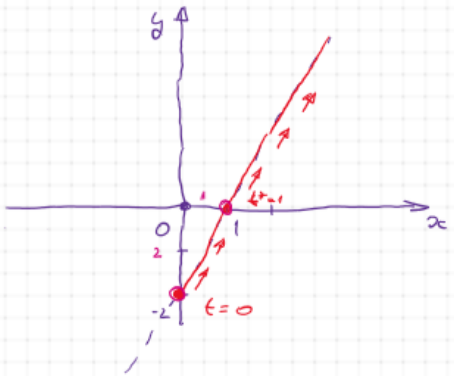
(1) $x = t^2$

(2) $y = -2 + 2t^2$ → $y = -2 + 2x$ | ∧∩

ПРАВА

OK $t \geq 0$

(1) $x \geq 0$



$t^* = ?$ → на x осу

$y(t^*) = 0$ → $y(t^*) = -2 + 2t^{*2} = 0$

$t^{*2} = 1$

$t^* = 1$; $t^* \neq -1$

$\dot{x}(t) = 2t$

$\dot{y}(t) = 4t$

$v(t) = \sqrt{4t^2 + 16t^2} = \sqrt{20t^2}$

$v(t) = 2\sqrt{5}\,t$

$\ddot{x}(t) = 2 = const$

$\ddot{y}(t) = 4 = const$

$a(t) = \sqrt{2^2 + 4^2}$

$a(t) = 2\sqrt{5} = const$

$R_k(t) = \infty$

$$s(t) = \pm \int_{t_0=0}^{t} v(t)\,dt$$

$$s(t) = + \int_0^t 2\sqrt{5}\,t\,dt = 2\sqrt{5} \cdot \frac{t^2}{2}\Big|_0^t = \sqrt{5}\,t^2$$

$P[t_0 = 0,\ t^* = 1] = |s(1) - s(0)| = |\sqrt{5} - 0| = \sqrt{5}$

Zadatak 5

1. Kretanje tačke dato je u polarnim koordinatama jednačinama:

$$r = k\left(1 + \cos\left(\frac{\pi}{2}t\right)\right)$$

$$\theta = \frac{\pi}{2}t$$

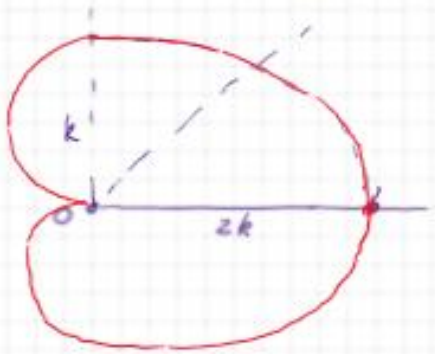
gde je k pozitivna konstanta, (θ u rad, r u m).
Odrediti jednačinu trajektorije, brzinu, ubrzanje i poluprečnik krivine u trenutku t =1 s.

1. Kretanje tačke dato je u polarnim koordinatama jednačinama:

$$r = k\left(1+\cos\left(\frac{\pi}{2}t\right)\right)$$

$$\varphi = \frac{\pi}{2}t$$

gde je k pozitivna konstanta, (φ u rad, r u m).
Odrediti jednačinu trajektorije, brzinu, ubrzanje i poluprečnik krivine u trenutku $t = 1$ s.



(1) $r = k + k\cos\left(\frac{\pi}{2}t\right)$

(2) $\varphi = \frac{\pi}{2}t$

$\in \wedge$ → t

$r = k + k\cos(\varphi)$

KARDIOIDA

Ok $t \geq 0$

(2) $\varphi \geq 0$

(1) $0 \leq r \leq 2k$

$\varphi = 0 \rightarrow r = 2k$

$\varphi = \frac{\pi}{4} \rightarrow r = k + k\frac{\sqrt{2}}{2}$

$\varphi = \frac{\pi}{2} \rightarrow r = k$

$\varphi = \pi \rightarrow r = 0$

(1) $r(t) = k + k\cos\left(\frac{\pi}{2}t\right)$

(2) $\varphi(t) = \frac{\pi}{2}t$

$\dot{r}(t) = -k\frac{\pi}{2}\sin\left(\frac{\pi}{2}t\right)$

$\dot{\varphi}(t) = \frac{\pi}{2}$

$\ddot{r}(t) = -k\frac{\pi^2}{2^2}\cos\left(\frac{\pi}{2}t\right)$

$\ddot{\varphi}(t) = 0$

$$v_r = \dot{r} = -k\frac{\pi}{2}\sin\left(\frac{\pi}{2}t\right)$$

$$v_c = r\dot{\varphi} = \left(k + k\cos\left(\frac{\pi}{2}t\right)\right)\cdot\frac{\pi}{2}$$

$$v = \sqrt{v_r^2 + v_c^2} =$$

$$a_r = \ddot{r} - r\dot{\varphi}^2 = -k\frac{\pi^2}{4}\cos\left(\frac{\pi}{2}t\right) - \left(k + k\cos\left(\frac{\pi}{2}t\right)\right)\frac{\pi^2}{4}$$

$$a_c = r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = 2\cdot\left(-k\frac{\pi}{2}\sin\left(\frac{\pi}{2}t\right)\right)\frac{\pi}{2}$$

$$a = \sqrt{a_r^2 + a_c^2} =$$

$t_1 = 1$

$$v_r(1) = -k\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) = -k\frac{\pi}{2}$$

$$v_c(1) = \left(k + k\cos\frac{\pi}{2}\right)\frac{\pi}{2} = k\frac{\pi}{2}$$

$$v(1) = \sqrt{v_r^2(1) + v_c^2(1)} = \sqrt{\frac{k^2\pi^2}{4} + \frac{k^2\pi^2}{4}} = \frac{k\pi}{\sqrt{2}} = \frac{\sqrt{2}}{2}k\pi$$

$$a_r(1) = -k\frac{\pi^2}{4}\cos\frac{\pi}{2} - \left(k + k\cos\frac{\pi}{2}\right)\frac{\pi^2}{4} = -k\frac{\pi^2}{4}$$

$$a_c(1) = 2\left(-k\frac{\pi}{2}\sin\frac{\pi}{2}\right)\frac{\pi}{2} = -k\frac{\pi^2}{2}$$

$$a(1) = \sqrt{a_r^2(1) + a_c^2(1)} = \sqrt{\cdots}$$

$$R_k = \ldots$$

Zadatak 6

1. Kretanje tačke u ravni zadato je parametarskim jednačinama kretanja:

$$r(t) = t$$

$$\varphi(t) = 2t.$$

Odrediti trajektoriju kretanja tačke i nacrtati je. Odrediti položaj, brzinu i ubrzanje tačke kao i poluprečnik krivine trajektorije u početnom trenutku t_1=1.

1. Kretanje tačke u ravni zadato je parametarskim jednačinama kretanja:

$r(t) = t$

$\varphi(t) = 2t.$

Odrediti trajektoriju kretanja tačke i nacrtati je. Odrediti položaj, brzinu i ubrzanje tačke kao i poluprečnik krivine trajektorije u početnom trenutku $t_1 = 1$.

$r(t) = t \qquad \dot{r}(t) = 1 \qquad \ddot{r}(t) = 0$

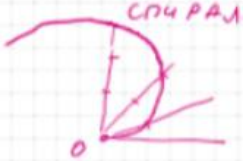
$\varphi(t) = 2t \qquad \dot{\varphi}(t) = 2 \qquad \ddot{\varphi}(t) = 0$

(1) $r = t$

(2) $\varphi = 2t$

$\left.\begin{matrix}\\ \\ \end{matrix}\right\} \xrightarrow[t]{\text{EL.}}$ (2) $\rightarrow t = \frac{\varphi}{2} \rightarrow$ (1) $\boxed{r = \frac{1}{2}\varphi}$

АРХИМЕДОВА СПИРАЛА



$v_r = \dot{r} = 1$

$v_c = r\dot{\varphi} = 2t$

$v(t) = \sqrt{1 + 4t^2}$

$a_r = \ddot{r} - r\dot{\varphi}^2 = -4t$

$a_c = r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = 4$

$a(t) = \sqrt{16t^2 + 16} = 4\sqrt{t^2+1}$

$t_1 = 1$

$v_r(1) = 1$

$v_c(1) = 2 \cdot 1 = 2$

$v(1) = \sqrt{5}$

$a_r(1) = -4$

$a_c(1) = 4$

$a(1) = 4\sqrt{2}$

$R_K(1) = \frac{v^2(1)}{a_N(1)} = ?$

$* \quad v(t) = \sqrt{1+4t^2} \rightarrow \dot{v}(t) = \frac{4t}{2\sqrt{1+4t^2}} \rightarrow \dot{v}(1) = \frac{4}{\sqrt{5}} \rightarrow$

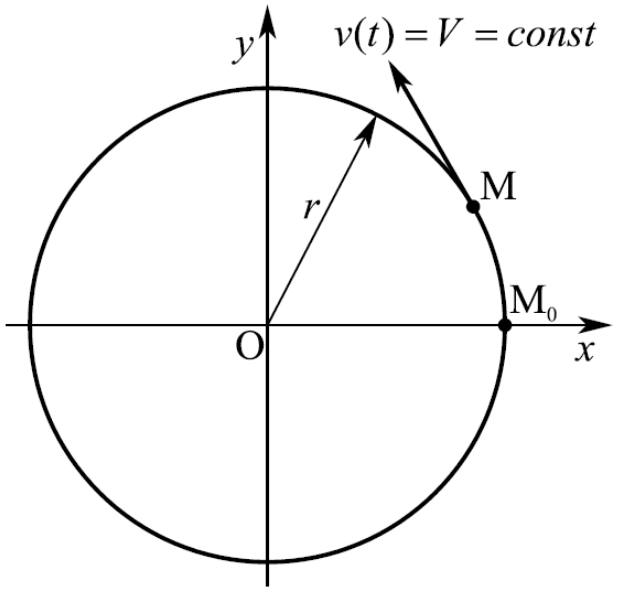
$a_T^2(1) = \dot{v}^2(1) = \left(\frac{4}{\sqrt{5}}\right)^2 = \frac{16}{5}$

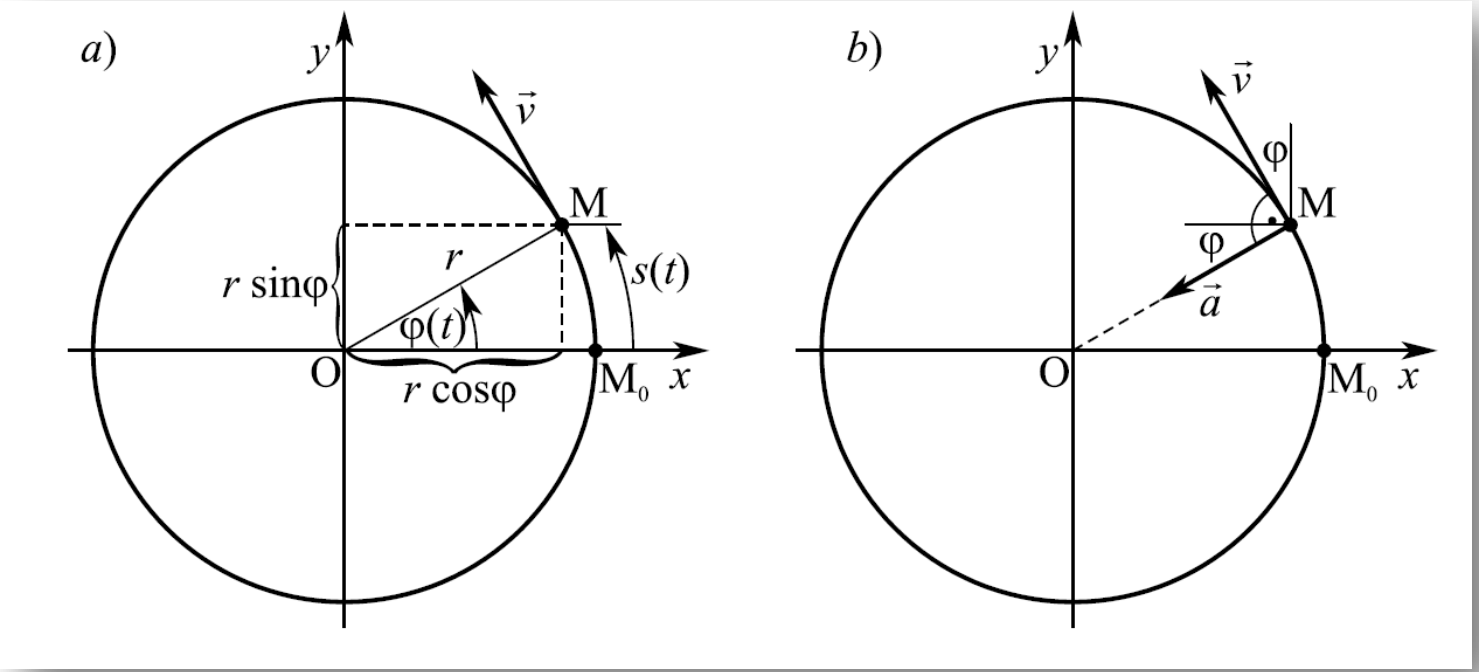
$a_N(1) = \sqrt{a^2(1) - a_T^2(1)} = \sqrt{(4\sqrt{2})^2 - \frac{16}{5}} = \sqrt{32 - \frac{16}{5}} = \sqrt{\frac{144}{5}} = \frac{12}{\sqrt{5}}$

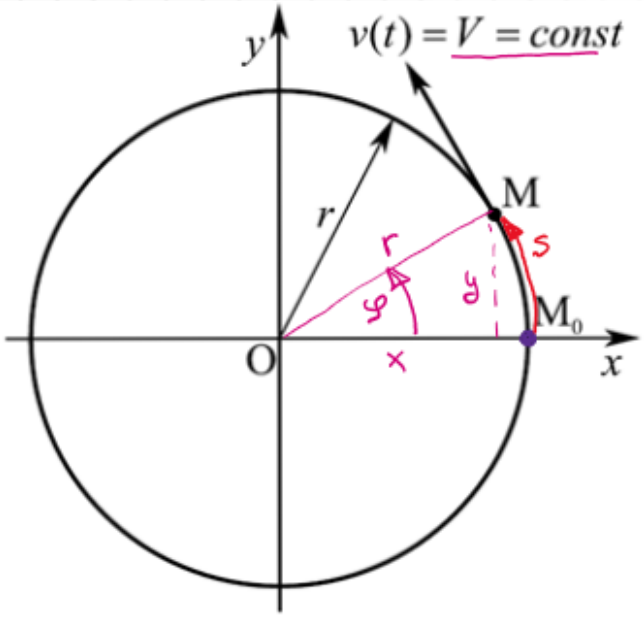
$R_K(1) = \frac{v^2(1)}{a_N(1)} = \frac{(\sqrt{5})^2}{\frac{12}{\sqrt{5}}} = \frac{5\sqrt{5}}{12}$

Zadatak 7

Zadatak 2.6 Tačka se kreće po kružnici, poluprečnika r, brzinom konstantnog intenziteta $v(t) = V = const$. Odrediti konačne jednačine kretanja tačke $x(t)$ i $y(t)$, ako je koordinatni početak Dekartovog koordinatnog sistema u centru kružnice, a tačka započinje kretanje iz položaja: $x(0) = r, y(0) = 0$. Odrediti i ubrzanje tačke pri ovom kretanju.




a)
y
$\vec{v}$
M
r
r sinφ
φ(t)
s(t)
O
r cosφ
M0
x
b)
y
$\vec{v}$
φ
M
φ
$\vec{a}$
O
M0
x



$x(t) = ?$ $y(t) = ?$

$x = r\cos\varphi$

$y = \sin\varphi$

$s = r\varphi \rightarrow \varphi = \frac{s}{r}$

$\dot{s} = v = V = const$

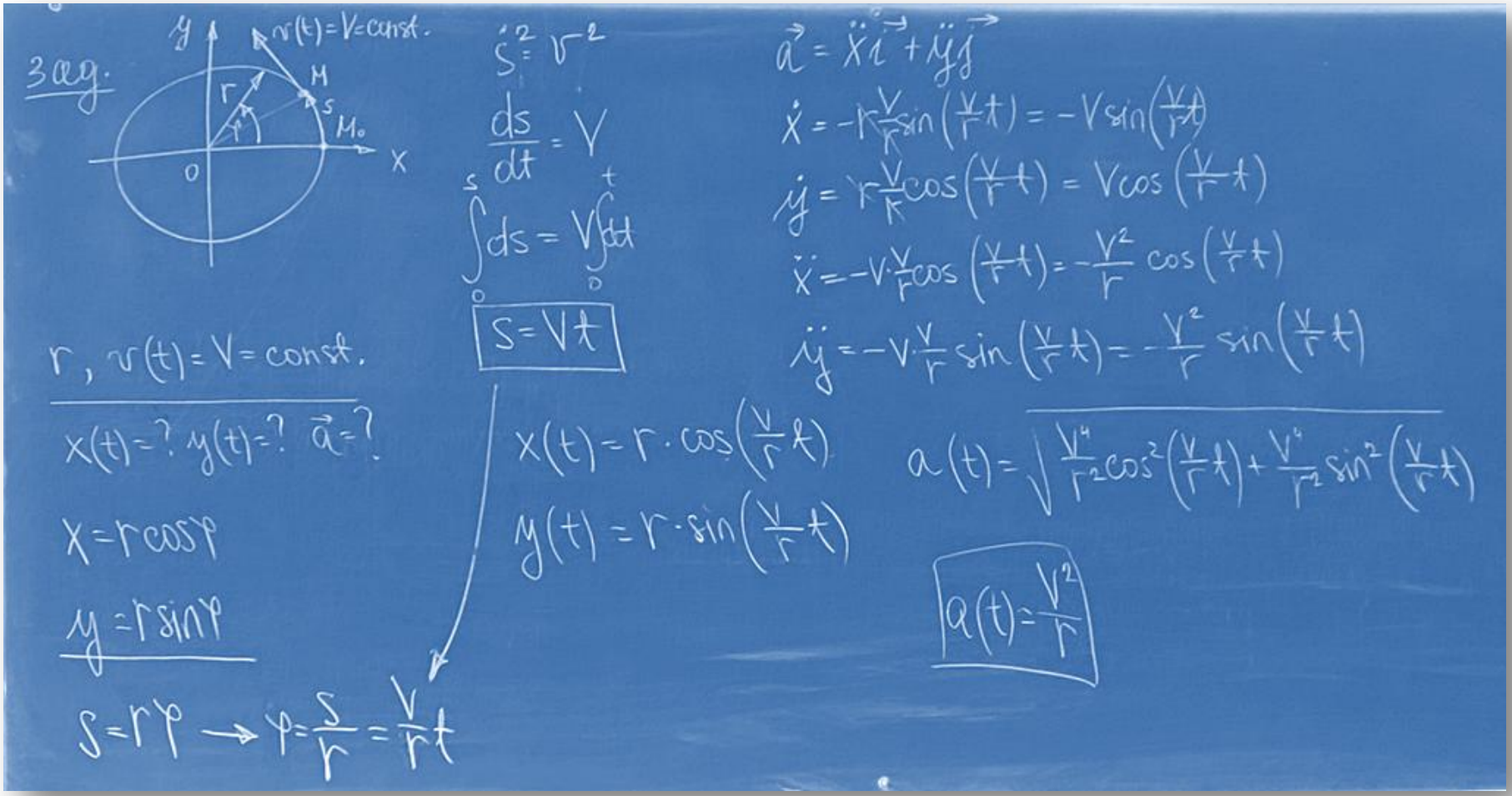
$\frac{ds}{dt} = V \rightarrow \int_{s(0)=0}^{s} ds = V\int_{0}^{t} dt$

$s\Big|_0^s = V\,t\Big|_0^t \rightarrow s - 0 = V(t - 0)$

$s = Vt \rightarrow \varphi = \frac{V}{r}t$

$x(t) = r\cos\left(\frac{V}{r}t\right)$

$y(t) = r\sin\left(\frac{V}{r}t\right)$

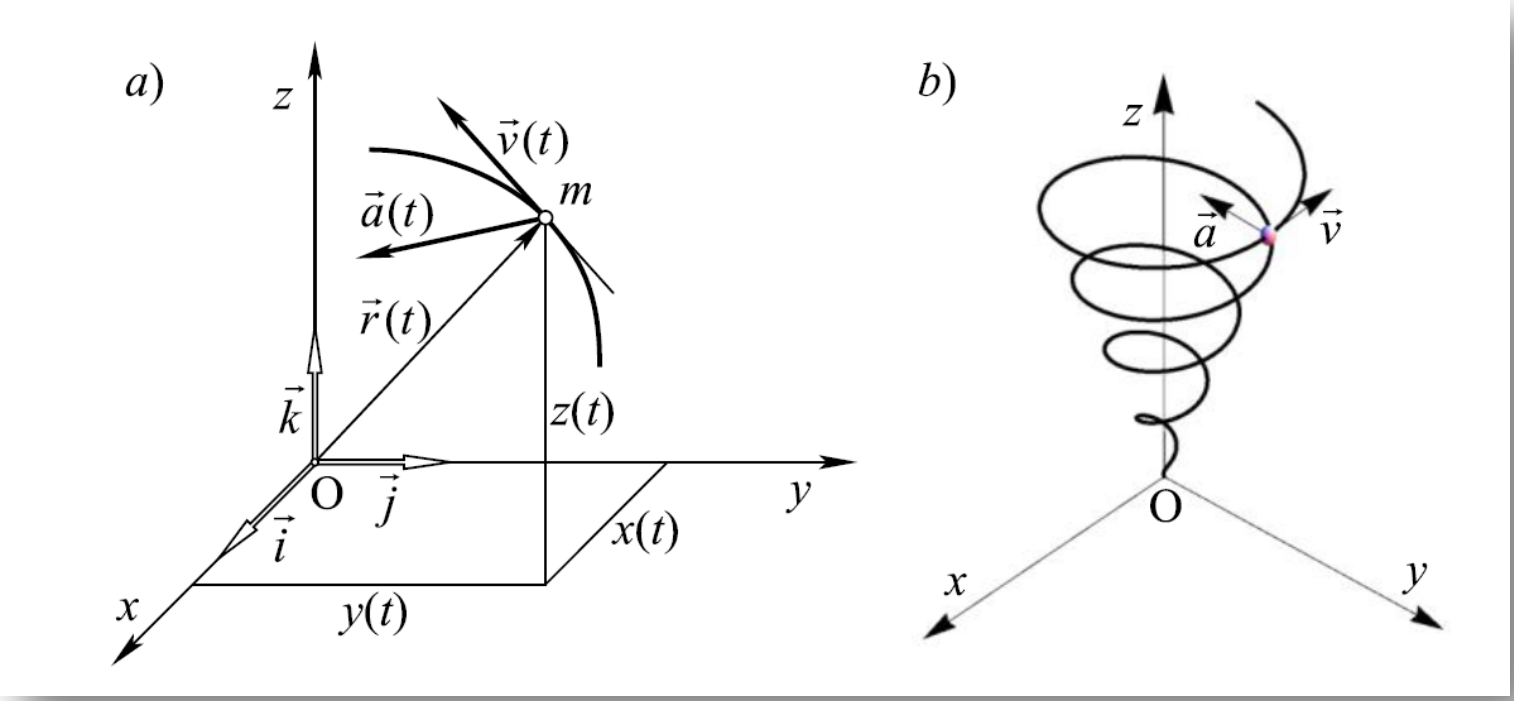

3 aeg.
y
v(t)=V=const.
M
s
M₀
r
φ
0
x
r, v(t)=V=const.
x(t)=? y(t)=? a=?
x=r cosφ
y=r sinφ
s=rφ → φ=s/r=V/r t
s²=v²
ds/dt=V
∫ds=V∫dt
s=Vt
x(t)=r·cos(V/r t)
y(t)=r·sin(V/r t)
a=ẍi+ÿj
ẋ=-r V/r sin(V/r t)=-V sin(V/r t)
ẏ=r V/r cos(V/r t)=V cos(V/r t)
ẍ=-V·V/r cos(V/r t)=-V²/r cos(V/r t)
ÿ=-V·V/r sin(V/r t)=-V²/r sin(V/r t)
a(t)=√(V⁴/r² cos²(V/r t)+V⁴/r² sin²(V/r t))
a(t)=V²/r

Zadatak 8

Zadatak 2.8 Kretanje tačke je opisano parametarskim jednačinama

$$x(t) = kt\cos t\,, y(t) = kt\sin t\,, z(t) = pt$$

Odrediti brzinu i ubrzanje tačke u proizvoljnom trenutku vremena t.


a)
z
$\vec{v}(t)$
m
$\vec{a}(t)$
$\vec{r}(t)$
$\vec{k}$
z(t)
O
$\vec{j}$
y
$\vec{i}$
x(t)
x
y(t)
b)
z
$\vec{a}$
$\vec{v}$
O
x
y

$$\dot{x}(t) = k\cos t - kt\sin t \qquad \ddot{x}(t) = -k\sin t - k\sin t - kt\cos t$$

$$\dot{y}(t) = k\sin t + kt\cos t \qquad \ddot{y}(t) = k\cos t + k\cos t - kt\sin t$$

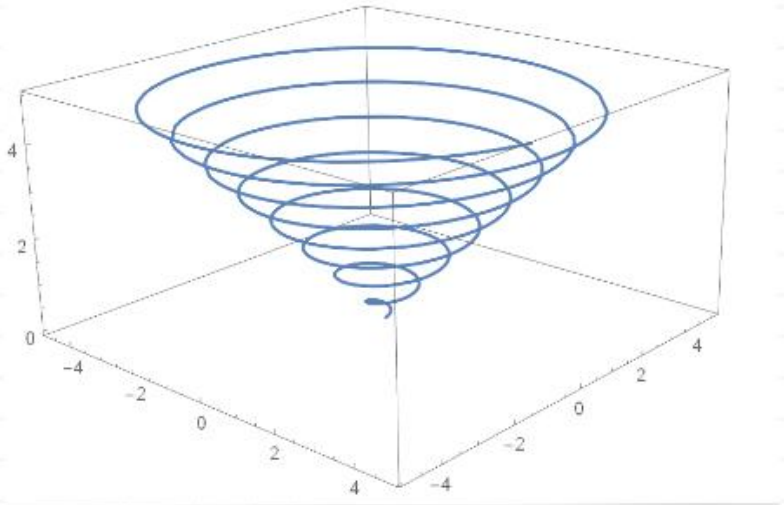
$$\dot{z}(t) = p \qquad \ddot{z}(t) = 0$$

$$x = kt\cos t \;/^2$$

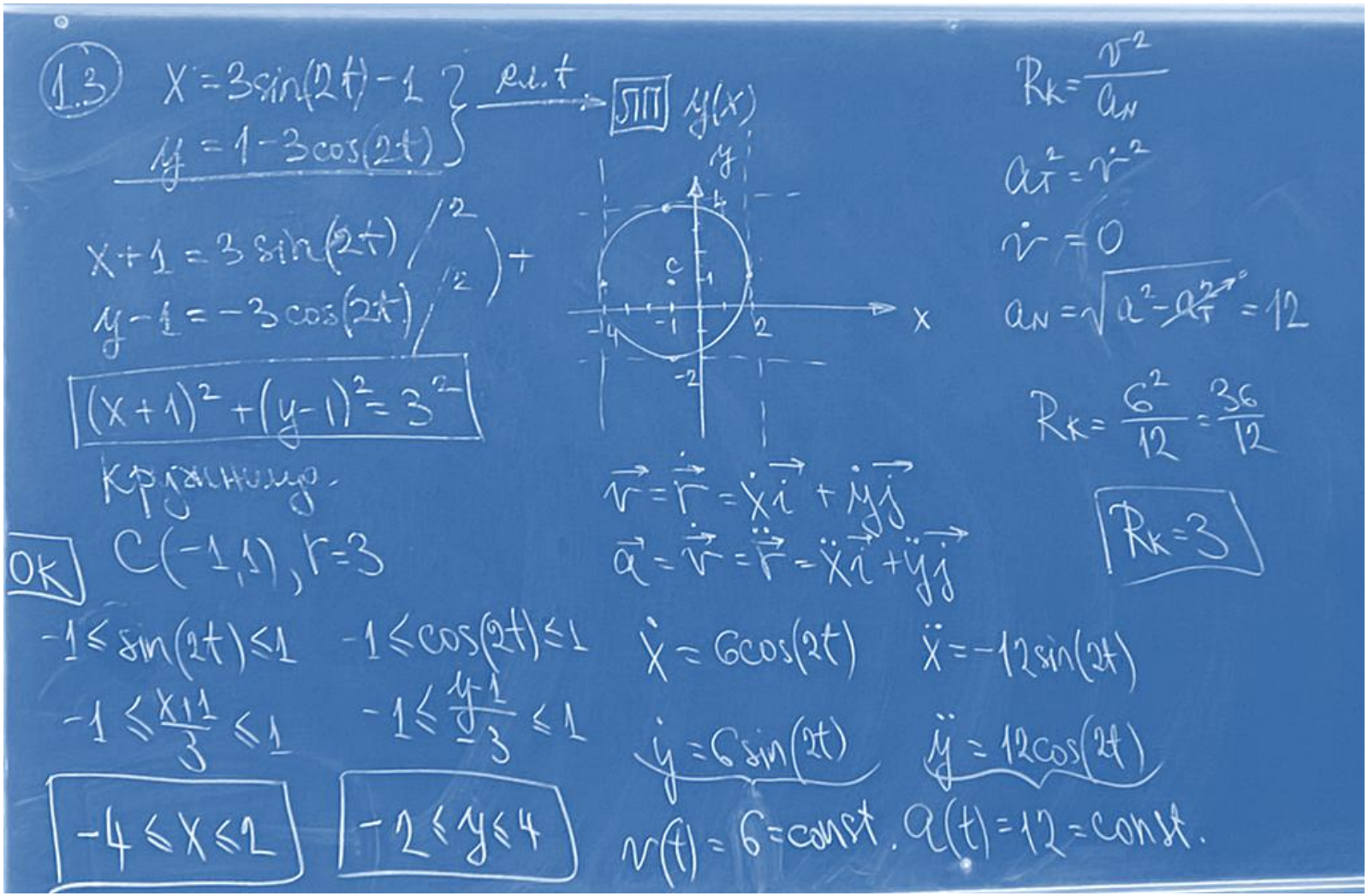
$$y = kt\sin t \;/^2$$

$$+ \qquad x^2 + y^2 = k^2t^2$$

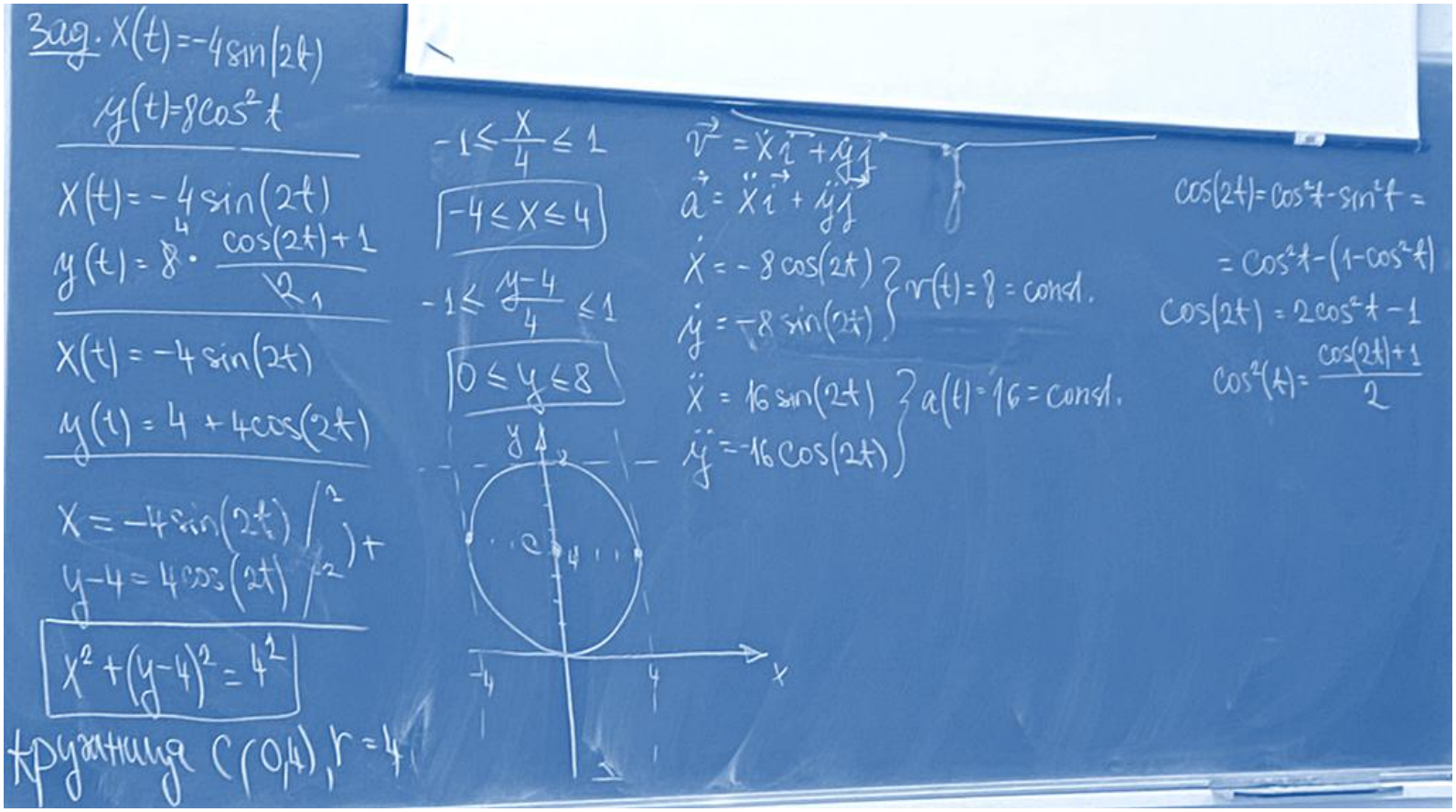
$$\sqrt{x^2+y^2} = kt$$



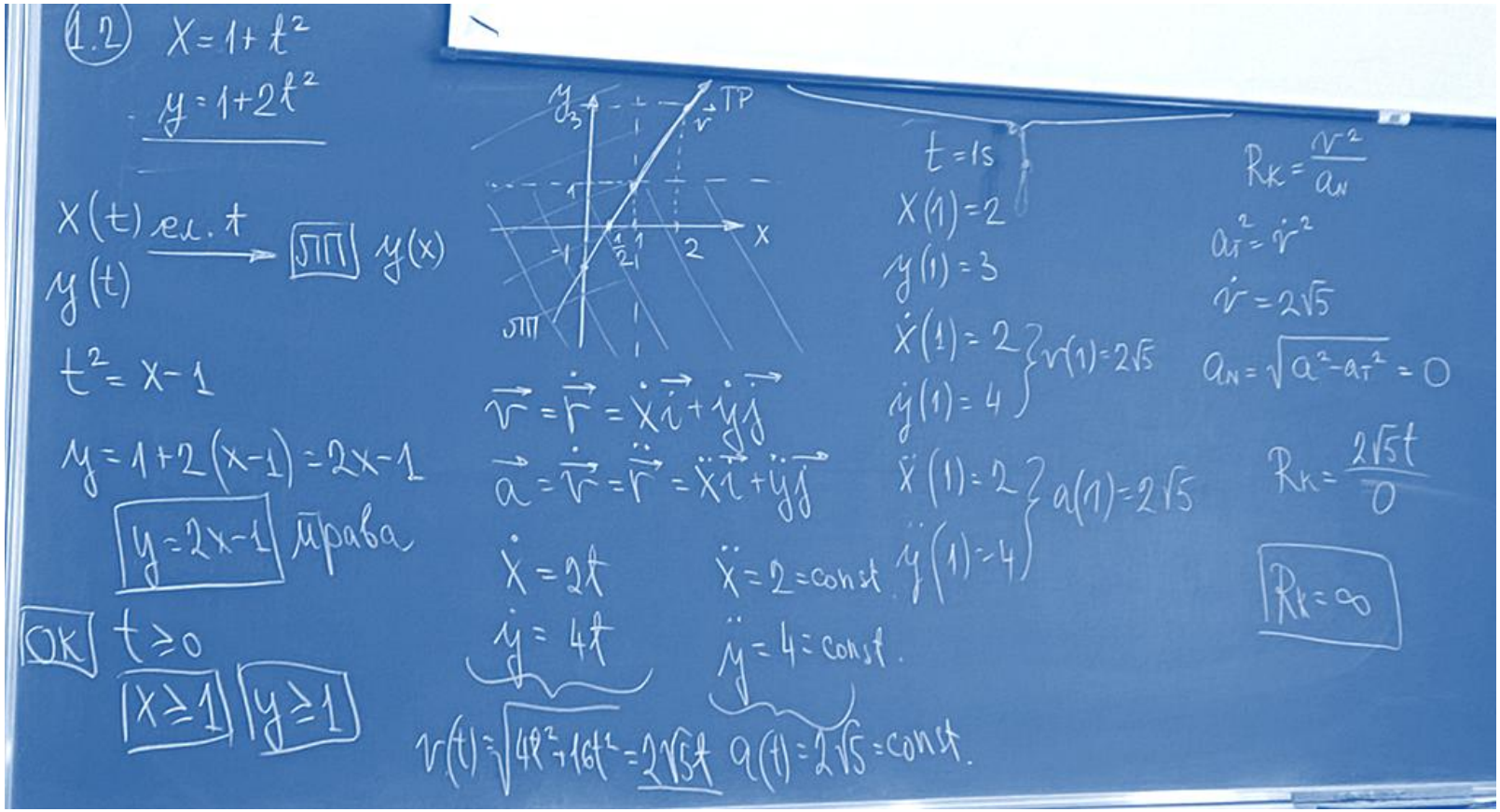
Zadatak 9


1.3
x = 3sin(2t) − 1
y = 1 − 3cos(2t)
el. t
JIII y(x)
(x+1)² + (y−1)² = 3²
kružnica.
C(−1, 1), r = 3
OK
−1 ≤ sin(2t) ≤ 1
−1 ≤ cos(2t) ≤ 1
−4 ≤ x ≤ 2
−2 ≤ y ≤ 4
ẋ = 6cos(2t)
ẍ = −12sin(2t)
ẏ = 6sin(2t)
ÿ = 12cos(2t)
v(t) = 6 = const.
a(t) = 12 = const.
Rk = v²/aN
aT = v̇
v̇ = 0
aN = √(a² − aT²) = 12
Rk = 6²/12 = 36/12
Rk = 3

Zadatak 10


Zad. $x(t) = -4\sin(2t)$
$y(t) = 8\cos^2 t$
$x(t) = -4\sin(2t)$
$y(t) = 8 \cdot \frac{\cos(2t)+1}{2}$
$x(t) = -4\sin(2t)$
$y(t) = 4 + 4\cos(2t)$
$x = -4\sin(2t) \mid^2$
$y-4 = 4\cos(2t) \mid^2$ +
$x^2 + (y-4)^2 = 4^2$
kružnica $C(0,4), r = 4$
$-1 \le \frac{x}{4} \le 1$
$-4 \le x \le 4$
$-1 \le \frac{y-4}{4} \le 1$
$0 \le y \le 8$
$\vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j}$
$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j}$
$\dot{x} = -8\cos(2t)$
$\dot{y} = -8\sin(2t)$
$v(t) = 8 = \text{const.}$
$\ddot{x} = 16\sin(2t)$
$\ddot{y} = -16\cos(2t)$
$a(t) = 16 = \text{const.}$
$\cos(2t) = \cos^2 t - \sin^2 t =$
$= \cos^2 t - (1 - \cos^2 t)$
$\cos(2t) = 2\cos^2 t - 1$
$\cos^2(t) = \frac{\cos(2t)+1}{2}$
y
x
C
4
-4
4

Zadatak 11


1.2
$x = 1 + t^2$
$y = 1 + 2t^2$
$x(t)$ el. t → ЈП $y(x)$
$y(t)$
$t^2 = x - 1$
$y = 1 + 2(x-1) = 2x - 1$
$y = 2x - 1$ права
OK $t \geq 0$
$x \geq 1$ $y \geq 1$
$\vec{v} = \vec{r} = \dot{x}\vec{i} + \dot{y}\vec{j}$
$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} = \ddot{x}\vec{i} + \ddot{y}\vec{j}$
$\dot{x} = 2t$
$\dot{y} = 4t$
$\ddot{x} = 2 = \text{const}$
$\ddot{y} = 4 = \text{const}$
$v(t) = \sqrt{4t^2 + 16t^2} = 2\sqrt{5}t$
$a(t) = 2\sqrt{5} = \text{const}$
ЈП
TP
$t = 1s$
$x(1) = 2$
$y(1) = 3$
$\dot{x}(1) = 2$
$\dot{y}(1) = 4$
$v(1) = 2\sqrt{5}$
$\ddot{x}(1) = 2$
$\ddot{y}(1) = 4$
$a(1) = 2\sqrt{5}$
$R_K = \frac{v^2}{a_N}$
$a_T^2 = \dot{v}^2$
$\dot{v} = 2\sqrt{5}$
$a_N = \sqrt{a^2 - a_T^2} = 0$
$R_K = \frac{2\sqrt{5}t}{0}$
$R_K = \infty$

Šta smo naučili?

7. Kinematika tačke - prirodni koordinatni sistem
8. Kretanje tačke po kružnici

Mehanika 2 (Kinematika)
Vežbe 3

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Novi Sad, 2023.